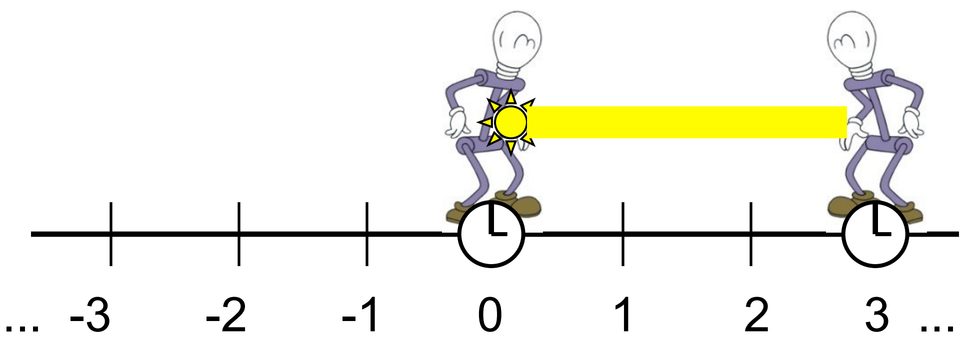
Everything should be made as simple as possible,
but not simpler

-A. Einstein

SR1

Synchronizing clocks



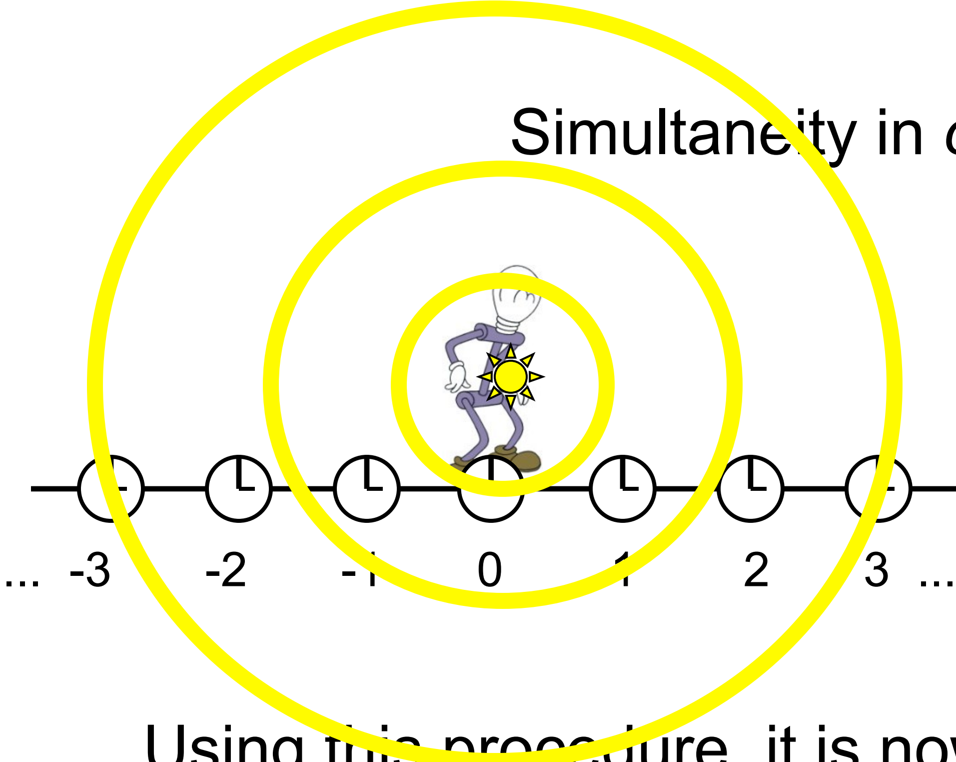
At the origin, at three o’clock, the clock sends out a light signal to tell everybody it’s three o’clock.

Time passes as the signal gets to the clock at x = 3m.

When the signal arrives, the clock at x=3m is set to 3:00 *plus the 10 ns delay*.

SR2

Simultaneity in *one* frame



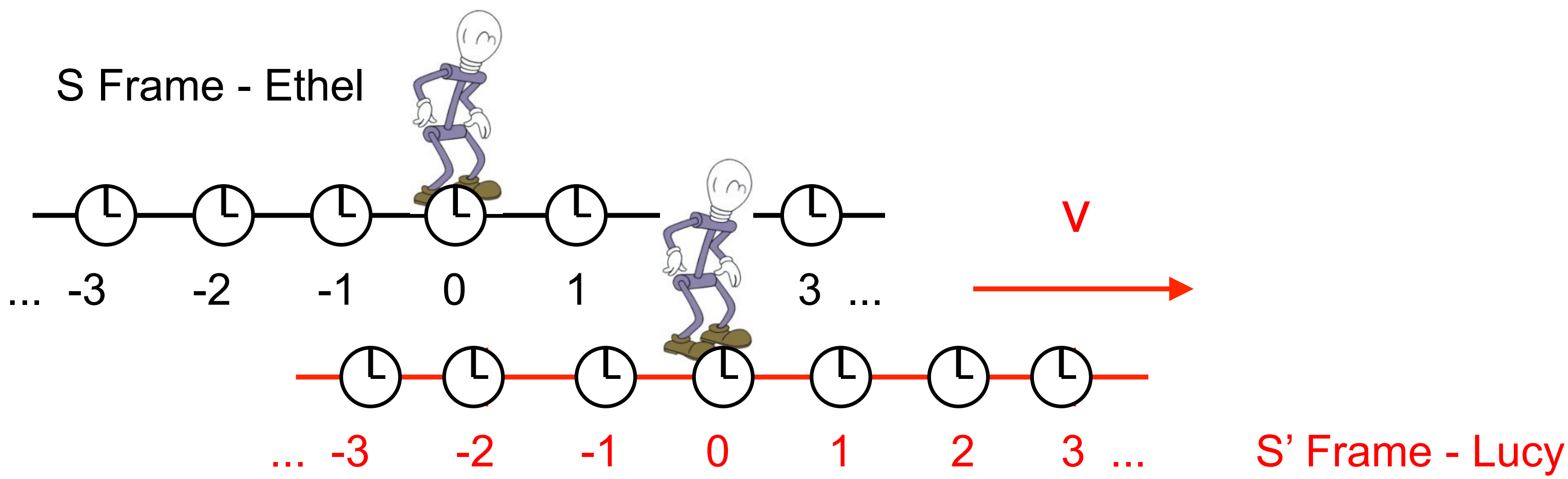
Using this procedure, it is now possible to say that all the clocks in a given inertial reference frame read the same time.
Even if I don’t go out there to check it myself.

