


402

## Electricity and Magnetism II

AC Circuits & Complex Numbers  
Clicker Questions




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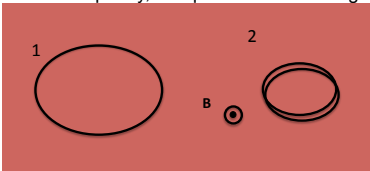
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Loop 1 sits in a uniform field  $\mathbf{B}$  which is increasing in magnitude. Loop 2 has the SAME LENGTH OF WIRE looped (coiled) to make two (smaller) loops. (The 2 loops are connected appropriately, think of it as the start of a solenoid)

How do the induced EMFs compare?

HINT: Don't answer too quickly, it requires some thinking!



A)  $EMF(1)=4 EMF(2)$       B)  $EMF(1) = 2 EMF(2)$   
 C) They are both the same.  
 D)  $EMF(2)= 4 EMF(1)$       E)  $EMF(2) = 2 EMF(1)$

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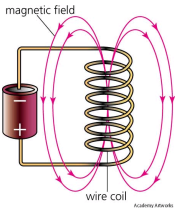
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Consider a simpler case of a solenoid



$$|\vec{B}_{inside}| = \mu_0 n I = \mu_0 \frac{N}{L'} I$$

Recall that the B-field inside a solenoid is uniform!

N = number of loops  
 L' = length of solenoid  
 \* Be careful with symbol L !

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$$|\vec{B}_{inside}| = \mu_0 n I = \mu_0 \frac{N}{L'} I$$

$$\Phi_B = N \iint \vec{B} \cdot d\vec{A} = NBA = N(\mu_0 n I) A$$

$$L = \frac{\Phi_B}{I} = \mu_0 N n A = \mu_0 A n^2 L'$$

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Two long solenoids, each of inductance  $L$ , are connected together to form a single very long solenoid of inductance  $L_{total}$ . What is  $L_{total}$ ?

A)  $2L$   
 B)  $4L$   
 C)  $8L$   
 D) none of these/don't know

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What do these inductors do in circuits?

Just recall that the EMF or Voltage across an inductor is:

$$\epsilon = -L \frac{di}{dt}$$

So, when we add them to circuits, we can apply the usual Kirchhoff's Voltage Law and include the inductors.

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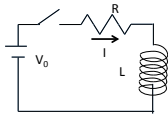
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<sup>MC</sup> The switch is closed at  $t=0$ .  
 What can you say about  $I(t=0^+)$ ?

A) Zero  
 B)  $V_0/R$   
 C)  $V_0/L$   
 D) Something else!  
 E) ???




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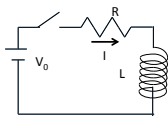
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<sup>MC</sup> The switch is closed at  $t=0$ .  
 What can you say about  $I(t=\infty)$ ?

A) Zero  
 B)  $V_0/R$   
 C)  $V_0/L$   
 D) Something else!  
 E) ???




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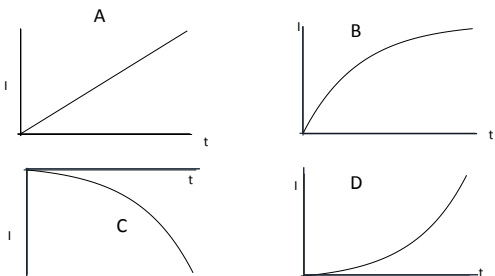
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<sup>MC</sup> The switch is closed at  $t=0$ .  
 Which graph best shows  $I(t)$ ?

E) None of these (they all have a serious error!)




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405

Consider a cubic meter box of uniform magnetic field of 1 Tesla and a cubic meter box of uniform electric field of 1 Volt/meter. Which box contains the most energy?

A. The box of magnetic field  
 B. The box of electric field  
 C. They are both the same  
 D. Not enough information given

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Rank in order, from largest to smallest, the time constants in the three circuits.  $\tau_1$ ,  $\tau_2$  and  $\tau_3$

A.  $\tau_1 > \tau_2 > \tau_3$   
 B.  $\tau_2 > \tau_1 > \tau_3$   
 C.  $\tau_2 > \tau_3 > \tau_1$   
 D.  $\tau_3 > \tau_1 > \tau_2$   
 E.  $\tau_3 > \tau_2 > \tau_1$

$$i(t) = i_0 e^{-t/\left(\frac{L}{R}\right)}$$

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The switch in the circuit below is closed at  $t=0$ .

