POLARIZATION

Class Activities: Polarization (1)

Whiteboard
Dipole field
Sketch the E field from an ideal dipole, given the formula (and from a real dipole)

Demo
Charged rods and water
(CU Demo # 5A40.40)
http://physicslearning.colorado.edu/website_new/Common/ViewDemonstration.asp?Topic=5&SubSubtopic=5A40.40&DemoCode=5A40.40
Demonstrate the deflection of water with a charged rod, indicating that it is polarizeable. Note the HW problem on the same topic.

Tutorial
Electric Fields in Matter
Paul van Kampen – Dublin University (Tutorials 9-16, page 14)
Electric Fields in Matter tutorial. Calculate polarization of hydrogen atom, potential near a dipole, expand for small r, put into spherical coordinates.

Whiteboard
Charged fluid
Imagine two fluids, red (positive) and blue (negative), each uniform, identical, in a rectangular shape (area A, height H). This fluid is made up from N "atoms"/m^3, and each "atom" (or unit) has available a charge q (which can separate/move). Imagine the red fluid moves UP the page, uniformly, a distance "d". For them to work out

1) How much charge Q appears on the top surface? (How much on the bottom? the sides?)
2) What is sigma on the top, in terms of the given variables (N, q, d, A, and/or H)
3) What is the polarization P in terms of those variables?
4) What is sigma on the top in terms of P?=
5) What if we displace the fluid that same distance "d", but at an angle theta with respect to the vertical. What are the answers above?

Purpose was for THEM to derive sigma(bound) = P dot nhat. Took about 10 minutes, 10/12 whiteboards we looked at afterwards had gotton through part 4 correctly, and a few had dealt with the theta story. Many had a hard time getting started, visualizing the story, deciding if "q" was all the info they needed or if there was some OTHER "charge" needed. Some were confused about whether or not "H" plays a role, (e.g. a couple thought the polarization P should be calculated by looking only at the top and bottom sheets, and the distance H between them)

Class Activities: Polarization (2)

Demo
Polarize coke can
Activities: I brought rods (+ and -), an empty coke can to attract, a small smooth plastic wine cork (which CAN be budged, although friction is high so it doesn't do much, but that's really part of the point) and I brought a small low friction pivot (from Mike Thomason, it's used to put a bar magnet on, but I just laid a 1 meter wooden stick on it, and was able to get enough torque to easily move it around by attraction through polarization) Also brought an electroscope to show sign of charges.

Demo
Faraday's Ice Pail
(CU Demo # 5B20.10)
Charge is transferred to a conductor. Glass jar, filled with conducting water, wrap in al foil and charge. Capacitor. Remove glass jar from Al foil and pour out water. Another one… the glass jar has an induced polarization and that's stored and it discharges into new one. Nice intro to polarization.

Griffiths by Inquiry (Lab 7): Dielectric materials

Griffiths by Inquiry (Lab 8): Dielectrics II

Tutorial
Polarization of hydrogen atom
Paul van Kampen – Dublin University (Tutorials 9-16, page 14)
In document "Tutorials 9-16"
Calculate polarization of hydrogen atom, potential near a dipole, expand for small r, put into spherical coordinates.

Are σ_b and ρ_b due to real charges?

A) Of course not! They are as fictitious as it gets! (Like in the 'method of images.')

B) Of course they are! They are as real as it gets! (Like σ and ρ in Chapter 2.)

C) I have no idea ☹

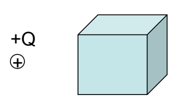
MD8-1

A stationary point charge +Q is near a block of polarization material (a linear dielectric). The net electrostatic force on the block due to the point charge is

A) attractive (to the left)

B) repulsive (to the right)

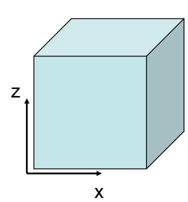
C) zero



4.1

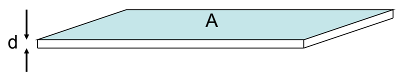
The cube below (side a) has uniform polarization $\mathbf{P}_0$
(which points in the z direction.)
What is the total dipole moment of this cube?