Now I know when events happen, even if I don’t find out until later (due to finite speed of light).

r4

SR3

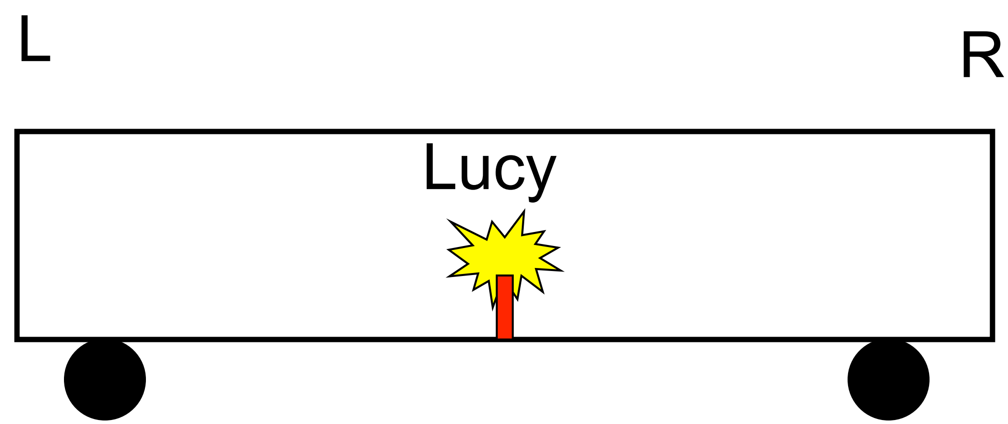
Simultaneity in *two* frames



A second frame has its own clocks, and moves past me. What happens now?

r5

CT-SR4



Lucy is the middle of a railroad car, and sets off a firecracker. (Boom, an event!) Light from the explosion travels to both ends of the car. Which end does it reach first?

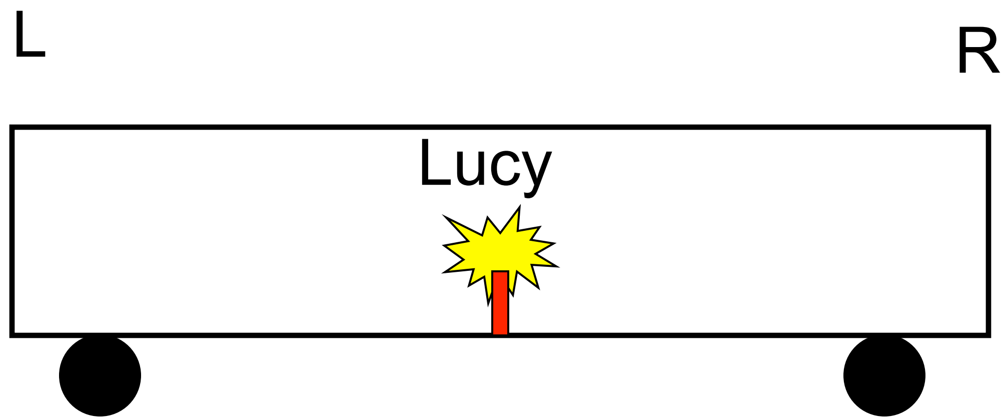
a) both ends at once
b) the left end, L
c) the right end, R

If you are reading these slides outside of class, when you get to a “concept question” (like the last slide), PAUSE, think about it, commit yourself to an answer. Don’t be in a rush to look to the next slide until you have THOUGHT about your reasoning!

No really!

Have you got an answer for the previous concept test yet?

CT-SR4

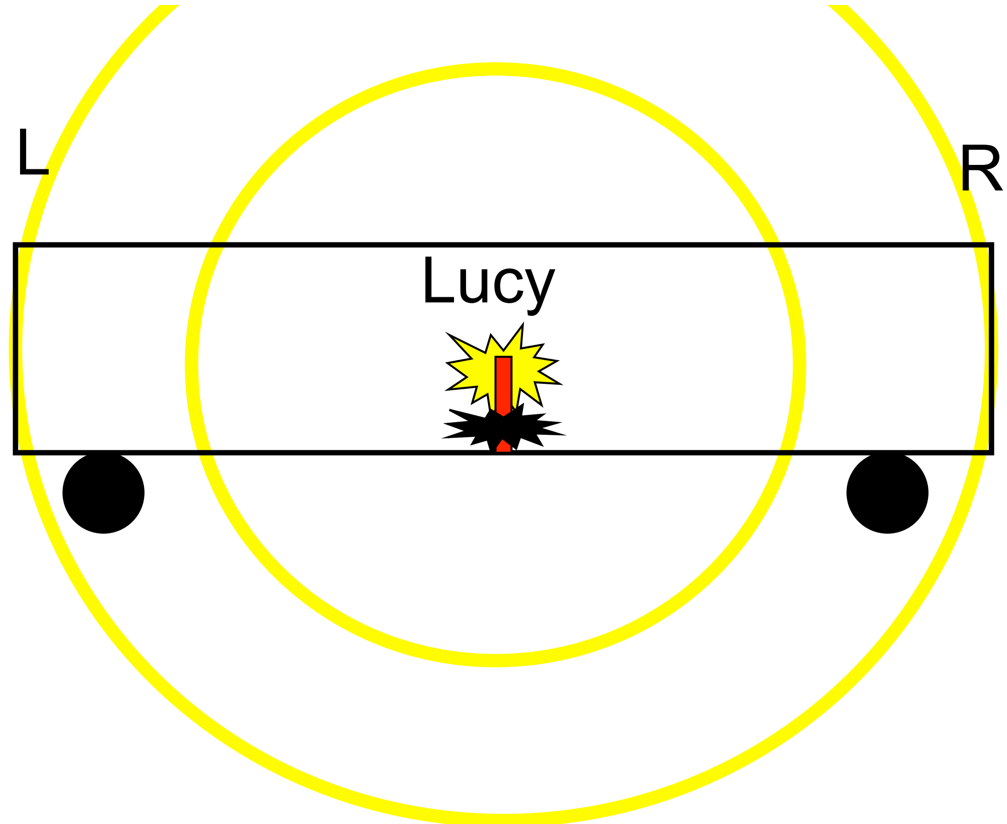


Lucy is the middle of a railroad car, and sets off a firecracker. (Boom, an event!) Light from the explosion travels to both ends of the car. Which end does it reach first?

