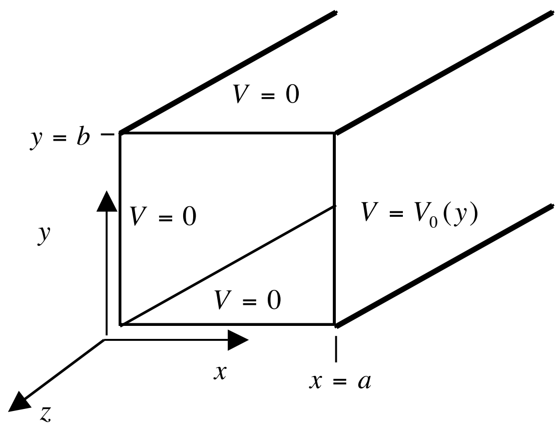
✭ TUTORIAL 5: BREAKING DOWN THE STEPS ✭
SEPARATION OF VARIABLES

Part 1: Laplace's Equation and Separation of Variables

Within a very long, rectangular, hollow pipe, there are no electric charges. The walls of this pipe are kept at a known voltage (they are known because in a lab, you can control them).

Three of the walls are grounded: $V(x=0,y,z)=0$; $V(x,y=0,z)=0$; $V(x,y=b,z)=0$

The fourth wall maintains a potential that varies with y: $V(x=a,y,z)=V_o(y)$ which will be specified later.



In order to find out the voltage inside the pipe, you will need to solve Laplace's equation:

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

i. What does it mean to "separate variables" of $V(x,y,z)$. What advantage is there to using that approach here?

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ii. Plug the separated form of V into Laplace's equation. After doing this, you should have several terms.

- Simplify as much as possible.
- Are any of the terms zero in this case?
- What must be true about the remaining terms in order to satisfy Laplace's equation?
- Write down the ordinary differential equations you need to solve to find V.

iii. The boundary conditions on the pipe are listed below. Which boundary condition(s) allow you to determine the direction (x or y) that must have sinusoidal behavior?

1. $V(x,y=0,z)=0$
2. $V(x,y=b,z)=0$
3. $V(x=0,y,z)=0$
4. $V(x=a,y,z)=V_o$

Write down and modify your general expression for the voltage everywhere inside the pipe so that it satisfies the first three boundary conditions. **Do *not* apply the 4th boundary condition yet**.

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Part 2: Fourier's Trick

i. Having applied boundary conditions 1-3 (three grounded walls), what does your answer look like? (write down what you had in the space after the C_n in the equation below)

$$V(x,y,z) = \sum_n C_n$$

ii. Now apply the Boundary Condition on the fourth wall: $V(x=a,y,z) = V_o$

What is the voltage everywhere inside the pipe? To find it:

1. Evaluate the equation above at the $x=a$ boundary.

2. To use Fourier's Trick, we multiply both sides of the equation for voltage on the last boundary by $\mathrm{Sin}(\frac{m\pi y}{b})$ and integrate over one cycle. How does this help us solve for the voltage? What are the limits of integration?

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3. Perform the integration. What happens to the infinite sum after you perform the integration?

4. Now find all the C_n 's and write down V(x,y,z).

Optional (if you have time): Can you invent a different boundary condition on the fourth wall, i.e. V(x=a,y, z) is a particular FUNCTION of y, not just a constant, such that you can solve for the final constant in your general solution quickly and easily without integrating anything at all? If so, do it. If not, explain why.

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