A) zero
B) $a^3 P_0$
C) P_0
D) P_0 / a^3
E) $2 P_0 a^2$



MD8-2

A slab of thickness d and area A has volume charge density $\rho = Q / V$ and surface charge density $\sigma = Q / A$.



The relation between ρ and σ is

A) $\sigma = \rho$

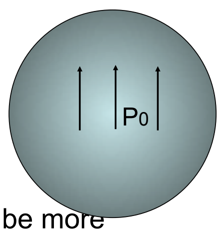
B) $\sigma = d\,\rho$ C) $d\,\sigma = \rho$

D) $\sigma = V\,\rho$ E) $V\,\sigma = \rho$

4.1 alt

The sphere below (radius a) has uniform polarization $\mathbf{P}_0$ (which points in the z direction.)

What is the total dipole moment of this sphere?



A) zero

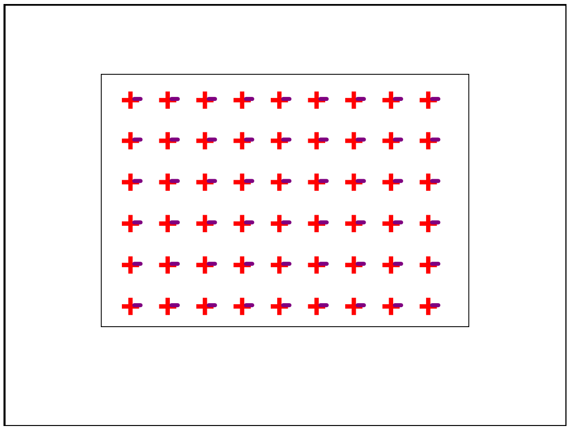
B) $P_0\, a^3$

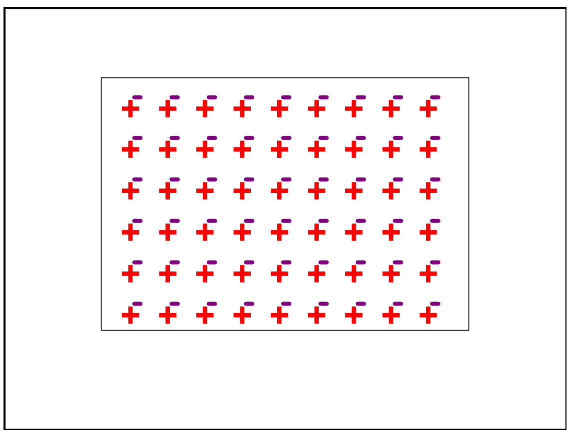
C) $4\pi a^3\, P_0/3$

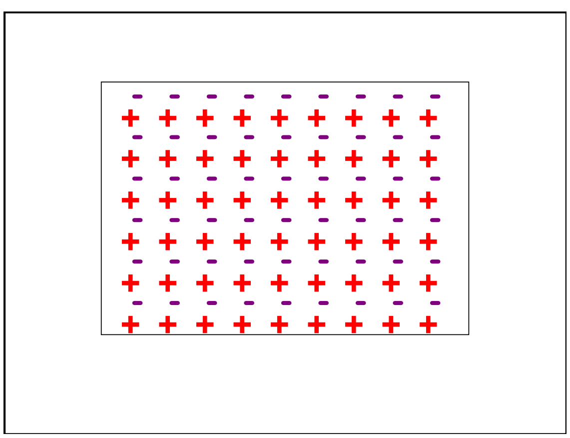
D) P_0

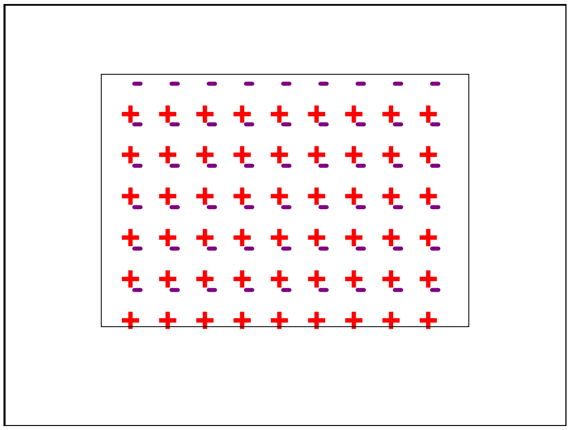
E) None of these/must be more complicated