a) both ends at once
b) the left end, L
c) the right end, R

These events are simultaneous in Lucy's frame.

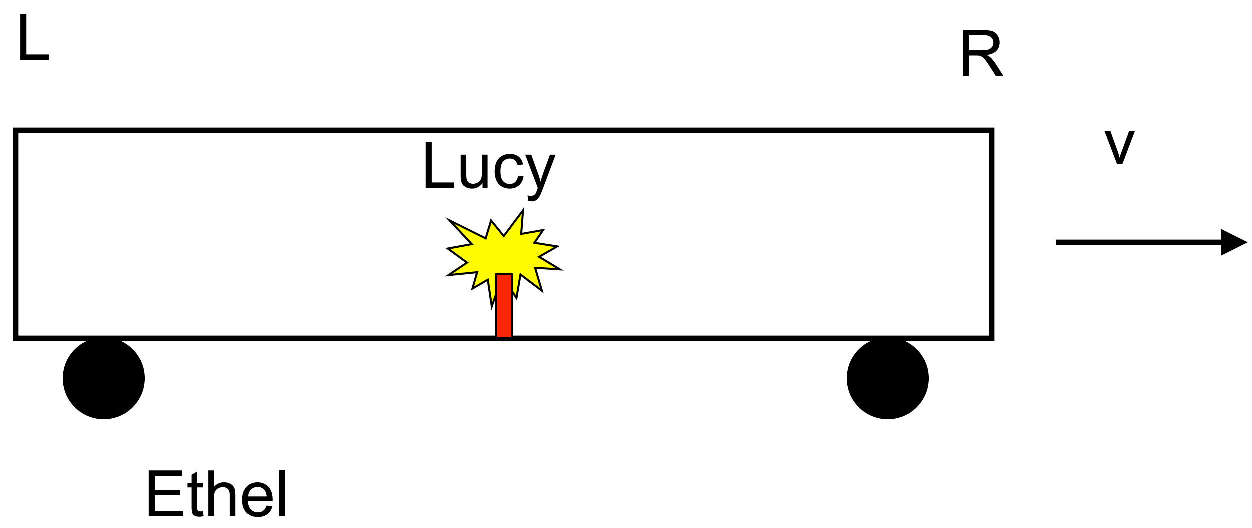
SR5



Sure! After the firecracker explodes, a spherical wave front of light is emitted.
A little while later, it reaches both ends of the car.

Sometime *later*, Lucy finds out about it – but that's a different story. The synchronized clocks are all that matter.

CT-SR6

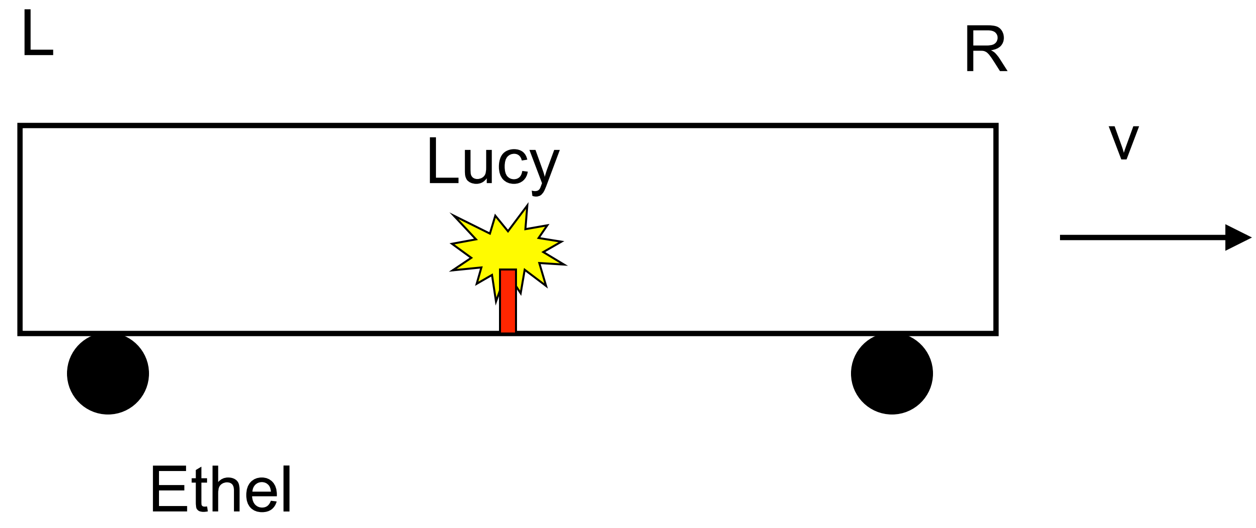


Lucy's friend Ethel is standing still next to the tracks, watching the train move to the right. According to Ethel, which end of the train car does the light reach first?

a) both ends at once
b) the left end, L
c) the right end, R

(That was a concept question –
did you decide on an answer?)

