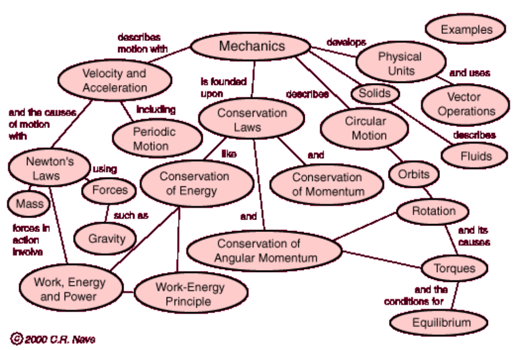
LORENTZ FORCE



Chapter 3 and 4 Concept Map

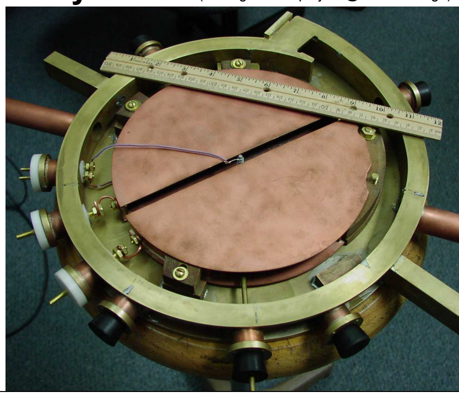
Make a concept map showing the relationship between the following quantities and concepts:

Electric Field **E**
Voltage V
Displacement **D**
Polarization **P**
Charge Density **ρ**
Permittivity ε_0
Bound Charge Density $\boldsymbol{\rho}_b$

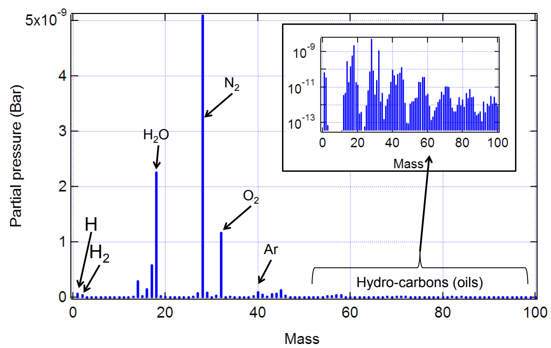
Laplace's equation
Multipole expansion
Uniqueness theorem
Method of Images
Dipole moment

If you have time you can include:
Linear Dielectric
Poisson's Equation
Susceptibility χ_e
Relative Permittivity ε

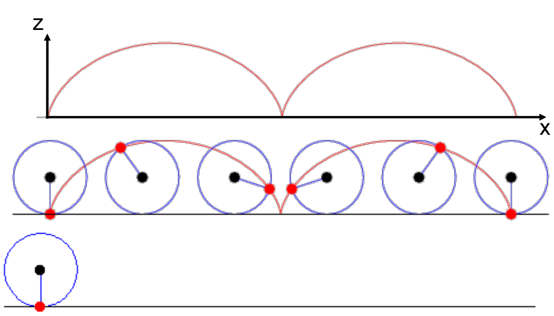
Cyclotron (undergraduate project @ Knox College)



Mass spectrometer

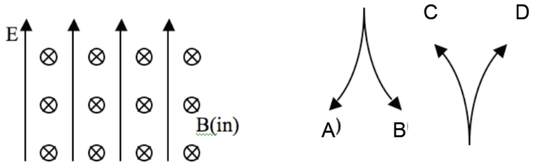


Cycloid



Source: Woframe Research, Inc.

A proton (q=+e) is released from rest in a uniform **E** and uniform **B**. **E** points up, **B** points into the page. Which of the paths will the proton initially follow?



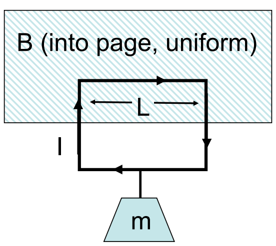
E. It will remain stationary

(To think about: what happens after longer times?)

5.4

A wire loop in a B field has a current I.
The mass is "levitated" by the magnetic force F_{mag}=ILB. If you increase the current , does the magnetic force do positive work on the mass?

A) Yes
B) No



CURRENTS & CHARGE CONTINUITY

Class Activities: current (1)

Visual Explanation
Current Density and the Mississippi
I motivated J by thinking about the "flow of the Mississippi" compared to "flow of Boulder Creek", and characterizing flow as total current (Mississippi clearly vastly bigger) but what about "water flowing at me through this circle I am making with my fingers". Then perhaps Boulder Creek even wins - so there's some OTHER quantity to characterize flow, which motivates our definition of "current density" as current/area. (Then rotate the circle to show them that it's really perpendicular area needed to DEFINE this current density).