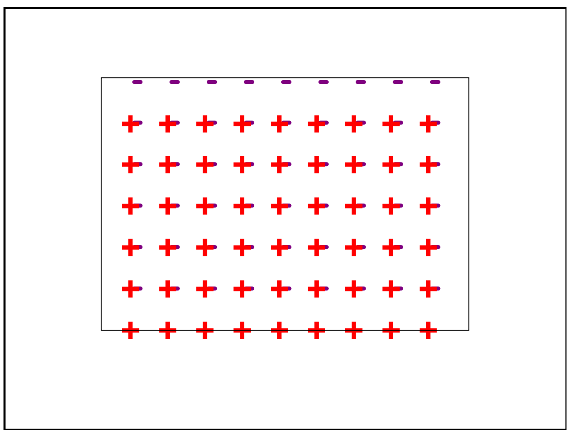
FIELD OF POLARIZED OBJECT

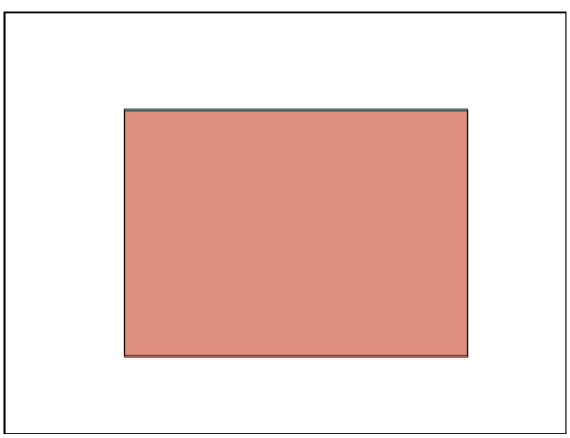


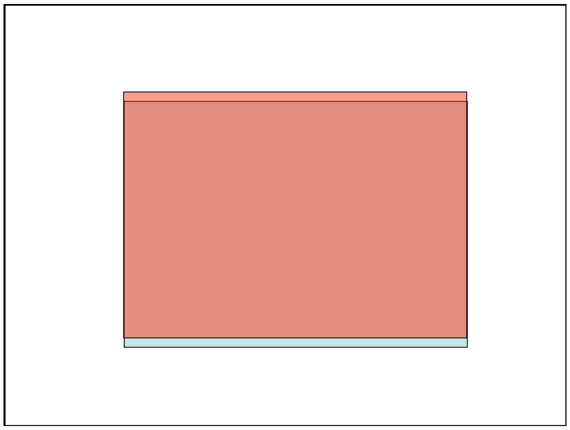


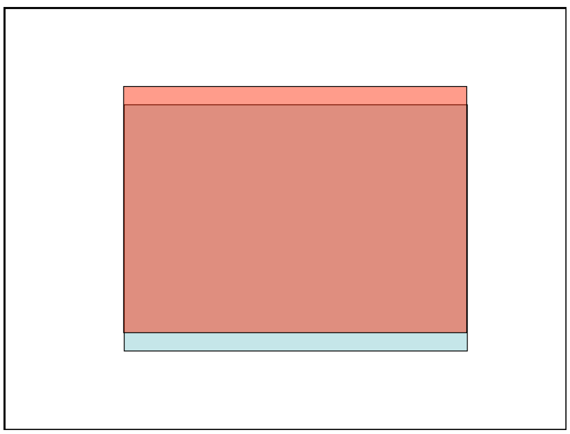


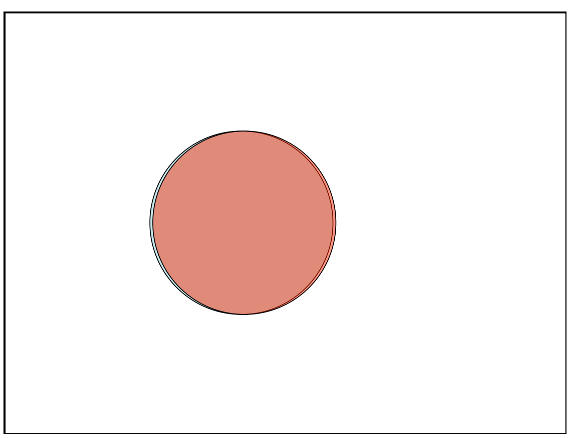


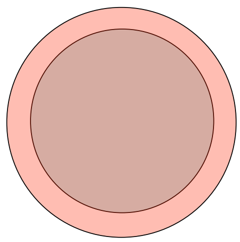






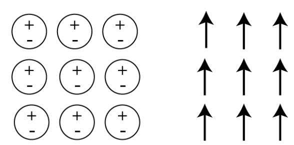






4.3

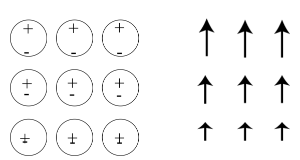
In the following case, is the bound surface and volume charge zero or nonzero?



A. $\sigma_b = 0,\ \rho_b \neq 0$
B. $\sigma_b \neq 0,\ \rho_b \neq 0$
C. $\sigma_b = 0,\ \rho_b = 0$
D. $\sigma_b \neq 0,\ \rho_b = 0$

4.3 b

In the following case, is the bound surface and volume charge zero or nonzero?

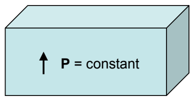


A. $\sigma_b = 0,\ \rho_b \neq 0$
B. $\sigma_b \neq 0,\ \rho_b \neq 0$
C. $\sigma_b = 0,\ \rho_b = 0$
D. $\sigma_b \neq 0,\ \rho_b = 0$

MD8-3

A linear dielectric in the shape of a rectangular block has a uniform polarization **P** (due to an external E-field) parallel to an edge, as shown. How many of the sides of the block have a non-zero surface charge density?