CT-SR6

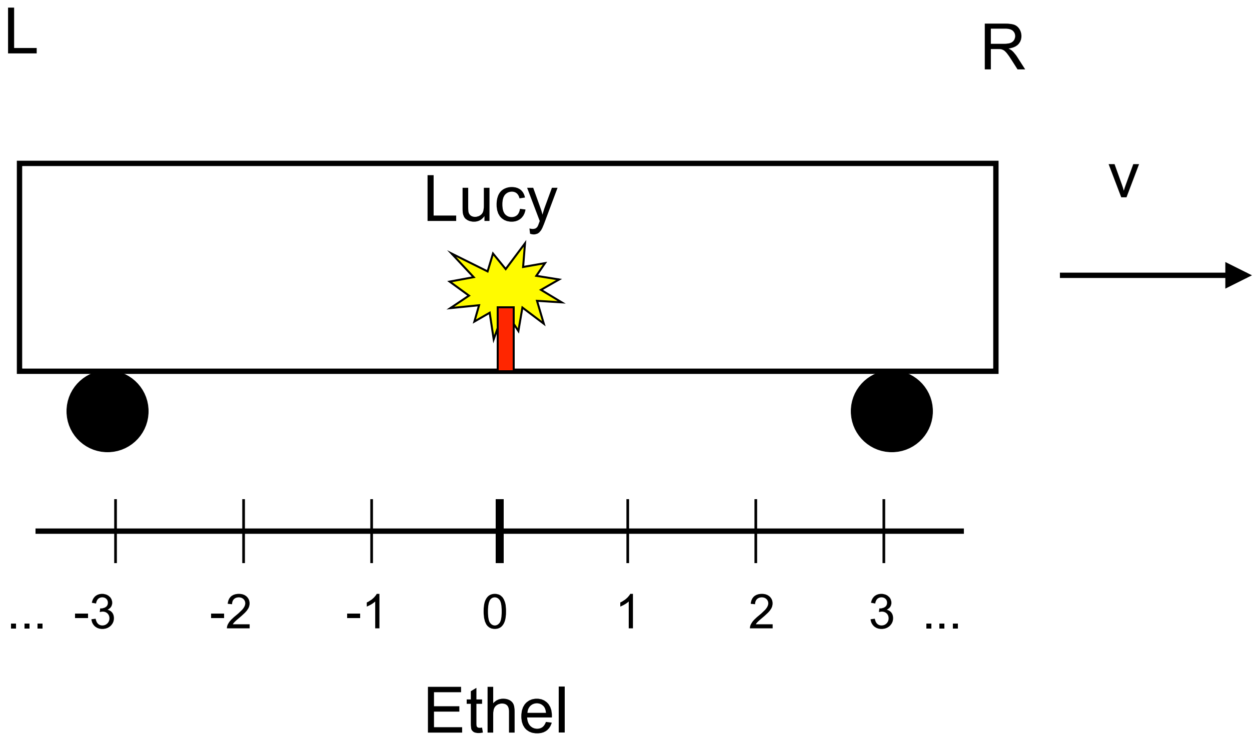


Lucy's friend Ethel is standing still next to the tracks, watching the train move to the right. According to Ethel, which end of the train car does the light reach first?

a) both ends at once
b) the left end, L
c) the right end, R

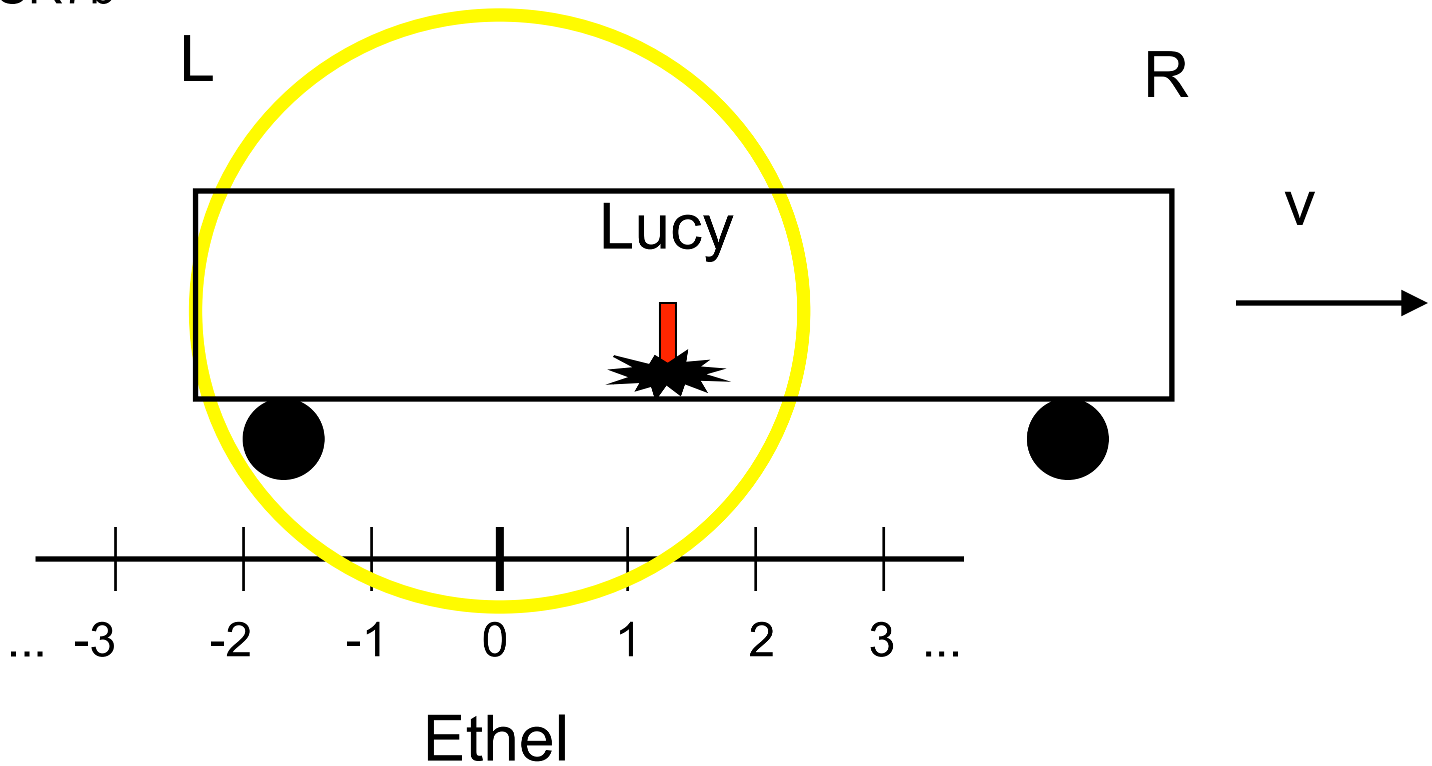
In Ethel's frame, these events are *not* simultaneous.