Discussion
Where is the current in a wire?
Also, nice discussion built on student question, would current in a wire be only on the edges (since we've learned excess charge goes there). Lots of things to talk about - wires needn't have excess charge, visualize the "negative electron fluid" displacing in an E field, and so on…

Class Activities: Current (2)

Discussion
Chunks of current
Also good class discussion about "is it meaningful to talk about a tiny chunk of current" (like we talk about a tiny chunk of charge) Issues of current conservation, but also vector nature came up (it bugged some that dI has to stretch over SOME length, but dq is truly a point)

Kinesthetic
Steady current activity.
Oregon State University (not on website)
Tell students: each of you is a charge, make the imaginary B field meter fluctuate. They have to move around the room. Now tell them to keep moving but move in such a way that B field meter doesn't fluctuate. They have to go around her. Do they have to be in a circle, all same distance or not? When is the idealization of current density appropriate (as opposed to discrete charges)? They're moving but all one behind others, steady current. Then how do you measure current? Linear current density? Something per unit length. Linear mass density is mass/length, and charge density is charge per unit length, so they assume linear current density is current per unit length... but it's not! This is an a-hah moment. For a surface current they'll ask "how do you measure a surface current" as they're walking around. How much current is here? They keep walking while they're thinking, how would we measure this. Then walk in surface density, shoulder to shoulder. You count how many go through per second through gate. Talk about the dimension of gate. Linear current = 0D gate. Surface current = 1D gate. You can use a meter stick for a 1D gate, but don't put it perpendicular to the current. Talk about how total current is a flux). Only component of velocity perpendicular to gate matters. Linear current density is lnear charge density times velocity. That makes language make more sense, dimensionally.

A student argues that the current through a wire flows throughout its volume, you:

A) Agree, resistance is inversely proportional to cross sectional area, not circumference
B) Disagree, it must flow only on the surface of the wire because the negative charges repel each other
C) Agree for different reasons
D) Disagree for different reasons

5.5
Positive ions flow right through a liquid, negative ions flow left.
The spatial density and speed of both ions types are identical.
Is there a net current through the liquid?

A) Yes, to the right
B) Yes, to the left
C) No
D) Not enough information given

5.7
Current I flows down a wire (length L) with a square cross section (side *a)*
If it is uniformly distributed over the entire wire area, what is the magnitude of the volume current density *J*?

A) $J = I/a^2$ B) $J = I/a$

C) $K = J/(4a)$ D) $J = I/(a^2L)$

E) None of the above

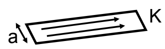
5.6
Current I flows down a wire (length L) with a square cross section (side *a)*
If it is uniformly distributed over the outer surfaces only, what is the magnitude of the surface current density *K*?

A) $K = I/a^2$ B) $K = I/a$

C) $K = I/(4a)$ D) $K = I/(a^2L)$

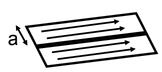
E) None of the above

5.8 A "ribbon" (width a) of surface current flows (with surface current density K)
Right next to it is a second identical ribbon of current.
Viewed collectively, what is the new total surface current density?



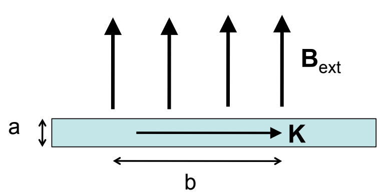
5.8 A "ribbon" (width a) of surface current flows (with surface current density K)
Right next to it is a second identical ribbon of current.
Viewed collectively, what is the new total surface current density?

A) K
B) 2K
C) K/2
D) Something else



ERK5-1 A "ribbon" (width a) with uniform surface current density K passes through a uniform magnetic field $\mathbf{B}_{ext}$. Only the length b along the ribbon is in the field. What is the magnitude of the force on the ribbon?

A) KB
B) aKB
C) abKB
D) bKB/a
E) KB/(ab)



5.10 Which of the following is a statement of charge conservation?

A) $\frac{\partial \rho}{\partial t} = -\int \vec{\mathbf{J}} \bullet d\vec{\mathbf{l}}$

B) $\frac{\partial \rho}{\partial t} = -\iint \vec{\mathbf{J}} \bullet d\vec{\mathbf{A}}$

C) $\frac{\partial \rho}{\partial t} = -\iiint (\nabla \bullet \vec{\mathbf{J}})\, d\tau$

D) $\frac{\partial \rho}{\partial t} = -\nabla \bullet \vec{\mathbf{J}}$

E) Not sure/can't remember

Until now, you have been told that magnetic fields loop around a current carrying wire. But how do you know that there are no other components? Show (mathematically) which of the below B-field components are/not possible.

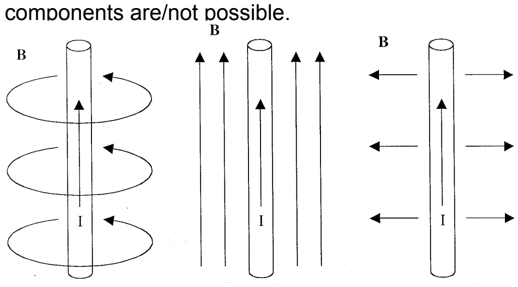


Figure a. Figure b. Figure c.

