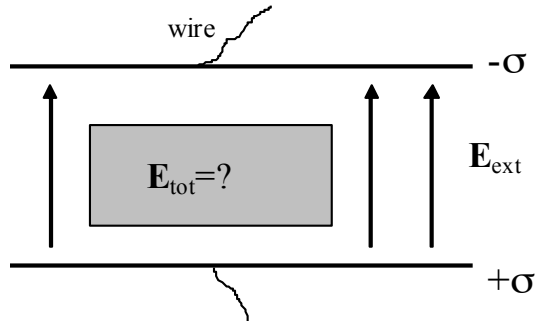


★ “LEC-TORIAL:” SQUINTING CLOSELY AT PLASTIC ★  
***Polarization & Bound Charge***

Part 1 – Polarization and Bound Charge

A slab of plastic is placed within a charged capacitor.

Before inserting the plastic, there is a uniform electric field inside the capacitor,  $E_{\text{ext}}$ . Let’s explore the properties of a dielectric to find the (total) electric field  $E_{\text{tot}}$  inside the plastic.



- i. (Together as a class:) *Extreme close-up!* At right, sketch a cartoon of the first few rows of atoms inside the plastic. Think of many negative charges “ $-q$ ,” each bound to a fixed positive core “ $+q$ ,” with a spring constant  $k$ . Each “ $-q$ ” charge is displaced from the core by a distance  $\Delta z$ .
  - a. Write an expression for the magnitude of the dipole moment  $\mathbf{p}$  of each “atom”:



- ii. (Together as a class:) *Not-so-extreme close-up.* Back to a model with smooth, continuous charge distributions.... Draw a sketch of the plastic showing where the net charge is positive, negative, and zero.



- iii. (In your groups:) If there are  $N$  atoms per unit volume in the plastic (each with a  $+q$  and a  $-q$  charge) write simple expressions for:
  - a. the volume charge densities  $\rho^+$  (this is some sort of “smeared” average volume charge density arising just from the  $+q$ ’s) and  $\rho^-$
  - b. the volume dipole moment density (total dipole moment per unit volume), a.k.a. the *polarization*  $\mathbf{P}$ . (**Note:** this is a vector! Which way does it point here?)

- iv. (In your groups:) When using a continuous model of charge densities, you should have found that the top surface of the plastic has an overall net charge found within a thin layer of area  $A$  and thickness  $\Delta z$  just inside the surface. Find the *surface charge density* of this thin layer.
- v. (In your groups:) The surface-charge density you found above is called the *bound-charge density*,  $\sigma_B$ . Rewrite your expression so that it is written in terms of  $\mathbf{P}$ .
- vi. (Together as a class): Is your result for  $\mathbf{P}$  consistent with the relationship  $\sigma_B = \bar{\mathbf{P}} \cdot \hat{\mathbf{n}}$ ?  
Be sure to check **all surfaces** (top, bottom, and sides) of the plastic!

## Part 2 – Electric Fields in Dielectric Materials (or Insulators)

(All parts together as a class.) For our purposes here, there are *three* (!) E-fields to keep track of:

- The uniform electric field  $\mathbf{E}_{\text{ext}}$  inside the capacitor before we inserted the plastic.
  - The induced electric field  $\mathbf{E}_{\text{ind}}$  caused by the bound charge densities in the plastic.
  - The total electric field  $\mathbf{E}_{\text{tot}}$ .
- i. Write a *vector* equation relating all three fields:
- ii. Write a separate *vector* expression for the induced electric field,  $\mathbf{E}_{\text{ind}}$ , inside the plastic slab in terms of  $\mathbf{P}$ . (*Hints:* How is  $|\mathbf{E}_{\text{ind}}|$  related to  $\sigma_B$ ? How is its direction related to that of  $\mathbf{P}$ ?)
- iii. Combine your results from parts i and ii to write a *vector* equation relating  $\mathbf{E}_{\text{ext}}$ ,  $\mathbf{E}_{\text{tot}}$  and  $\mathbf{P}$ .
- iv. Griffiths defines a quantity  $\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$ , where  $\mathbf{D}$  is called the “electric displacement field,” or simply the “D field.” Look at your answer to part iii – think about how you might interpret the meaning of  $\mathbf{D}$ .