BASIC EQUATIONS OF ELECTRODYNAMICS

Maxwell's Equations

In General:

$$\vec{\nabla} \cdot \vec{E} = \frac{1}{\varepsilon_0}\rho$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

In Matter:

$$\vec{\nabla} \cdot \vec{D} = \rho_f$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t}$$

Auxiliary Fields

Definitions:

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

$$\vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M}$$

Linear Media:

$$\vec{P} = \varepsilon_0 x_e \vec{E}, \quad \vec{D} = \varepsilon \vec{E}$$

$$\vec{M} = x_m \vec{H}, \quad \vec{H} = \frac{1}{\mu}\vec{B}$$

Potentials

$$\vec{E} = -\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t}, \quad \vec{B} = \vec{\nabla} \times \vec{A}$$

Lorentz Force Law

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

Energy

Energy: $U = \frac{1}{2}\int \left(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2 \right) d\tau$

FUNDAMENTAL CONSTANTS

$\varepsilon_0 = 8.85 \times 10^{-12}\, C^2 / Nm^2$

(permittivity of free space)

$\mu_0 = 4\pi \times 10^{-7}\, N / A^2$

(permeability of free space)

$e = 1.60 \times 10^{-19}\, C$

(charge of the electron)

$m = 9.11 \times 10^{-31}\, kg$

(mass of the electron)

SPHERICAL AND CYLINDRICAL COORDINATES

Spherical

$x = r\sin\theta\cos\phi$

$y = r\sin\theta\sin\phi$

$z = r\cos\theta$

$\hat{x} = \sin\theta\cos\phi\hat{r} + \cos\theta\cos\phi\hat{\theta} - \sin\phi\hat{\phi}$

$\hat{y} = \sin\theta\sin\phi\hat{r} + \cos\theta\sin\phi\hat{\theta} + \cos\phi\hat{\phi}$

$\hat{z} = \cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}$

Cylindrical

$x = s\cos\phi$

$y = s\sin\phi$

$z = z$

$\hat{x} = \cos\phi\hat{s} - \sin\phi\hat{\phi}$

$\hat{y} = \sin\phi\hat{s} + \cos\phi\hat{\phi}$

$\hat{z} = \hat{z}$

FUNDAMENTAL THEOREMS

Gradient Theorem: $\int_a^b (\nabla f) \cdot d\vec{l} = f(\vec{b}) - f(\vec{a})$

Divergence Theorem: $\int (\nabla \cdot \vec{A}) d\tau = \oint \vec{A} \cdot d\vec{a}$

Curl Theorem: $\int (\vec{\nabla} \times \vec{A}) \cdot d\vec{a} = \oint \vec{A} \cdot d\vec{l}$

VECTOR DERIVATIVES

Cartesian. $d\vec{l} = dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z};\quad d\tau = dx\,dy\,dz$

Gradient: $\vec{\nabla} t = \frac{\partial t}{\partial x}\hat{x} + \frac{\partial t}{\partial y}\hat{y} + \frac{\partial t}{\partial z}\hat{z}$

Divergence: $\vec{\nabla}\cdot v = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$

Curl: $\vec{\nabla}\times v = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\hat{x} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\hat{y} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\hat{z}$

Laplacian: $\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$

Spherical. $d\vec{l} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\varphi};$

$d\tau = r^2 \sin\theta\; dr\; d\theta\; d\phi$

Gradient: $\vec{\nabla} t = \frac{\partial t}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\phi}$

Divergence:

$$\vec{\nabla}\cdot v = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 v_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\, v_\theta\right) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

Curl:

$$\vec{\nabla}\times v = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}\left(\sin\theta\, v_\varphi\right) - \frac{\partial v_\theta}{\partial\varphi}\right]\hat{r} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\varphi} - \frac{\partial}{\partial r}\left(r\, v_\varphi\right)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\, v_\theta\right) - \frac{\partial v_r}{\partial\theta}\right]\hat{\varphi}$$