A) 1 B) 2 C) 4 D) 6 E) 0

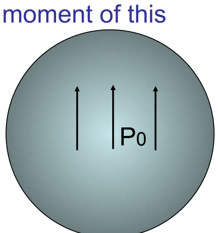


4.1 alt

The sphere below (radius a) has uniform polarization $\mathbf{P}_0$ (which points in the z direction.)

What is the total dipole moment of this sphere?

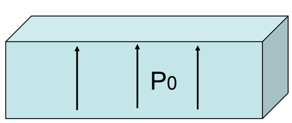
A) zero
B) $P_0\, a^3$
C) $4\pi a^3\, P_0/3$
D) P_0
E) None of these/must be more complicated



4.4

A dielectric slab (top area A, height h) has been polarized, with $\mathbf{P}=P_0$ (in the +z direction)

What is the surface charge density, σ_b, on the bottom surface?



A) 0
B) $-P_0$
C) P_0
D) $P_0\, A\, h$
E) $P_0\, A$

In your own words, define what we mean by "free charge", and "bound charge"

FORCE AND ENERGY

4.11 We argued that C goes UP by a factor of ε_r if you fill a capacitor with dielectric. What happens to the stored energy of a capacitor if it's filled with a dielectric?

A) It goes up

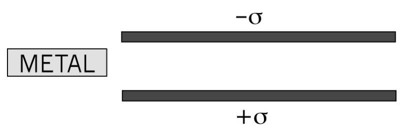
B) It goes down

C) It is unchanged

D)The answer depends on what else is "held fixed" (V? Q?)