SR7a



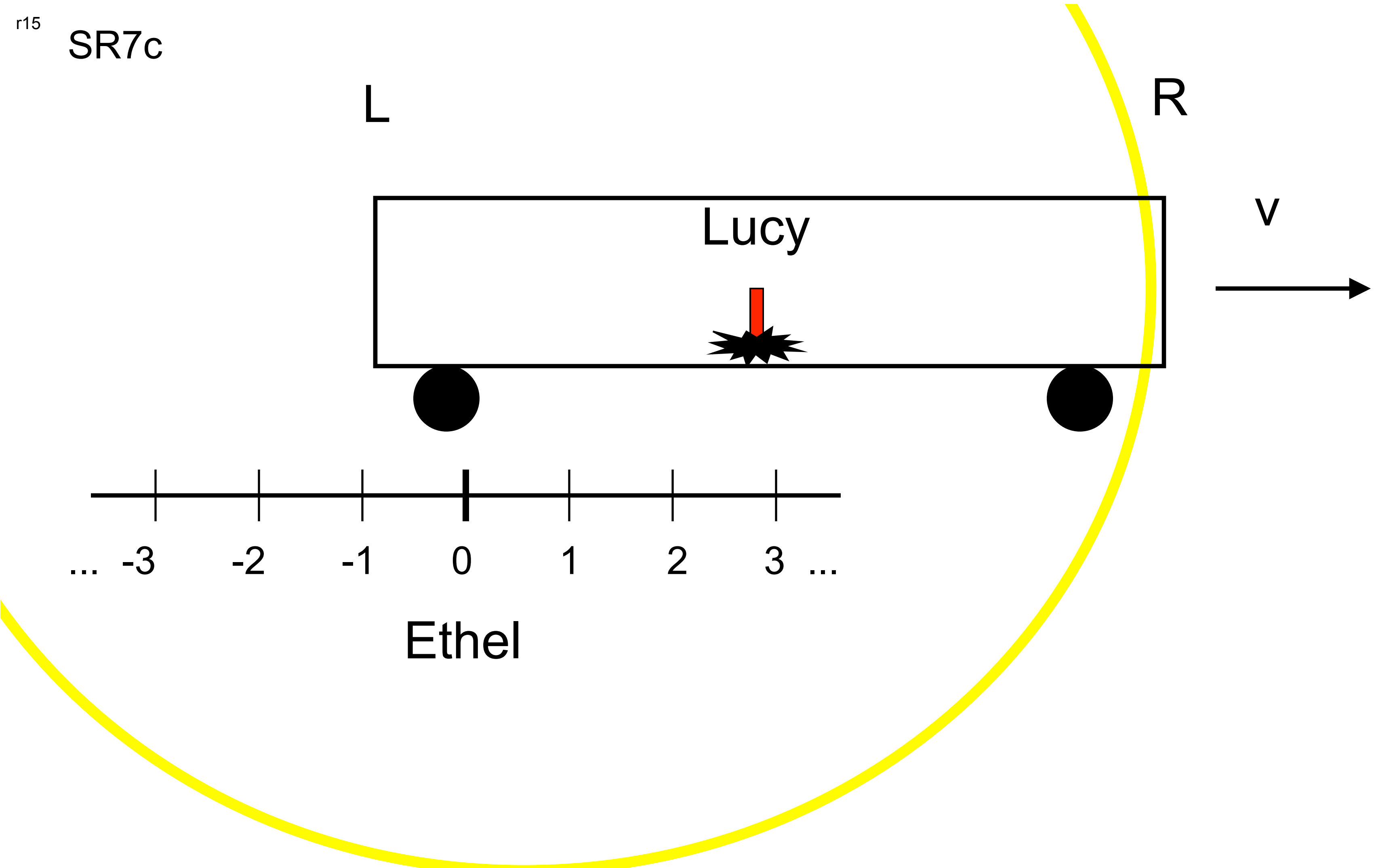
Suppose Lucy’s firecracker explodes at the origin of Ethel’s reference frame.

r14 SR7b



The light spreads out in Ethel’s frame from the point she saw it explode. Because the train car is moving, the light in Ethel’s frame arrives at the left end first.

r15 SR7c



Sometime later, in Ethel’s frame, the light catches up to the right end of the train (the light is going faster than the train).

SR8

An important conclusion

Given two events located at different positions:

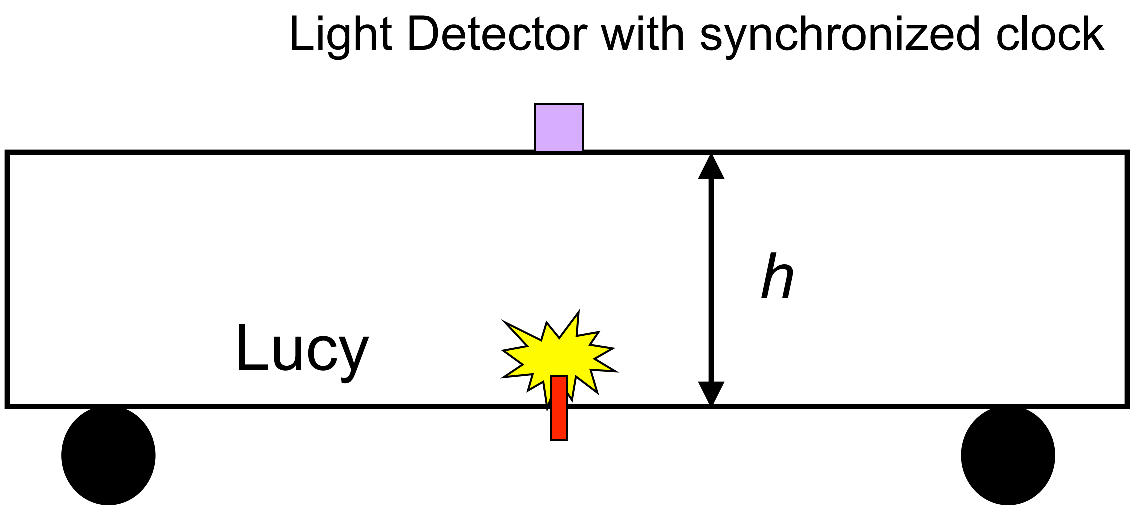
1) light hits the right end of the train car
2) light hits the left end of the train car

Lucy finds that the events are simultaneous.