Laplacian: $\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\varphi^2}$

Cylindrical. $d\vec{l} = ds\,\hat{s} + s\,d\phi\,\hat{\phi} + dz\,\hat{z};\quad d\tau = s\,ds\,d\phi\,dz$

Gradient: $\vec{\nabla} t = (\partial t/\partial s)\,\hat{s} \;+\; (1/s)(\partial t/\partial\varphi)\,\hat{\varphi} \;+\; (\partial t/\partial z)\,\hat{z}$

Divergence: $\vec{\nabla}\cdot v = \frac{1}{s}\frac{\partial}{\partial s}\left(s v_s\right) + \frac{1}{s}\frac{\partial v_\phi}{\partial\phi} + \frac{\partial v_z}{\partial z}$

Curl: $\vec{\nabla}\times v = \left[\frac{1}{s}\frac{\partial v_z}{\partial\phi} - \frac{\partial v_\phi}{\partial z}\right]\hat{s} + \left[\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right]\hat{\phi} + \frac{1}{s}\left[\frac{\partial}{\partial s}\left(s\, v_\phi\right) - \frac{\partial v_s}{\partial\phi}\right]\hat{z}$

Laplacian: $\nabla^2 t = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial t}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 t}{\partial\phi^2} + \frac{\partial^2 t}{\partial z^2}$

VECTOR IDENTITIES

Triple Products

(1) $\vec{A}\cdot\left(\vec{B}\times\vec{C}\right) = \vec{B}\cdot\left(\vec{C}\times\vec{A}\right) = \vec{C}\cdot\left(\vec{A}\times\vec{B}\right)$

(2) $\vec{A}\times\left(\vec{B}\times\vec{C}\right) = \vec{B}\left(\vec{A}\cdot\vec{C}\right) - \vec{C}\left(\vec{A}\cdot\vec{B}\right)$

Product Rules

(3) $\vec{\nabla}(fg) = f\left(\vec{\nabla}g\right) + g\left(\vec{\nabla}f\right)$

(4) $\vec{\nabla}\left(\vec{A}\cdot\vec{B}\right) = \vec{A}\times\left(\vec{\nabla}\times\vec{B}\right) + \vec{B}\times\left(\vec{\nabla}\times\vec{A}\right) + \left(\vec{A}\cdot\vec{\nabla}\right)\vec{B} + \left(\vec{B}\cdot\vec{\nabla}\right)\vec{A}$

(5) $\vec{\nabla}\cdot\left(f\vec{A}\right) = f\left(\vec{\nabla}\cdot\vec{A}\right) + \vec{A}\cdot\left(\vec{\nabla}f\right)$

(6) $\vec{\nabla}\cdot\left(\vec{A}\times\vec{B}\right) = \vec{B}\cdot\left(\vec{\nabla}\times\vec{A}\right) - \vec{A}\cdot\left(\vec{\nabla}\times\vec{B}\right)$

(7) $\vec{\nabla}\times\left(f\vec{A}\right) = f\left(\vec{\nabla}\times\vec{A}\right) - \vec{A}\times\left(\vec{\nabla}f\right)$

(8) $\vec{\nabla}\times\left(\vec{A}\times\vec{B}\right) = \left(\vec{B}\cdot\vec{\nabla}\right)\vec{A} - \left(\vec{A}\cdot\vec{\nabla}\right)\vec{B} + \vec{A}\left(\vec{\nabla}\cdot\vec{B}\right) - \vec{B}\left(\vec{\nabla}\cdot\vec{A}\right)$

Second Derivatives

(9) $\vec{\nabla}\cdot\left(\vec{\nabla}\times\vec{A}\right) = 0$

(10) $\vec{\nabla}\times(\vec{\nabla}f) = 0$

(11) $\vec{\nabla}\times\left(\vec{\nabla}\times\vec{A}\right) = \vec{\nabla}(\vec{\nabla}\cdot\vec{A}) - \nabla^2\vec{A}$