4.12

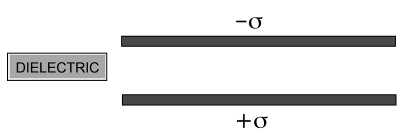
If we push this conductor inside the *isolated* capacitor, will it be drawn into the capacitor or repelled?



A. It gets sucked into the capacitor
B. It gets pushed out from the capacitor
C. I just don't know.

4.12 b

If we push this conductor inside the *isolated* capacitor, will it be drawn into the capacitor or repelled?



A. It gets sucked into the capacitor
B. It gets pushed out from the capacitor
C. I just don't know.

ELECTRIC DISPLACEMENT

4.5

We introduced "Electric Displacement" or "D" field: **D** $=\varepsilon_0$**E** + **P.**
If you put a dielectric in an external field E_{ext}, it polarizes, adding a new field, $E_{induced}$ (from the bound charges).
These superpose, making a total field E_{tot}.
Which of these three E fields is the "E" in the formula for D above?

A) E_{ext} B) $E_{induced}$ C) E_{tot}

ERK-3.1

Expand the following expression out to order $1/r^2$ for the case that r >> a and r>>b

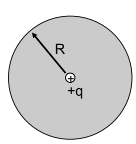
$$\frac{\sqrt{r^2+a^2}}{\sqrt{2r^2+b^2}}$$

4.7c-alt

We define **D** $=\varepsilon_0$**E** + **P**, with

$$\oint \vec{D}\cdot d\vec{a} = Q_{\text{free, enclosed}}$$

A point charge +q is placed at the center of a dielectric sphere (radius R). There are no other free charges anywhere. What is |D(r)|?



A) $q/(4\pi r^2)$ everywhere
B) $q/(4\pi\varepsilon_0 r^2)$ everywhere
C) $q/(4\pi r^2)$ for r<R, but $q/(4\pi\varepsilon_0 r^2)$ for r>R
D) None of the above, it's more complicated
E) We need more info to answer!

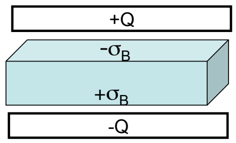
4.6

An vary large (effectively infinite) capacitor has charge Q. A neutral linear dielectric is inserted into the gap (and of course, it will polarize) . We want to find **D** *everywhere*

i) $\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$

ii) $\oint \vec{D} \cdot d\vec{a} = Q_{free}$

iii) $\oint \vec{E} \cdot d\vec{a} = Q / \varepsilon_0$



Which equation would *you* use first?
A) i B) ii C) iii
D) More than one of these would work OK.

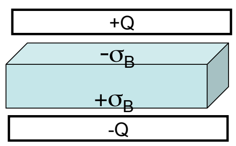
4.6 b

An ideal (large) capacitor has charge Q. A neutral dielectric is inserted into the gap (and of course, it will polarize) We want to find **E** *everywhere*

(i) $\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}$

(ii) $\oiint \vec{\mathbf{D}} \cdot \mathbf{d}\vec{\mathbf{A}} = Q_{free}$

(iii) $\oiint \vec{\mathbf{E}} \cdot \mathbf{d}\vec{\mathbf{A}} = Q/\varepsilon_0$

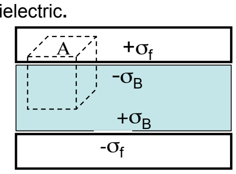


Which equation would *you* go to first?
A) i B) ii C) iii
D) Your call: *more* than 1 of these would work!
E) *Can't* solve, unless know the dielectric is linear!

MD8-4

An ideal (large) capacitor has charge Q. A neutral linear dielectric is inserted into the gap. We want to find **D** in the dielectric.

$\oint \vec{D} \cdot d\vec{a} = Q_{free}$



For the Gaussian pillbox shown, what is Qfree(enclosed)?