Is the total net charge in the universe conserved? How about the total mass?

A) Charge is conserved; total mass is conserved
B) Charge is conserved; total mass is not conserved
C) Charge is not conserved; total mass is conserved
D) Charge is not conserved; total mass is not conserved
E) Dude! How should I know?

Discussion

- Why does B follow the right hand rule? Is it contained in Ampere's Law?
- When you find the B field for a point in space near a long current carrying wire, what *could* B depend on? Given the form of Biot-Savart law, what would you GUESS?

BIOT SAVART LAW

Class Activities: Biot Savart

Discussion
Biot-Savart
Had them "think like an 18th century physicist" to *come up* with Biot-Savart.

Demonstration
Compass and dip angle
Brought a dip-compass needle to see the dramatic dip angle in the room (and brief discussion of geo-magnetic field).

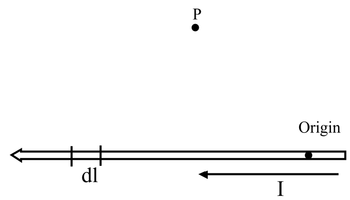
Tutorials
Magnetic Field due to a Spinning Ring of Charge" activity
Oregon State University
Working in small groups students are asked to consider a ring with charge Q, and radius R rotating about its axis with period T and create an integral expression for the magnetic field caused by this ring everywhere in space. Students also develop the power series expansion for the potential near the center or far from the ring.

Visualization
Stokes' Theorem
http://www.math.umn.edu/~nykamp/m2374/readings/stokesidea/

5.11 To find the magnetic field B at P due to a current-carrying wire we use the Biot-Savart law,

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} I \int \frac{d\vec{l} \times \hat{\mathfrak{R}}}{\mathfrak{R}^2}$$

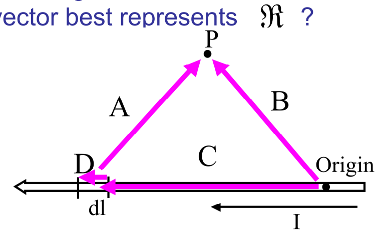
In the figure, with "dl" shown, what is $\vec{\mathfrak{R}}$?



5.11 To find the magnetic field B at P due to a current-carrying wire we use the Biot-Savart law,

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} I \int \frac{d\vec{l} \times \hat{\mathfrak{R}}}{\mathfrak{R}^2}$$

In the figure, with "dl" shown, which purple vector best represents $\mathfrak{R}$?

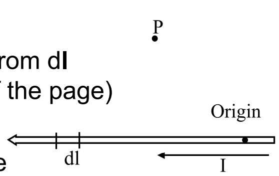


5.12 To find the magnetic field B at P due to a current-carrying wire we use the Biot-Savart law,

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} I \int \frac{d\vec{l} \times \hat{\mathfrak{R}}}{\mathfrak{R}^2}$$

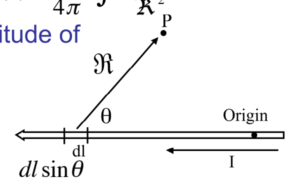
What is the *direction* of the infinitesimal contribution d**B**(P) created by current in d**l**?

A) Up the page
B) Directly away from d**l** (in the plane of the page)
C) Into the page
D) Out of the page
E) Some other direction



5.13 To find the magnetic field B due to a current-carrying wire, below, we use the Biot-Savart law, $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} I \int \frac{d\vec{l} \times \hat{\mathfrak{R}}}{\mathfrak{R}^2}$

What is the magnitude of $\frac{d\vec{l} \times \hat{\mathfrak{R}}}{\mathfrak{R}^2}$?



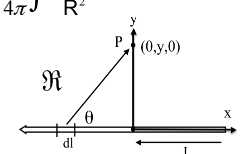
a) $\frac{dl \sin\theta}{\mathfrak{R}^2}$ b) $\frac{dl \sin\theta}{\mathfrak{R}^3}$

c) $\frac{dl \cos\theta}{\mathfrak{R}^2}$ d) $\frac{dl \cos\theta}{\mathfrak{R}^3}$ e) $\frac{dl}{\mathfrak{R}^2}$

(And, what's $\mathfrak{R}$ here?)

5.13m To find the magnetic field B due to a current-carrying wire, below, we use the Biot-Savart law, $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{I} dl \times \hat{R}}{R^2}$

What is the value of $\frac{\vec{I} dl \times \hat{R}}{R^2}$?



a) $\frac{Iydx'\hat{z}}{[(x')^2 + y^2]^{3/2}}$ b) $\frac{Ix'dx'\hat{z}}{[(x')^2 + y^2]^{3/2}}$

c) $\frac{-Ix'dx'\hat{z}}{[(x')^2 + y^2]^{3/2}}$ d) $\frac{-Iydx'\hat{z}}{[(x')^2 + y^2]^{3/2}}$ e) $\frac{-Idx'(y\hat{x} - x'\hat{y})}{[(x')^2 + y^2]^{3/2}}$

5.14

What is B at the point shown?