What is the initial rate of change of current ( $di/dt$ ) in the inductor, immediately after the switch is closed?

A) 0 A/s B) 0.5A/s  
 C) 1A/s D) 10A/s  
 E) None of these.

Hints: What is the initial current through the circuit?  
 Given that - what is the initial voltage across the resistor? The inductor?

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The switch in the circuit below is closed at  $t=0$ .

Hints: What is the initial current through the circuit?  
Given that - what is the initial voltage across the inductor?

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The switch in the circuit below is closed at  $t=0$ .

Hints: What is the initial current through the circuit?  
Given that - what is the initial voltage across the inductor?

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The switch in the circuit below is closed at  $t=0$ .

Hints: What is the initial current through the circuit?  
Given that - what is the initial voltage across the inductor?

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The switch in the circuit below is closed at  $t=0$ .

At  $t=0+$ ,  $V=?$

What is the initial rate of change of current  $di/dt$  in the inductor, immediately after the switch is closed ?

A) 0 A/s B) 0.5A/s  
C) 1A/s D) 10A/s  
E) None of these.

Hints: What is the initial current through the circuit?  
Given that - what is the initial voltage across the inductor?

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The switch in the circuit below is closed at  $t=0$ .

$V=10!!$  (briefly)

What is the initial rate of change of current  $di/dt$  in the inductor, immediately after the switch is closed ?

A) 0 A/s B) 0.5A/s  
C) 1A/s D) 10A/s  
E) None of these.

Hints: What is the initial current through the circuit?  
Given that - what is the initial voltage across the inductor?

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The switch in the circuit below is closed at  $t=0$ .

What is the initial rate of change of current  $di/dt$  in the inductor, immediately after the switch is closed ?

A) 0 A/s B) 0.5A/s  
**C) 1A/s** D) 10A/s  
E) None of these.

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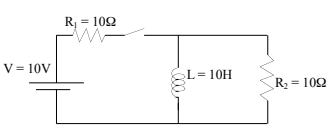
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An LR circuit is shown below. Initially the switch is open. At time  $t=0$ , the switch is closed. What is the current thru the inductor  $L$  immediately after the switch is closed (time =  $0^+$ )?



A) Zero  
 B) 1 A  
 C) 0.5A  
 D) None of these.

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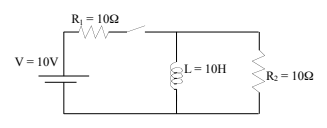
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An LR circuit is shown below. Initially the switch is open. At time  $t=0$ , the switch is closed. After a long time, what is the current from the battery?



A) 0A    B) 0.5A    C) 1.0A    D) 2.0A    E) None of these.

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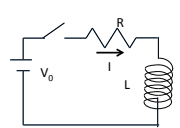
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The switch is closed at  $t=0$ . What can you say about the magnitude of  $\Delta V$  (across the inductor) at  $(t=0^+)$ ?



A) Zero  
 B)  $V_0$   
 C)  $L$   
 D) Something else!  
 E) ???

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The solution to an ODE is

$$I(t) = a \cos(\omega t) + b \sin(\omega t),$$

(with  $a$  and  $b$  still undetermined constants) Or equivalently,

$$I(t) = A \cos(\omega t + \phi)$$

(with  $A$  and  $\phi$  still undetermined constants)

Which expression connects the constants in these two forms?

- A)  $a = A \cos \phi$
- B)  $a = A \sin \phi$
- C) I can do this, but it's more complicated than either of the above!
- D) I'm not sure at the moment how to do this.
- E) It's a trick, these two forms are not equivalent!

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428

The solution to an ODE is

$$I(t) = a \cos(\omega t) + b \sin(\omega t),$$

(with  $a$  and  $b$  still undetermined constants) Or equivalently,

$$I(t) = A \cos(\omega t + \phi)$$

(with  $A$  and  $\phi$  still undetermined constants)

Which expression connects the constants in these two forms?

- A)  $A = a^2 + b^2$
- B)  $A = \sqrt{a^2 + b^2}$
- C) I can do this, but it's more complicated than either of the above!
- D) I'm not sure at the moment how to do this.

