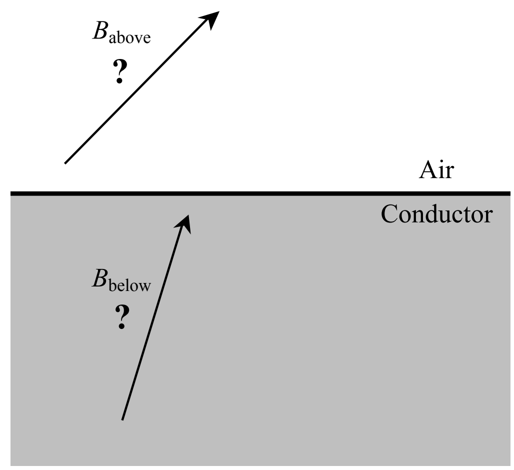
BOUNDARY CONDITIONS FOR THE MAGNETIC FIELD

I. Amperian loops and Gaussian pillboxes

Imagine zooming in toward the boundary of a conductor that is exposed to air. At the region we are zooming in on, assume that there could be a surface current density $\vec{K}$ at the conductor surface and a volume current density $\vec{J}$ within the interior of the conductor.

We'd like to know what might be different—and what might be the same—about the magnetic field *just above* the surface ($\vec{B}_{above}$) and that *just below* ($\vec{B}_{below}$).

(The diagram at right illustrates the situation at hand.)



A. On the diagram above right, draw a sketch imagine a Gaussian "pillbox" that straddles the interface between the air and the object. The endcaps of the pillbox, each of area ΔA, are parallel to the interface. The height of the pillbox is denoted as Δz.

Identify which quantity (or quantities) would be relevant for the Gaussian pillbox you have drawn, and state what would be true about the value of it (them).

$$\oint(\vec{\nabla}\bullet\vec{B})dV \;? \qquad \oint(\vec{\nabla}\times\vec{B})\bullet d\vec{a} \;? \qquad \oint\vec{B}\bullet d\vec{a} \;? \qquad \oint\vec{B}\bullet d\vec{s} \;?$$

B. Now imagine shrinking the pillbox in such a way that $\Delta z \to 0$, with the pillbox always containing an area ΔA of the surface the entire time.

On the basis of your answers above, we can conclude a useful relationship between $\vec{B}_{above}$ and $\vec{B}_{below}$, but only with respect to *either* the component of these fields **perpendicular** to the interface or the component that is **parallel** to it.

- Which component can we conclude a useful relationship about, and what exactly is that relationship?
- Why can we ignore the *other* component of the magnetic field in the process?

C. Now let's start over, except this time imagine an imaginary rectangular loop that "straddles" the boundary between the air and the conductor's surface. The loop should "straddle" the boundary between the air and the non-conducting object's surface. Two legs of the loop, each of length Δx, should be parallel to the interface. The other two legs, each of length Δz, should be perpendicular to the interface. In the space below, draw a sketch showing the imaginary loop and the boundary.

As you did for the imaginary loop, identify which quantity (or quantities) would be relevant for the imaginary loop you have drawn, and state what would be true about the value of it (them).

$\oint(\vec{\nabla}\bullet\vec{B})dV$? $\quad\quad$ $\oint(\vec{\nabla}\times\vec{B})\bullet d\vec{a}$? $\quad\quad$ $\oint\vec{B}\bullet d\vec{a}$? $\quad\quad$ $\oint\vec{B}\bullet d\vec{s}$?

D. Now imagine shrinking the loop now in such a way that $\Delta z \to 0$, and the loop continues to straddle the interface the whole time. Which component of the B- fields above and below the boundary can we conclude a useful relationship about, and what exactly is that relationship?
(*Big hint:* A rectangular loop like the one we are considering is sometimes called an *Amperian* loop. And, remember, we have both surface and volume currents.)

E. Recall how we modeled an infinitely long (ideal) solenoid as a cylinder with a uniform, azimuthal surface current density $\vec{K}$. (We found the magnitude of this current density was $K = nI$.) Show how your answers on this mini-tutorial are consistent with your findings about the magnetic fields just inside and just outside the solenoid (cylinder). Consider the components of the magnetic field that are perpendicular and parallel to the surface. Discuss your reasoning with your partners.

To think about tonight: recall the boundary conditions we determined for electric fields near the interface between two media. How are they different in form from those we just found for magnetic fields?

