✯ TUTORIAL 10: VISUALIZING VECTOR POTENTIAL ✯

Part 1 – Sketching Vector Potential

i. Notice that the equations defining **A** are mathematically analogous to Maxwell's equations

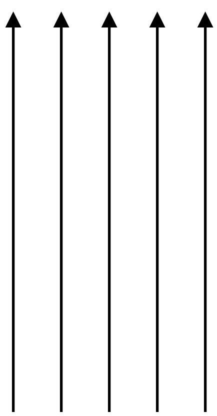
for **B**:

$$\nabla \cdot \vec{\mathbf{B}} = 0 \quad \Leftrightarrow \quad \nabla \cdot \vec{\mathbf{A}} = 0 \qquad \text{(Coulomb gauge)}$$
$$\nabla \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{J}} \quad \Leftrightarrow \quad \nabla \times \vec{\mathbf{A}} = \vec{\mathbf{B}}$$

Sketch **B** in Fig 1 (note this is a "cylindrical" volume with uniform **J**). Then, using the mathematical similarities above, sketch **A** in Fig 2:

Side view:

$\vec{J}(s \le a, \phi, z) = J_o \hat{z}$

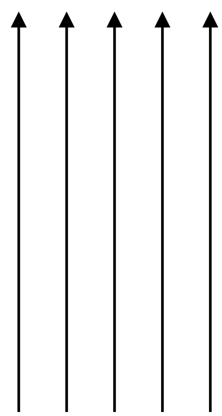


$\vec{J}(s > a, \phi, z) = 0$

Figure 1: Given **J**, sketch the **B** field.

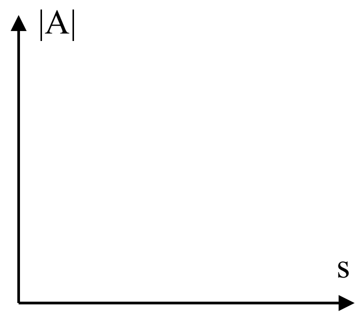
Side view:

$\vec{B}(s \le a, \phi, z) = B_o \hat{z}$



$\vec{B}(s > a, \phi, z) = 0$

Figure 2: Given **B**, sketch the **A** field.



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ii. One way to check your previous answer (conceptually) is using an Ampere's Law analogy. Ampere's Law tells you that the **J**-flux (or I_{encl}) is equal to $\oint \vec{B} \bullet d\vec{l}$.

What is a similar relationship between the vector potential and magnetic field?

Try using this "Ampere's Law analogy" to check your sketch of **A**.

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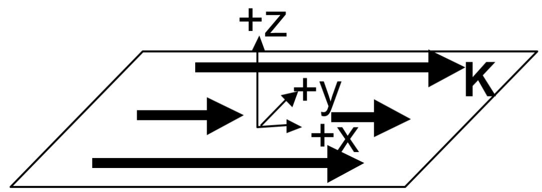
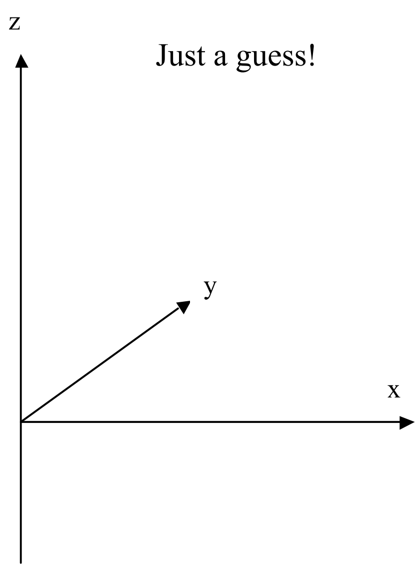
Part 2 – Calculating Vector Potential

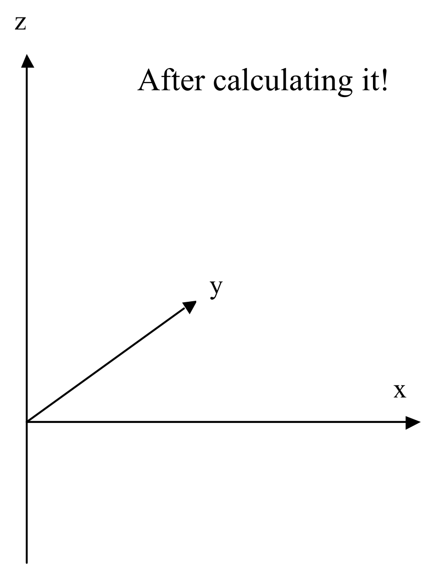


In a common homework question, you are asked to calculate the magnetic field produced by a uniform surface current: $K(z=0)=K_o\hat{x}$. The answer is:

$$\vec{B}(z>0)=\frac{-\mu_o K_o}{2}\hat{y}\,,\quad \vec{B}(z<0)=\frac{+\mu_o K_o}{2}\hat{y}$$

Sketch your best guess of what **A** looks like for the uniform surface current. Which components (x, y, or z) does **A** have (it might help to look at relationship between **A**, **B**, and **J** in the two examples in Part 1)? Which variables (x, y, or z) does **A** depend on?





i. Using your assumption for which components **A** has, and which variables **A** depends on, <u>calculate (or guess)</u> what **A** is.

   Check your answer by calculating the **B** field from your vector potential **A**.

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