Ethel (in a different reference frame) finds that they are *not* simultaneous.

And they're both right!

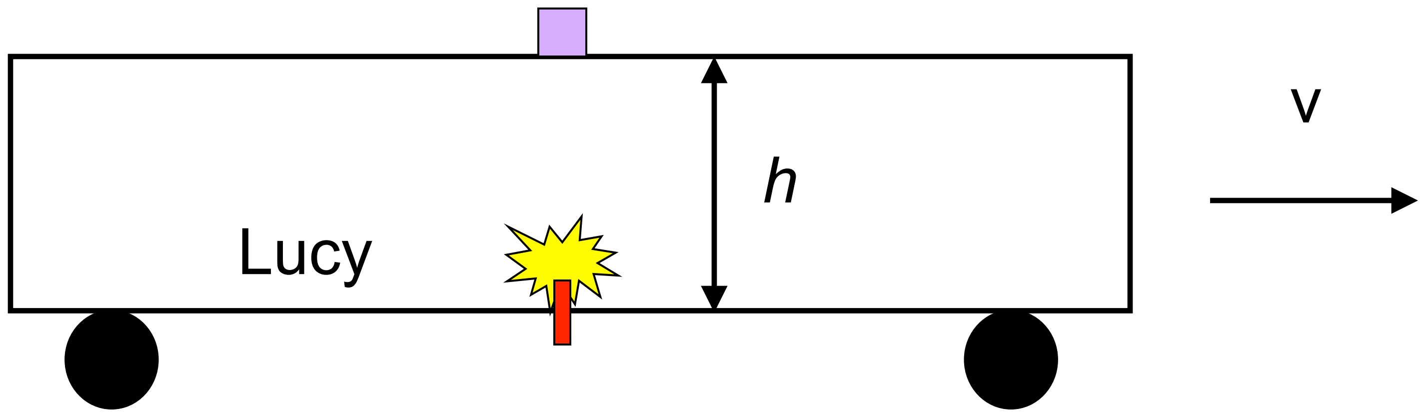
SR9



Event 1 – firecracker explodes
Event 2 – light reaches detector

In Lucy's frame, these events are distance h apart.

CT-SR10



Ethel

Now Ethel stands by the tracks and watches the train whiz by at speed v.

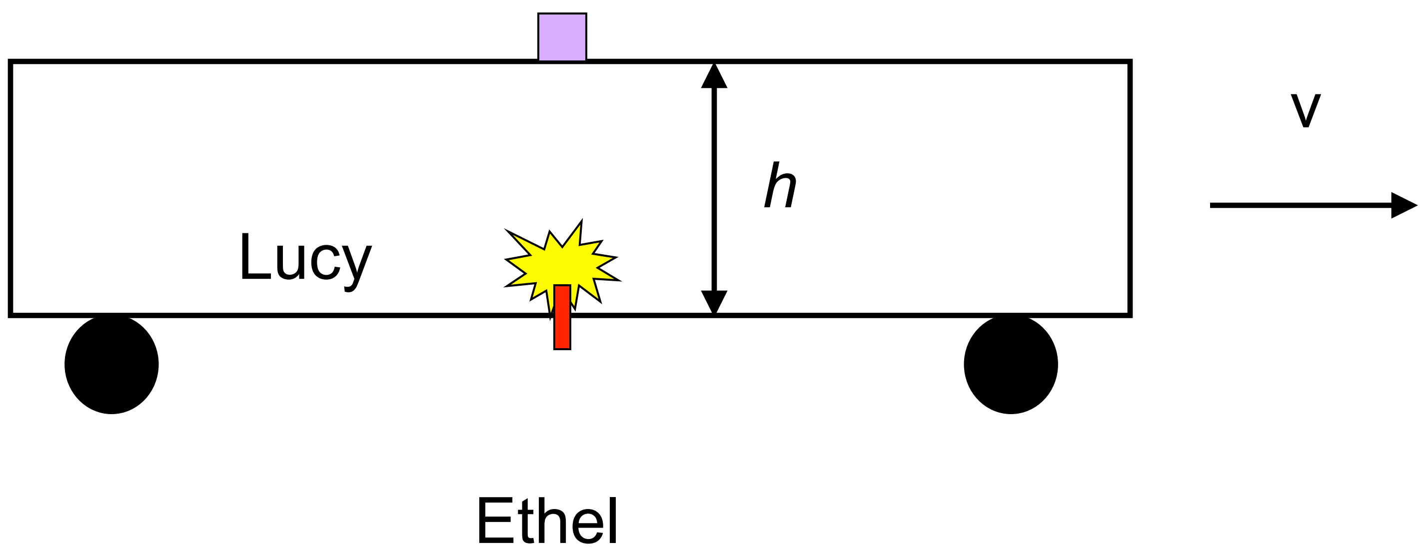
Event 1 – firecracker explodes

Event 2 – light reaches detector

In Ethel's frame, the distance between the two events is

a) Greater than in Lucy's frame
b) Less than in Lucy's frame
c) The same as in Lucy's frame

CT-SR10



Now Ethel stands by the tracks and watches the train whiz by at speed v.