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The complex exponential:  $e^{i\omega t}$

is useful in calculating properties of many time-dependent equations. According to Euler, we can also write this function as:

- A)  $\cos(i \omega t) + \sin(i \omega t)$
- B)  $\sin(\omega t) + i \cos(\omega t)$
- C)  $\cos(\omega t) + i \sin(\omega t)$
- D) MORE than one of these is correct
- E) None of these is correct!

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AC10

What is  $|2+i|$

A) 1  
B) Sqrt[3]  
C) 5  
D) Sqrt[5]  
E) Something else!

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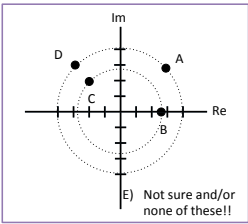
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AC11

Which point below best represents  $4e^{i3\pi/4}$  on the complex plane?



Challenge question: Keeping the general form  $Ae^{i\theta}$ , do any OTHER values of  $\theta$  represent the SAME complex number as this? (If so, how many?)

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AC12

What is  $\frac{(1+i)^2}{(1-i)}$

A)  $e^{i\pi/4}$   
B) Sqrt[2]  $e^{i\pi/4}$   
C)  $e^{i3\pi/4}$   
D) Sqrt[2]  $e^{i3\pi/4}$   
E) Something else!

There are two obvious methods. 1) multiply it out ("rationalizing" the denominator)  
Or 2) First write numerator and denominator in standard  $Ae^{i\theta}$  form.  
Both work. Try it with method 2b

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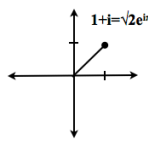
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AC13

What is  $(1+i)^2/(1-i)$



$1+i = \sqrt{2}e^{i\pi/4}$

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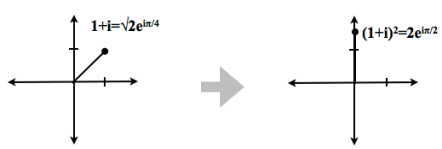
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AC14

What is  $(1+i)^2/(1-i)$



$1+i = \sqrt{2}e^{i\pi/4}$

$(1+i)^2 = 2e^{i\pi/2}$

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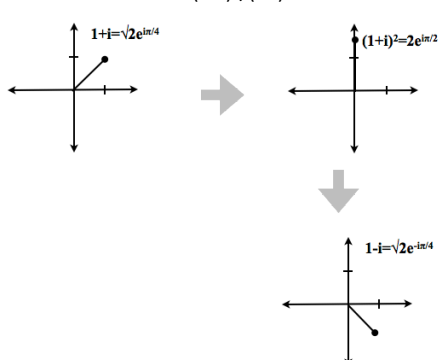
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AC15

What is  $(1+i)^2/(1-i)$



$1+i = \sqrt{2}e^{i\pi/4}$

$(1+i)^2 = 2e^{i\pi/2}$

$1-i = \sqrt{2}e^{-i\pi/4}$

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AC26

What is  $(1+i)^2/(1-i)$

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AC27

AC voltage  $V$  and current  $I$  vs time  $t$  are as shown:

The graph shows that..

A)  $I$  leads  $V$  (  $I$  peaks before  $V$  peaks )  
 B)  $I$  lags  $V$  (  $I$  peaks after  $V$  peaks )  
 C) Neither

$I$  leads  $V$  =  $I$  peaks before  $V$  peaks  
 $I$  lags  $V$  =  $I$  peaks after  $V$  peaks

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AC28

Suppose  $\hat{V} = V_0 e^{j\omega t}$  and  $\hat{I}$  are complex solutions of this equation:

$$\hat{V} = \hat{I}R + L \frac{d\hat{I}}{dt}$$

Is it always true that the real parts of these complex variables are solutions of the equation?

$$\text{Re}[\hat{V}] \stackrel{?}{=} \text{Re}[\hat{I}]R + L \frac{d}{dt} \text{Re}[\hat{I}]$$

A) Yes, always    B) No, not always

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AC19

$$I_0 e^{i\delta} = \frac{V_0}{i\omega L} = \frac{V_0}{\omega L} e^{i\delta}$$

The phase angle  $\delta =$

- A) 0
- B)  $+\pi/2$
- C)  $-\pi/2$
- D)  $+\pi$
- E)  $-\pi$

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AC20

$$V = IZ = I|Z|e^{+j\pi/2}$$

$$I = \frac{V}{Z} = \frac{V}{|Z|} e^{-j\pi/2}$$

Which is the correct current phasor?