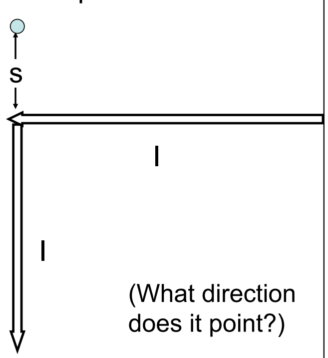
A) $\frac{\mu_0}{\pi s} I$

B) $\frac{\mu_0}{2\pi s} I$

C) $\frac{\mu_0}{4\pi s} I$

D) $\frac{\mu_0}{8\pi s} I$

E) None of these

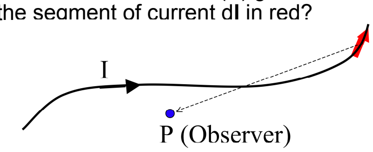


(What direction does it point?)

5.16 com

What do you expect for direction of **B**(P)?

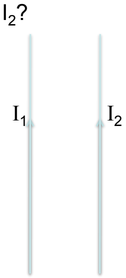
How about direction of d**B**(P) generated JUST by the segment of current d**l** in red?



A) **B**(p) in plane of page, ditto for d**B**(P, by red)
B) **B**(p) into page, d**B**(P, by red) into page
C) **B**(p) into page, d**B**(P, by red) out of page
D) **B**(p) complicated - has mult component (*not* ⊥ or || to page), ditto for d**B**(P, by red)
E) Something else!!

I have two very long, parallel wires each carrying a current I_1 and I_2, respectively. In which direction is the force on the wire with the current I_2?

A) Up
B) Down
C) Right
D) Left
E) Into or out of the page



(How would your answer change if you would reverse the direction of the currents?)

5.15

To find the magnetic field B due to a current-carrying loop, we use the Biot-Savart law

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} I \int \frac{d\vec{l} \times \hat{\mathfrak{R}}}{\mathfrak{R}^2}$$

What is the magnitude of $\frac{d\vec{l} \times \hat{\mathfrak{R}}}{\mathfrak{R}^2}$?

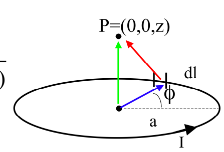
A) $\frac{dl \sin\phi}{z^2}$

B) $\frac{dl}{z^2}$

C) $\frac{dl \sin\phi}{(z^2 + a^2)}$

D) $\frac{dl}{(z^2 + a^2)}$

E) Something quite different!



(Which colored arrow is $\mathfrak{R}$? **r**? **r**'?)

5.15 b

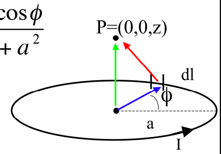
To find the magnetic field B due to a current-carrying loop, we use the Biot-Savart law, $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} I \int \frac{d\vec{l} \times \hat{\mathfrak{R}}}{\mathfrak{R}^2}$

What is the d**B**$_z$ (the contribution to the vertical component of **B** from this dl segment?)

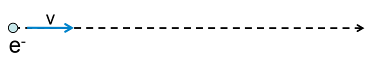
A) $\frac{dl}{z^2+a^2}\frac{a}{\sqrt{z^2+a^2}}$ B) $\frac{dl}{z^2+a^2}$

C) $\frac{dl}{z^2+a^2}\frac{z}{\sqrt{z^2+a^2}}$ D) $\frac{dl\cos\phi}{z^2+a^2}$

E) Something quite different!



An electron is moving in a straight line with constant speed v. What approach would you choose to calculate the B-field generated by this electron?



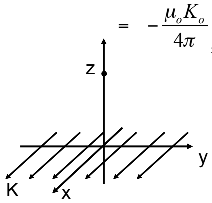
A) Biot-Savart
B) Ampere's law
C) Either of the above.
D) Neither of the above.

MD10-1

We have derived the integral expression for the B-field a distance z from a current sheet in the z = 0 plane:

$$\vec{B}(0,0,z) = \frac{\mu_o}{4\pi}\iint_{xy} da \frac{\vec{K}\times\vec{\Re}}{\Re^{3/2}}$$

$$= -\frac{\mu_o K_o}{4\pi}\iint_{z=0\ \text{plane}} dx'\, dy' \frac{z\,\hat{y} + y\,\hat{z}}{(x'^2+y'^2+z^2)^{3/2}}$$



The B-field has
A) y-component only
B) z-component only
C) y and z-components
D) x, y, and z-components

DIVERGENCE & CURL OF B; STOKES THEOREM

Class Activities: Stokes' Thm, Div, Curl. Ampere. (1)

Visualization
Stokes' Theorem
http://www.math.umn.edu/~nykamp/m2374/readings/stokesidea/

Demonstration
Loop and arrow
Also brought a small "loop with arrow" which turned out to be a useful prop throughout class. One thing I did near the end was hold the compass near the loop, and pointed out B is NOT zero, and B dot dl is NOT zero, at various points around the loop... so, what if we integrate? Got the class to discuss that it must be zero ("backside of loop" cancelling with front) and that this was correct, since there's no current in the room...

