

★ TUTORIAL 3: ELECTRIC POTENTIAL ★

i) Consider a **uniformly charged (solid) sphere** with total charge Q and radius R , centered at the origin. You solved for $E(r)$ in a recent homework, and should have found the following:

$$E(r) = \begin{cases} \frac{Q}{4\pi\epsilon_0 R^3} r & \text{for } r < R \\ \frac{Q}{4\pi\epsilon_0 r^2} & \text{for } r > R \end{cases}$$

Which aspects of this formula now seem “obvious” to you?

What’s the principle of physics used to generate this formula?

(If you’re not sure of any details, take the time at home to rederive it from scratch!)

ii) The definition of voltage (potential) says $V(r) = -\int_0^r \vec{E} \cdot d\vec{L}$.

Choosing $V(\infty)=0$ (i.e. setting “0”= ∞ in this formula), this is easy enough to find if $r > R$:

$$\text{If } r > R, V(r) = -\int_{\infty}^r \vec{E} \cdot d\vec{L} = -\int_{\infty}^r \frac{Q}{4\pi\epsilon_0 r^2} dr = +\frac{Q}{4\pi\epsilon_0} \frac{1}{r} \Big|_{\infty}^r = +\frac{Q}{4\pi\epsilon_0 r}$$

- Don’t just accept the details - check my math please!

Talk with your neighbors, or us, about any piece that seem at all confusing to you.

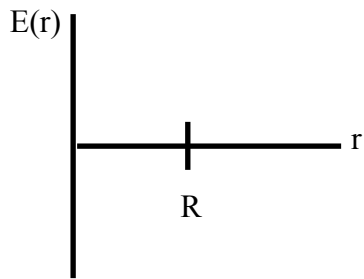
iii) If $r < R$, I need to break that integral up into two pieces (**why?**), giving:

$$\begin{aligned} \text{If } r < R, V(r) &= -\int_{\infty}^r \vec{E} \cdot d\vec{L} \\ &= -\int_{\infty}^R \vec{E} \cdot d\vec{L} - \int_R^r \vec{E} \cdot d\vec{L} \\ &= -\int_{\infty}^R \frac{Q}{4\pi\epsilon_0 r^2} dr - \int_R^r \frac{Q}{4\pi\epsilon_0 R^3} r dr \\ &= +\frac{Q}{4\pi\epsilon_0 R} - \frac{Q}{4\pi\epsilon_0 R^3} \frac{r^2}{2} \Big|_R^r \\ &= \frac{Q}{4\pi\epsilon_0} \left(\frac{3}{2R} - \frac{r^2}{2R^3} \right) \end{aligned}$$

Work through all 5 lines. Make sure you see what is going on at each step .

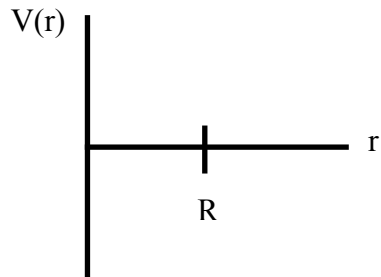
Talk with your neighbors, or ask us about anything that seem at all confusing to you!

iv) In the space below, sketch $E(r)$ and $V(r)$. (E should be easy. V is harder, be careful!)



- Is $E(r)$ continuous? Is $V(r)$? Should they be? **Why/why not?**

- Are their derivatives continuous? Should they be?

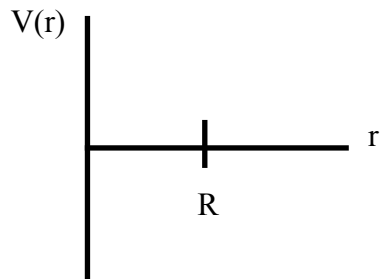


- Do the signs make sense *everywhere*? **Explain!**

- Is the behavior of $E(r)$ and $V(r)$ at large r correct?

- What's the "slope" of $V(r)$ at the origin? (What should it be?)

Now let **the origin** ($r=0$) be your choice of zero voltage, and re-sketch this V as a function of r .



Explain how(if) the choice of zero voltage at the origin changes the electric field.