E) None of these

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AC21

What is the total impedance of this circuit?

$Z_{total} =$

- A)  $R + j\left(\omega L + \frac{1}{\omega C}\right)$
- B)  $R + j\left(\omega L - \frac{1}{\omega C}\right)$
- C)  $\frac{1}{R} + \frac{1}{j\omega L} + j\omega C$
- D)  $\frac{1}{R + \frac{1}{j\omega L} + j\omega C}$
- E) None of these

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MC22

What is  $\text{Re} \left[ \frac{e^{i\omega t}}{1+i} \right]$

- A)  $\frac{1}{\sqrt{2}} \cos(\omega t + \pi/4)$   
 B)  $\frac{1}{\sqrt{2}} \cos(\omega t - \pi/4)$   
 C)  $\frac{1}{2} \cos(\omega t + \pi/4)$   
 D)  $\frac{1}{2} \cos(\omega t - \pi/4)$   
 E) Not sure/ something entirely different!

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MC23

Suppose you have a circuit driven by a voltage  
 $V(t) = V_0 \cos(\omega t)$ ,  
 and you observe the resulting current is  
 $I(t) = I_0 \cos(\omega t - \pi/4)$ .

Would you say the current is  
 A) leading  
 B) lagging  
 the voltage by 45 degrees?

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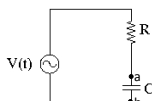
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MC24

A simple RC circuit is driven by an AC power supply with an emf described by

$$V(t) = \begin{cases} 0, & t < 0 \\ V_0 \cos \omega t, & t > 0 \end{cases}$$



The voltage across the capacitor ( $V_a - V_b$ ) just after  $t=0$  is

- A. 0  
 B.  $V_0$   
 C.  $-V_0$   
 D. Not enough information given

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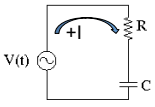
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AC25

A simple RC circuit is driven by an AC power supply with an emf described by

$$V(t) = \begin{cases} 0, & t < 0 \\ V_0 \cos \omega t, & t > 0 \end{cases}$$


The current through the capacitor just after  $t=0$  is

A. 0  
 B.  $V_0/R$   
 C.  $-V_0/R$   
 D. Not enough information given

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AC26

Given a capacitance,  $C$ , and a resistance,  $R$ , the units of the product,  $RC$ , are:

A) Amps  
 B) Volts\*seconds  
 C) seconds  
 D) 1/seconds.  
 E) I do know the answer, but can't prove it in the 60 seconds I'm being given here...

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AC27

The ac impedance of a RESISTOR is:

A) Dependent on voltage drop across the resistor.  
 B) Dependent on current flowing into the resistor.  
 C) Both A) and B)  
 D) None of the above.  
 E) ???

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AC28

The ac impedance of a capacitor is:

- A) Dependent on the magnitude of the voltage drop across the capacitor.
- B) Dependent on the magnitude of the current flowing into the capacitor.
- C) Both A) and B)
- D) None of the above.
- E) ??

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AC29

The ac impedance of an inductor is:

- A) Dependent on voltage drop across and/or current through the inductor.
- B)  $Z_L = i\omega L$
- C)  $Z_L = 1/i\omega L$
- D) None of the above.

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AC30

Two LR circuits driven by an AC power supply are shown below.

Which circuit is a low pass filter?  
 ("Low pass" means low freq. inputs yield strong output, but high frequency input is "blocked", you get no output. So "low pass" filters reduce high frequencies, and passes the low frequencies...)

- A. The left circuit
- B. The right circuit
- C. Both circuits
- D. Neither circuit
- E) ???

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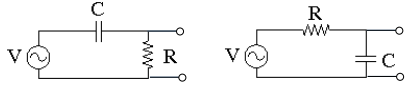
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AC31

Two RC circuits driven by an AC power supply are shown below.



Which circuit is a high pass filter?

A. The left circuit  
 B. The right circuit  
 C. Both circuits  
 D. Neither circuit  
 E. Not enough information given

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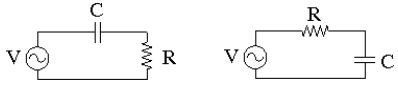
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AC32

Two RC circuits driven by an AC power supply are shown below.



Which circuit is a high pass filter?

A. The left circuit  
 B. The right circuit  
 C. Both circuits  
 D. Neither circuit

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