Demonstration
Ampere's Law loop
I had a prop (a strip of paper, white on one side, yellow on the other) which I could twist to show the concept test idea about "direction/sign of current through a loop"

Tutorial
Current-carrying wire
Paul van Kampen – Dublin University (Tutorials 9-16, page 21)
Use Biot-Savart and Ampere's Law near current carrying wire. Calculate force. Then do force on square loop.

Tutorials
Ampere's Law activity
Oregon State University
Students working in small groups practice using Ampere's Law to determine the electric field due to several current distributions. Students practice using the symmetry arguments necessary to use Ampere's Law.
Informal interviews have shown that students will mimic the words "by symmetry" without really knowing what they mean. Also, they tend not to realize that the magnetic field can vary in both magnitude and direction-independently. This is a really valuable place to slowdown the pace and get students thinking about the geometry of what is going on.

Class Activities: Stokes' Thm, Div, Curl, Ampere (2)

Whiteboards
Boundary conditions on B
One persistent difficulty that students have is an inability to recreate the mathematical steps to determine the boundary conditions on the parallel and perpendicular components of B. After watching 3 continuous semesters of this course, I strongly recommend having a whiteboard or worksheet activity where students are asked to derive those boundary conditions given a surface current.

Whiteboard
Griffiths "B" Triangle
Had them write out the triangle, took ~5 minutes. (There's still one "leg" they haven't gotten, some figured it out on the fly, it's a homework problem due Wed)

Computer Visualizations
B fields, circulation, flux
Java applets allowing you to see 3D magnetic fields, and do surface and line integrals to determine ciruculation and flux.
http://www.falstad.com/vector3dm/

Computer Animation
Cycloid
I Googled "cycloid" and pull up Mathematica's webpage, it has a very nice animation of the cycloid.

Context rich problems
http://groups.physics.umn.edu/physed/Research/CRP/on-lineArchive/crmff.html

Tutorial
Magnetic Field Continuity across a Boundary
Oregon State University
Students working in small groups use Maxwell's equations to determine the continuity of the magnetic field across a charged surface

Amperian loop analysis:

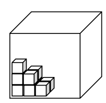
Consider the infinite uniform current sheet **K** flowing in the x direction.

I. Which variables (x, y, z) can B depend on?
II. Which vector components (B_x, B_y, B_z) can be non-zero?
Give your reasoning for each variable and component.

III. What loop would you use to find **B**? Why?

MD11-3

A large cube of volume V is made up of many smaller cubes. Each smaller cube (labeled with index i) has volume v_i, so that



$$V = \sum_i v_i$$

The flux of the vector field **F** over small cube i is $\oint_i \vec{E} \cdot d\vec{a}$

If we add up all the fluxes over all the little cubes will this equal the flux over the big cube ?

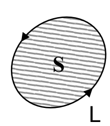
$$\sum_i \left(\oint_i \vec{E} \cdot d\vec{a} \right) \overset{??}{=} \oint_V \vec{E} \cdot d\vec{a}$$

A) Yes B) No
C) Depends on the vector field F

MD11-1

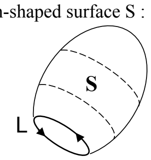
Stoke's Theorem says that for a surface S bounded by a perimeter L, any vector field **B** obeys

$$\int_S \nabla \times \vec{B} \cdot d\vec{a} = \oint_{L(S)} \vec{B} \cdot d\vec{l}$$

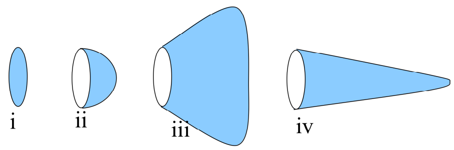


Does Stoke's Theorem apply for ***any*** surface S bounded by a perimeter L, even one such as this balloon-shaped surface S :

A) Yes
B) No
C) Sometimes

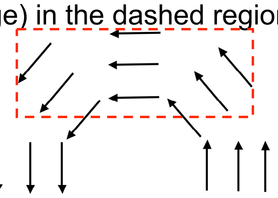


5.16 Rank order $\left|\iint \vec{\mathbf{J}} \bullet d\vec{\mathbf{A}}\right|$ (over blue surfaces) where **J** is uniform, going left to right:



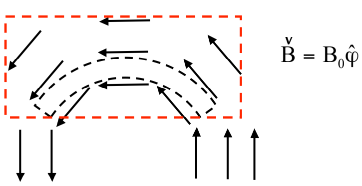
A) iii > iv > ii > i
B) iii > i > ii > iv
C) i > ii > iii > iv
D) Something else!!
E) Not enough info given!!

5.17 a If the arrows represent a B field (note that |B| is the same everywhere), is there a nonzero **J** (perpendicular to the page) in the dashed region?



