## * TUTORIAL 1 * <br> Coulomb's Law and "script $r$ "

## Part 3 - Charged bike tires

In the year 2240, a bicyclist, named Steve, gets lost east of Boulder and gets a flat tire. Steve pulls off the tire and then consults his Tricorder to find out what life forms are nearby. However, the flat tire has somehow been charged with uniform charge density $\lambda$. The Tricorder complains that the electric field from the tire is very annoying.


Your goal is to calculate the electric field produced by the electrically-charged tire (ring of line-charge density $\lambda$ ).
i. The origin is at the center of the tire. Label the diagram with points $(x, y, z)$ and ( $\mathrm{x}^{\prime}, \mathrm{y}^{\prime}, \mathrm{z}^{\prime}$ )
ii. Now label the three vectors: $\vec{r}, \vec{r}^{\prime}$, and $\overrightarrow{\boldsymbol{r}}$.
iii. Write down a formal integral expression for the electric field. Be very explicit about all "short hand symbols" that appear in that integral (What does script-r-hat mean here, specifically?) What choice of coordinates would be most convenient?

The integral you just wrote down has symbols in it which we need to be very clear about: iv. Express the Cartesian components of $\overrightarrow{\boldsymbol{\tau}}=\left[\tau_{\mathrm{x}}, \tau_{\mathrm{y}}, \tau_{z}\right]$ from parts (i) - (iii) (This is now the vector from a little "chunk" of charge dq, as shown, to the tricorder at P) using cylindrical variables. (Don't try to write $\overrightarrow{\boldsymbol{\tau}}$ in a cylindrical coordinate basis.
Write each of the Cartesian components of $\overrightarrow{\boldsymbol{\tau}}$ using appropriate cylindrical variables.)

v. Now you can manipulate your integral from iii into a form that could (at least in principle) be solved by a dumb computer or calculator. This will require expressing script-r in your chosen coordinate system, and writing out what dq is (again in terms of your chosen coordinate system)
vi. Now, simplify by putting the Tricorder on the z-axis, and evaluate that integral!

Some things to think about: Are the units correct for an electric field?
What are the limits for very large $z$, and $z=0$. Do these answers make sense?
Make a sketch of $\mathrm{E}_{\mathrm{z}}$ as a function of $z$, including both positive and negative $z$ axes.