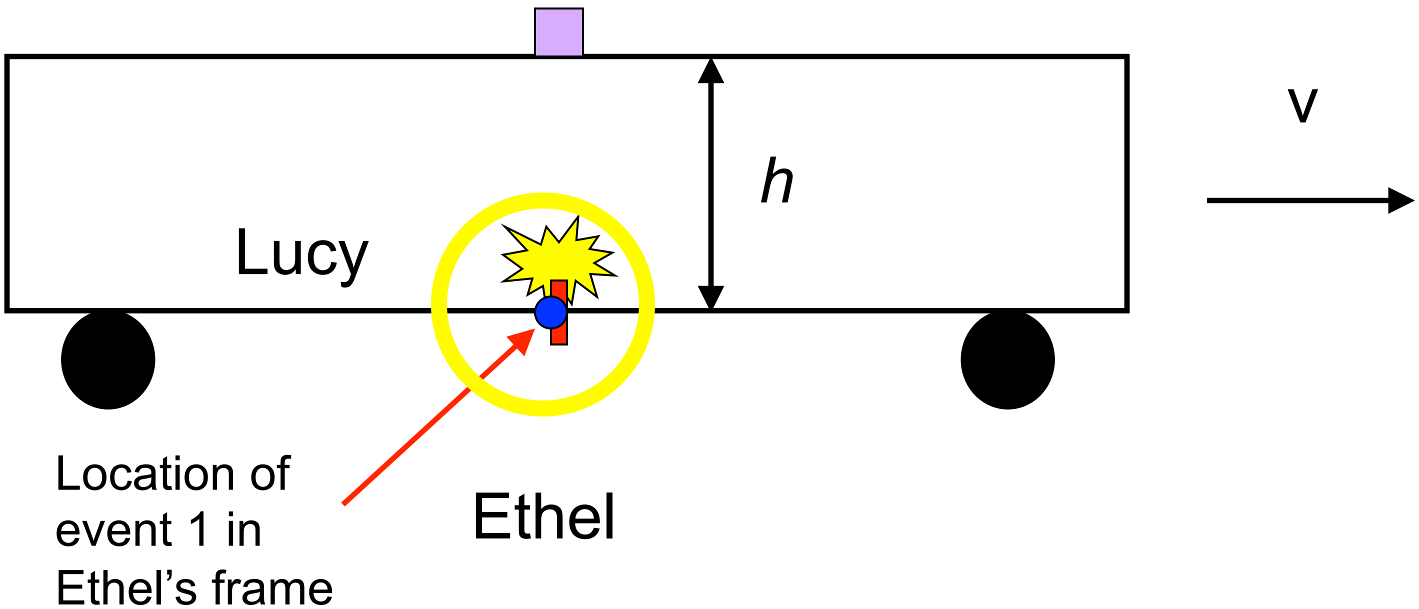
Event 1 – firecracker explodes

Event 2 – light reaches detector

In Ethel's frame, the distance between the two events is

a) Greater than in Lucy's frame
b) Less than in Lucy's frame
c) The same as in Lucy's frame

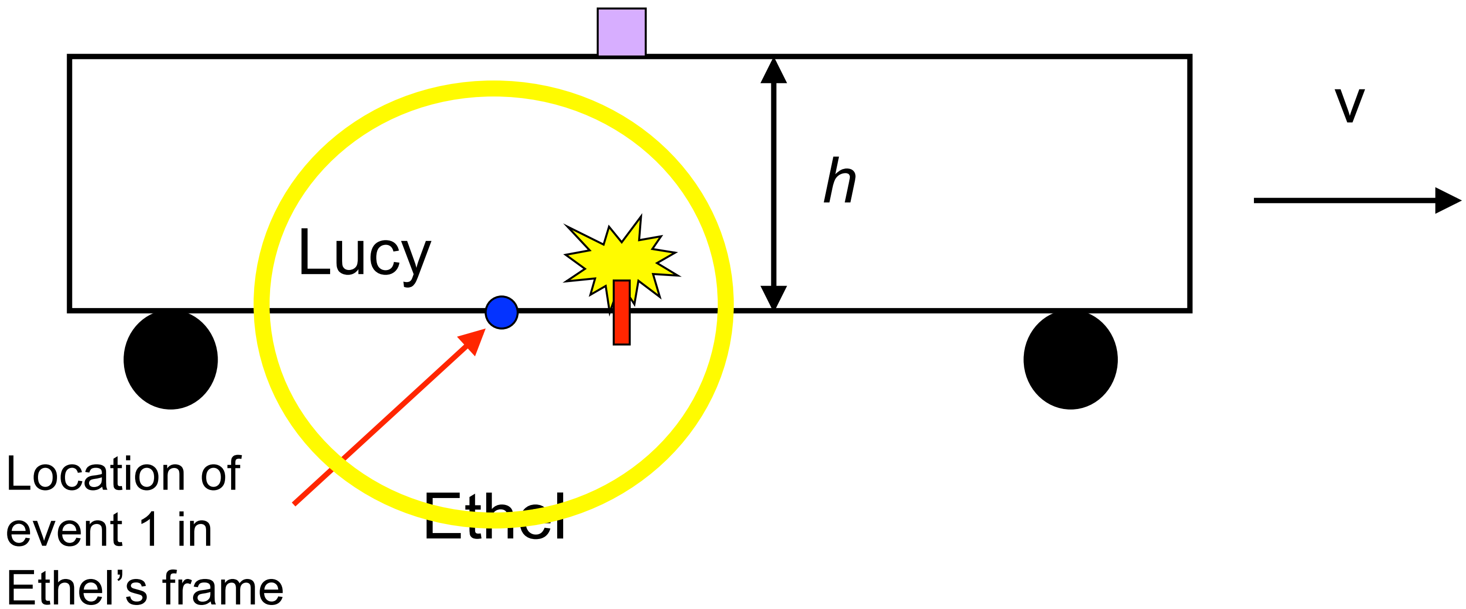
SR11a



Sure! These events happen at *different x coordinates* in Ethels' frame.