A. Yes
B. No
C. Need more information to decide

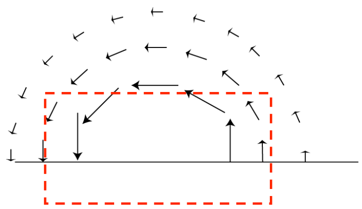
5.17 b If the arrows represent a B field (note that |B| is the same everywhere), is there a nonzero **J** (perpendicular to the page) in the dashed region?



A. Yes
B. No
C. Need more information to decide

5.17
c

If the arrows represent a B field, is there a **J** (perpendicular to the page) in the dashed region?

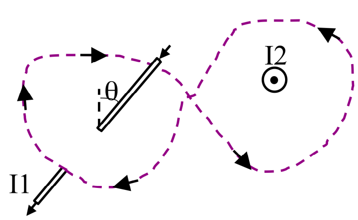


A. Yes
B. No
C. Need more information to decide

AMPERE'S LAW

5.22

What is $\oint \vec{\mathbf{B}} \bullet d\vec{\mathbf{l}}$ around this purple (dashed) Amperian loop?



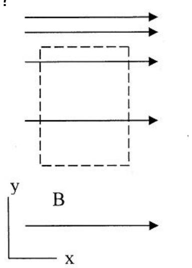
A) $\mu_0\,(|I_2| + |I_1|)$
B) $\mu_0\,(|I_2| - |I_1|)$
C) $\mu_0\,(|I_2| + |I_1|\sin\theta)$
D) $\mu_0\,(|I_2| - |I_1|\sin\theta)$

5.19 The magnetic field in a certain region is given by

$$\vec{B}(x,y) = C y \hat{x}$$

(C is a positive constant) Consider the imaginary loop shown. What can you say about the electric current passing through the loop?

A. must be zero
B. must be nonzero
C. Not enough info

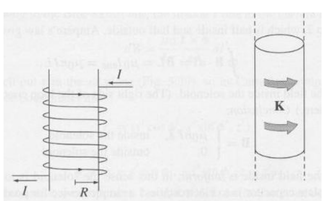


Maxwell's Equations

Differential form		Integral form
$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$	⟷	$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0}$
$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	⟷	$\oint \vec{E} \cdot d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{a}$
$\nabla \cdot \vec{B} = 0$	⟷	$\oint \vec{B} \cdot d\vec{a} = 0$
$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$	⟷	$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc} + \mu_0 \varepsilon_0 \int \frac{\partial \vec{E}}{\partial t}$

Terms in boxes are zero in the static case: dB/dt = 0, dE/dt = 0

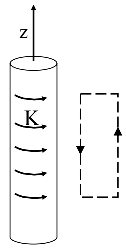
5.20
A solenoid has a total of N windings over a distance of L meters. We "idealize" by treating this as a surface current running around the surface.
What is K?



A) I B) NI C) I/L D) I N/L
E) Something else...

MD11-3

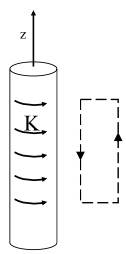
An infinite solenoid with surface current density K is oriented along the z-axis. Apply Ampere's Law to the rectangular imaginary loop in the yz plane shown. What does this tell you about Bz, the z-component of the **B**-field outside the solenoid?



A) Bz is constant outside
B) Bz is zero outside
C) Bz is not constant outside
D) It tells you nothing about Bz

MD11-3

An infinite solenoid with surface current density K is oriented along the z-axis. Apply Ampere's Law to the rectangular imaginary loop in the yz plane shown.
We can safely assume that B(s→∞)=0.
What does this tell you about the **B**-field outside the solenoid?



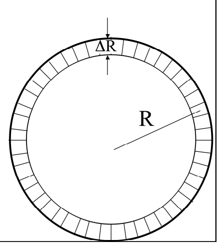
A) |B| is a non-zero constant outside
B) |B| is zero outside
C) |B| is not constant outside
D) We still don't know anything about |B|

5.21
a

A thin toroid has (average) radius R and a total of N windings with current I.
We "idealize" this as a surface current running around the surface.
What is K, approximately?

A) I/R B) I/(2 π R)
C) NI/R D) NI/(2 π R)

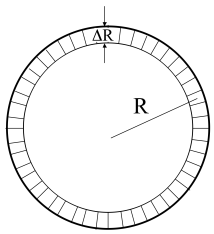
E) Something else



5.21b

What direction do you expect the B field to point?

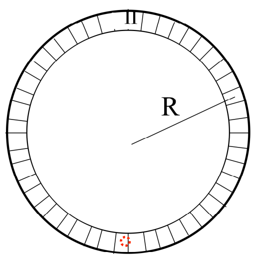
A) Azimuthally
B) Radially
C) In the z direction (perp. to page)
D) Loops around the rim
E) Mix of the above...