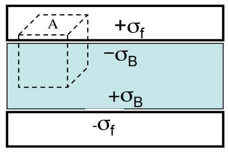
A) $\sigma_f A$ B) $-\sigma_B A$ C) $\sigma_f A - \sigma_B A$
D) $\sigma_f A + \sigma_B A$ E) Something else

What is |**D**| in the dielectric?
A) σ_f B) $2\sigma_f$ C) $\sigma_f / 2$
D) $\sigma_f + \sigma_b$ E) Something else

MD8-4

An ideal (large) capacitor has charge Q.
A neutral linear dielectric is inserted into the gap.
We want to find **D** in the dielectric.

$$\oint\!\!\oint \vec{D} \cdot d\vec{a} = Q_{free}$$



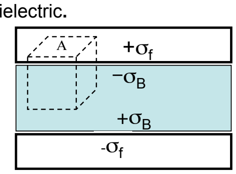
For the Gaussian pillbox shown, what is $Q_{free,enclosed}$?

A) $\sigma_f A$ B) $-\sigma_B A$ C) $(\sigma_f - \sigma_B)A$
D) $(\sigma_f + \sigma_B)A$ E) Something else

MD8-4

An ideal (large) capacitor has charge Q.
A neutral linear dielectric is inserted into the gap.
We want to find **D** in the dielectric.

$$\oint\!\!\oint \vec{D} \cdot d\vec{a} = Q_{free}$$



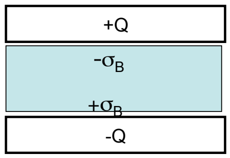
What is |**D**| in the dielectric?

A) σ_f B) $2\sigma_f$ C) $\sigma_f / 2$
D) $\sigma_f + \sigma_b$ E) Something else

-- WEEK 9 --
Electric displacement D

4.6d

An ideal (large) capacitor has charge Q.
A neutral linear dielectric is inserted into the gap.
Now that we have **D** in the dielectric,
what is **E** inside the dielectric ?

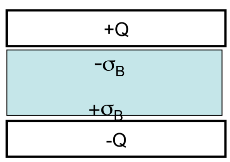


A) **E** = **D** $\varepsilon_0 \varepsilon_r$
B) **E** = **D**/$\varepsilon_0 \varepsilon_r$
C) **E** = **D** ε_0
D) **E** = **D**/ε_0
E) Not so simple! Need another method

4.6e

An ideal (large) capacitor has charge Q.
A neutral *linear* dielectric is inserted into the gap (with given dielectric constant)
Now that we have **D** in the dielectric,

what is **E** in that *small gap above* the dielectric ?

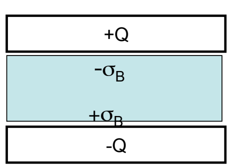


A) **E** = **D** ε
B) **E** = **D**/ε
C) **E** = **D** ε_0
D) **E** = **D**/ε_0
E) Not so simple! Need another method

4.6f

An ideal (large) capacitor has charge Q.
A neutral linear dielectric is inserted into the gap
(with given dielectric constant)

Where is E **dis**continuous?



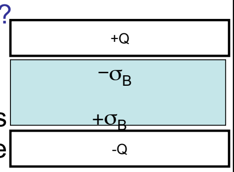
i) near the free charges on the plates
ii) near the bound charges on the dielectric surface

A) i only B) ii only
C) both i and ii (but nowhere else)
D) both i and ii but also other places
E) none of these/other/???

4.6g An ideal (large) capacitor has charge Q. A neutral *linear* dielectric is inserted into the gap (with given dielectric constant)

Where is D discontinuous?

i) near the free charges on the plates
ii) near the bound charges on the dielectric surface

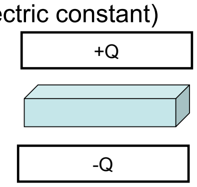


A) i only B) ii only
C) both i and ii (but nowhere else)
D) both i and ii but also other places
E) none of these/other/???

4.6g An ideal (large) capacitor has charge Q. A neutral *linear* dielectric is inserted into the gap (with given dielectric constant)

Where is D discontinuous?

i) near the free charges on the plates
ii) near the bound charges on the dielectric surface



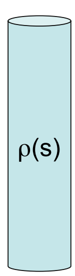
A) i only B) ii only
C) both i and ii (but nowhere else)
D) both i and ii but also other places
E) none of these/other/???