5.21 c

What Amperian loop would you draw to find B "inside" the Torus (region II)

A) Large "azimuthal" loop
B) Small loop in region II
C) Smallish loop from region II to outside (where B=0)
D) Like A, but perp to page
E) Something entirely different



5.21 d

Two long coaxial solenoids each carry current I but in opposite directions.

The inner solenoid (radius a) has n1 turns per unit length, and the outer one (radius b) has n2.

Find B (i) inside the solenoid, (ii) between them, and (iii) outside both.

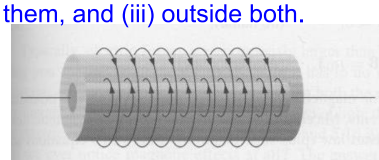
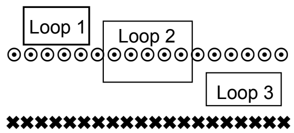


Figure 5.42

5.23



In the case of the infinite solenoid we used loop 1 to argue that the **B**-field outside is zero. Then we used loop 2 to find the **B**-field inside. What would loop 3 show?

a) The **B**-field inside is zero
b) It does not tell us anything about the **B**-field
c) Something else

BOUNDARY CONDITIONS

MAGNETIC VECTOR POTENTIAL

Class Activities: Vector Potential

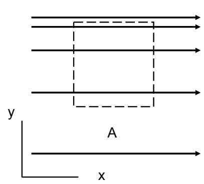
Writing
What is A?
Started with a writing exercise, basically "what is the A field, how is it used" (see my powerpoints for the wording) Gave ~3 minutes for that.

Tutorial
Magnetic Vector Potential due to a Spinning Charged Ring" activity
Oregon State University
Working in small groups students are asked to consider a ring with charge Q, and radius R rotating about its axis with period T and create an integral expression for the vector potential caused by this ring everywhere in space. Students also develop the power series expansion for the potential near the center or far from the ring.

5.19 The vector potential in a certain region is given by

$$\vec{A}(x, y) = C\,y\,\hat{x}$$

(C is a positive constant) Consider the imaginary loop shown. What can you say about the magnetic field in this region?



A. B is zero
B. B is non-zero, parallel to z-axis
C. B is non-zero, parallel to y-axis
D. B is non-zero, parallel to x-axis

$$\nabla \times \vec{E} = 0 \rightarrow \boxed{\vec{E} = -\nabla V}$$

Can add a constant 'c' to V without changing **E** ("Gauge freedom"): ∇c = 0, ∀c = const.

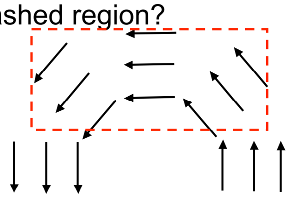
$$\vec{\nabla} \cdot \vec{B} = 0 \rightarrow \boxed{\vec{B} = \vec{\nabla} \times \vec{A}}$$

Can add any vector function '**a**' with ∇x**a**=0 to **A** without changing **B** ("Gauge freedom")
∇ x (**A**+**a**) = ∇ x **A** + ~~∇ x **a**~~ = ∇ x **A** = **B**
0

Do you want me to do more example showing how to find A given J?

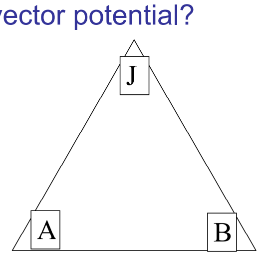
A) Yes
B) No

5.24 If the arrows represent the vector potential **A** (note that |A| is the same everywhere), is there a nonzero **B** in the dashed region?



A. Yes
B. No
C. Need more information to decide

Compare the magnetostatic triangle (p. 240) with the electrostatic triangle (pg. 87). How is the potential similar/different to the vector potential?



5.25

$$\nabla^2 \vec{\mathbf{A}} = -\mu_0 \vec{\mathbf{J}}$$

In Cartesian coordinates, this means:

$\nabla^2 \mathrm{A}_x = -\mu_0 \mathrm{J}_x$, etc.

Does it also mean, in spherical coordinates, that $\nabla^2 \mathrm{A}_r = -\mu_0 \mathrm{J}_r$?

A) Yes
B) No

5.25 b

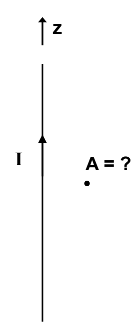
$$\vec{\mathbf{A}}(\vec{r}) = \frac{\mu_0}{4\pi} \iiint \frac{\vec{\mathbf{J}}(r')}{\Re} d\tau'$$

Can you calculate that integral using spherical coordinates?

A) Yes, no problem
B) Yes, r' can be in spherical, but **J** still needs to be in Cartesian components
C) No.

MD12-3

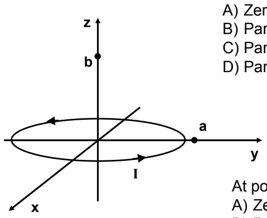
The vector potential **A** due to a long straight wire with current I along the z-axis is in the direction parallel to:



A) $\hat{z}$
B) $\hat{\varphi}$ (azimuthal)
C) $\hat{s}$ (radial)

MD12-4a,b

A circular wire carries current I in the xy plane. What can you say about the vector potential **A** at the points shown?

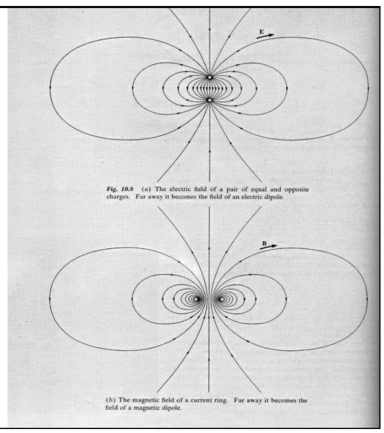


At point a, the vector potential A is:
A) Zero
B) Parallel to x-axis
C) Parallel to y-axis
D) Parallel to z-axis

At point b, the vector potential A is:
A) Zero
B) Parallel to x-axis
C) Parallel to y-axis
D) Parallel to z-axis

E-field around electric dipole

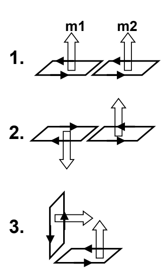
B-field around magnetic dipole (current loop)



From Purcell, Electricity and Magnetism

MD12-5

Two magnetic dipoles **m**1 and **m**2 are oriented in three different ways.



Which ways produce a dipole field at large distances?

A) None of these
B) All three
C) 1 only
D) 1 and 2 only
E) 1 and 3 only

5.26

What is $\oint \vec{\mathbf{A}} \cdot d\vec{l}$

A) The current density **J**
B) The magnetic field **B**
C) The magnetic flux Φ_B
D) It's none of the above, but is something simple and concrete
E) It has no particular physical interpretation at all

5.27 Suppose A is azimuthal, given by

$$\vec{\mathbf{A}} = \frac{c}{s}\hat{\varphi}$$

What can you say about curl(**A**)?

A) curl(**A**)=0 everywhere

B) curl(**A**) = 0 everywhere except at s=0.

C) curl(**A**) is nonzero everywhere

D) ???

Writing assignment

On paper (don't forget your name!) in your own words (by yourself):

What is the idea behind the magnetic vector potential?
What does it accomplish, why might we care about it?
In what ways is it like (or NOT like!) the electric potential?

5.28

Choose all of the following statements that are implied by $\oint\!\!\oint \vec{B} \cdot d\vec{a} = 0$ (for any closed surface you like)

(I) $\vec{\nabla} \cdot \vec{B} = 0$
(II) $B^{//}_{above} = B^{//}_{below}$
(III) $B^{\perp}_{above} = B^{\perp}_{below}$

A) (II) only
B) (III) only
C) (I) and (II) only
D) (I) and (III) only
E) All of the above

5.28 b

In general, which of the following are continuous as you move past a boundary?

A) **A** B) Not all of **A**, just A_{perp}

C) Not all of **A**, just $A_{//}$

D) Nothing is guaranteed to be continuous regarding **A**

DIPOLES, MULTIPOLES

This is the formula for an ideal magnetic dipole:

$$\vec{\mathbf{B}} = \frac{c}{r^3}(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta})$$

What is different in a sketch of a *real* (physical) magnetic dipole (like, a small current loop)?

5.29 The formula from Griffiths for a magnetic dipole at the origin is:

$$\vec{\mathbf{A}}(\vec{\mathrm{r}}) = \frac{\mu_0}{4\pi}\frac{\hat{\mathrm{m}}\times\hat{\mathrm{r}}}{\mathrm{r}^2}$$

Is this the *exact* vector potential for a flat ring of current with **m**=I**a**, or is it approximate?

A) It's exact

B) It's exact if |r| > radius of the ring

C) It's approximate, valid for large r

D) It's approximate, valid for small r

5.30 The leading term in the vector potential multipole expansion involves $\oint d\vec{\mathbf{l}}'$

What is the magnitude of this integral?

A) R

B) 2 π R

C) 0

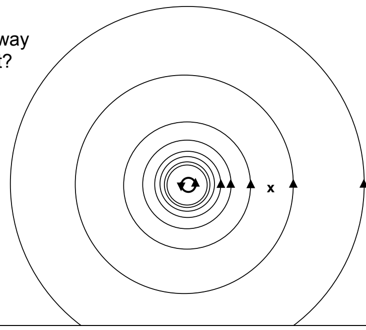
D) Something entirely different/it depends!

MD12-6

In the plane of a magnetic dipole, with magnetic moment **m (out)**, the vector potential **A** looks like kinda like this with A ~ $1/r^2$

At point x, which way does curl(**A**) point?

A) Right
B) Left
C) In
D) Out
E) Curl is zero



(See Chapter 6 concept tests for force and torque on dipole questions.)