4.8

An infinitely long, solid non-conducting dielectric rod has been injected ("doped") with a fixed, known charge distribution $\rho(s)$. (The material responds, polarizing internally)

When computing D in the rod, do you treat this $\rho(s)$ as the "free charges" or "bound charges" or neither?

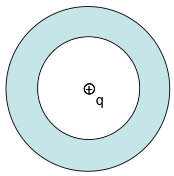
A) "free charge"
B) "bound charge"
C) Neither of these - $\rho(s)$ is some combination of free and bound
D) Something else.



MD8-5

A point charge +q is placed at the center of a neutral, linear, dielectric shell. Can D be computed from its divergence?

$$\oint \vec{D}\cdot d\vec{a} = Q_{free}$$

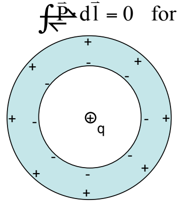


A) Yes
B) No
C) Depends on the inner radius of the dielectric.

MD8-6

A point charge +q is placed at the center of a neutral, linear, dielectric shell. The shell polarizes due to the point charge.
Is the curl of the polarization **P** zero everywhere?

$$\oint \vec{P}\cdot d\vec{l} = 0 \quad \text{for every possible loop?}$$

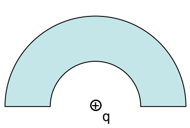


A) Yes
B) No
C) Depends on the inner radius of the dielectric.

MD8-7

A point charge +q is placed at the center of a neutral, linear, dielectric **hemispherical** shell. Can D be computed from its divergence?

$$\oint \vec{D}\cdot d\vec{a} = Q_{free}$$



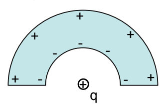
A) Yes
B) No
C) Depends on the inner radius of the dielectric.

MD8-8

A point charge +q is placed at the center of a neutral, linear, dielectric shell. The shell polarizes due to the point charge.

Is the curl of the polarization **P** zero everywhere?

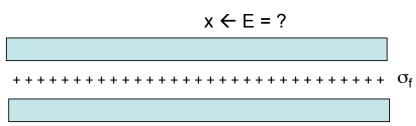
$\oint \vec{P} \cdot d\vec{l} = 0$ for every possible loop?



A) Yes
B) No
C) Depends on the inner radius of the dielectric.

MD8-9

An infinite plane of charge with surface charge density σ_f is between two infinite slabs of neutral linear dielectric (of dielectric constant ε), as shown. The "bare" E-field, due only to the plane of free charge, has magnitude $E_o = \sigma_f / 2\varepsilon_0$



What is the magnitude of the E-field in the space above the top dielectric at the point x ?

A) $E = E_0$ B) $E > E_0$ C) $E < E_0$

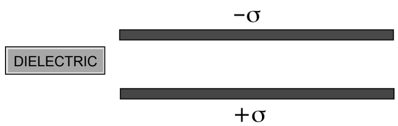
4.11

We argued that C goes UP by a factor of ε_r if you fill a capacitor with dielectric. What happens to the stored energy of a capacitor if it's filled with a dielectric?

A) It goes up
B) It goes down
C) It is unchanged
D) The answer depends on what else is "held fixed" (V? Q?)

4.12 b

If we push this dielectric inside the *isolated* capacitor, will it be drawn into the capacitor or repelled?



A. It gets sucked into the capacitor
B. It gets pushed out from the capacitor
C. I just don't know.

BOUNDARY VALUE PROBLEMS WITH DIELECTRICS

Class Activities

Whiteboards**
Snell's Law for Dielectrics
(a decent one!) I drew an E arrow approaching a boundary (angle theta1 with normal) and an E arrow leaving the boundary (angle theta2) epsilon is given (and different in both regions, both are linear dielectrics). There are no free charges in the region shown. Find tan(theta1)/tan(theta2). Gave them ~10 minutes for this, about half finished. (Followup question - does the E vector point more "towards the normal" in the lower, or higher dielectric region? Is this like Snell's law?)

4.10
a

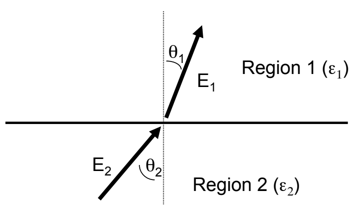
You have a straight boundary between two linear dielectric materials (ε_r has one value above, another below, the boundary) There are no free charges in the regions considered.

What is continuous across the boundary?

i) E(parallel) ii) E(perpendicular)
iii) D(parallel) iv) D(perpendicular)

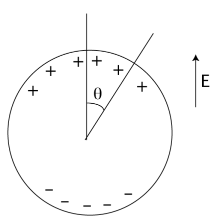
A) i and iii B) ii and iv
C) i and ii D) iii and iv
E) Some other combination!

Two different dielectrics meet at a boundary.
The E field in each region near the boundary is shown.
There are no free charges in the region shown.
What can we conclude about $\tan(\theta_1)/\tan(\theta_2)$?



4.2
a

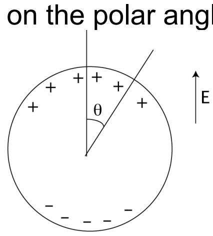
You put a conducting sphere in a uniform E-field. How do you expect the surface charge to depend on the polar angle (θ)?



a) Constant
b) $\cos(\theta)$
c) $\sin(\theta)$
d) Nothing simple, it will have to be an infinite series of sin's and cos's with coefficients.

4.2 b

Now what if the sphere is a dielectric? How do you expect the bound surface charge to depend on the polar angle (θ)?



a) Constant
b) $\cos(\theta)$
c) $\sin(\theta)$
d) Nothing simple, it will have to be an infinite series of sin's and cos's with coefficients.

Boundary Value Exercise

- A multi-step question:
- For a dielectric sphere in an electric field E0
- 1 – draw sketch, including E and charge sigma-bound
- 2 – write down boundary conditions
- 3 – What equation will you use to solve E inside (ie, what solution to Laplace)
- 4 – Tell me the steps you will go through without doing calculation
- 5 – How would you expect E inside to depend on E0

4.10 b

You have a boundary between two linear dielectric materials (ε_r has one value above, another below, the boundary) There are no free charges in the regions considered. Which formula will the voltage satisfy at the boundary?

A) $V\big|_{out} - V\big|_{in} = 0$

B) $V\big|_{out} - V\big|_{in} = \frac{-\sigma_{tot}}{\varepsilon_0}$

C) $\varepsilon_{out} V\big|_{out} - \varepsilon_{in} V\big|_{in} = 0$

D) $\varepsilon_{out} V\big|_{out} - \varepsilon_{in} V\big|_{in} = -\frac{\sigma_{tot}}{\varepsilon_0}$

E) None of these, or MORE than one...

4.10 You have a boundary between two linear dielectric materials (ε_r has one value above, another below, the boundary) There are no free charges in the regions considered. Which formula will the voltage satisfy at the boundary?

A) $\left.\frac{\partial V}{\partial n}\right|_{out} - \left.\frac{\partial V}{\partial n}\right|_{in} = \frac{-\sigma_{free}}{\varepsilon_0}$ B) $\left.\frac{\partial V}{\partial n}\right|_{out} - \left.\frac{\partial V}{\partial n}\right|_{in} = \frac{-\sigma_{tot}}{\varepsilon_0}$

C) $\varepsilon_{out}\left.\frac{\partial V}{\partial n}\right|_{out} - \varepsilon_{in}\left.\frac{\partial V}{\partial n}\right|_{in} = -\sigma_{free} = 0$ D) $\varepsilon_{out}\left.\frac{\partial V}{\partial n}\right|_{out} - \varepsilon_{in}\left.\frac{\partial V}{\partial n}\right|_{in} = -\sigma_{bound}$

E) None of these, or MORE than one...