Event 1 – firecracker explodes

SR11b

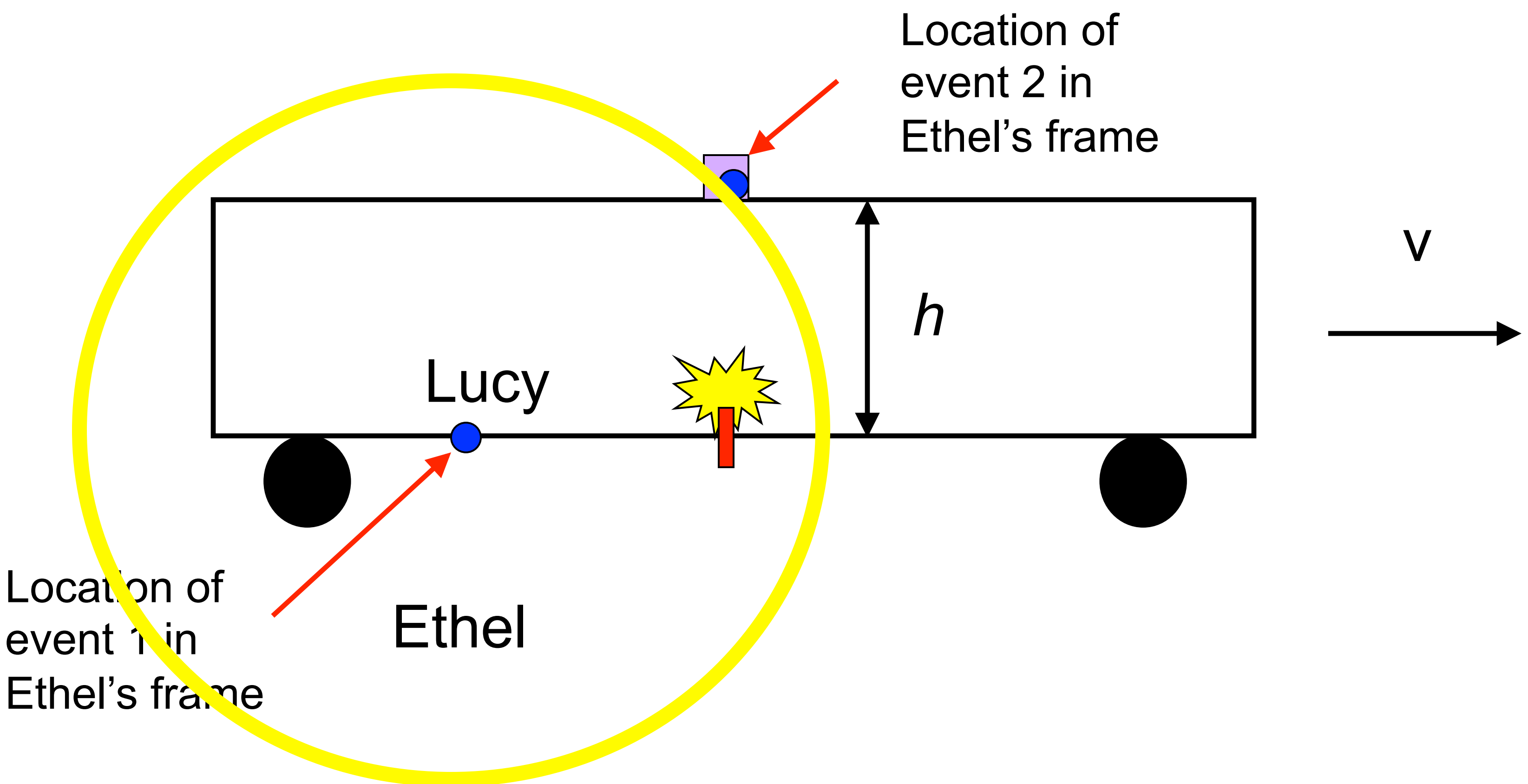


Sure! These events happen at *different x coordinates* in Ethels' frame.

Event 1 – firecracker explodes

r22

SR11c

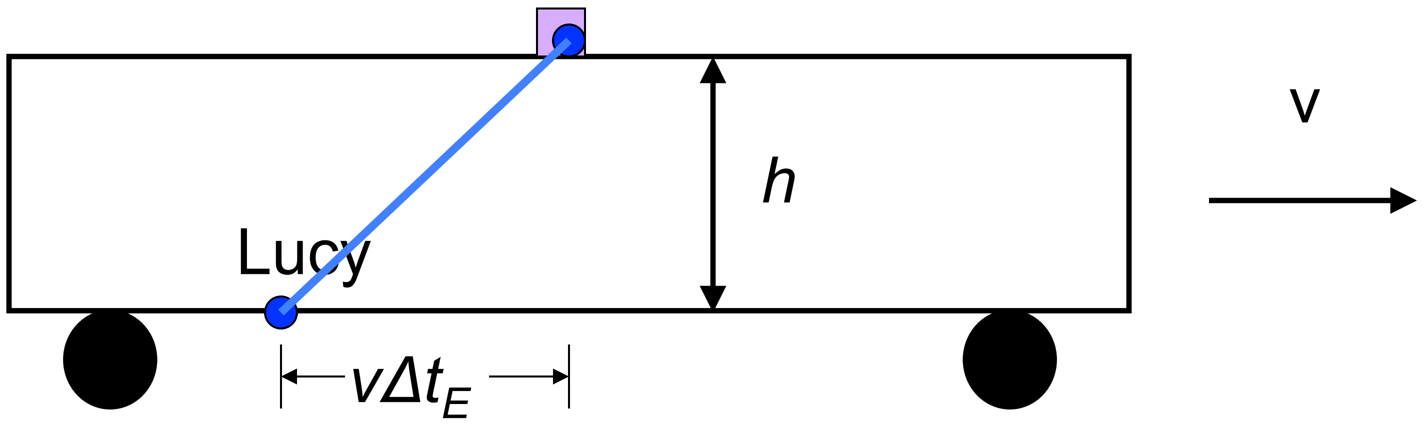


Sure! These events happen at *different x coordinates* in Ethels' frame.

Event 1 – firecracker explodes

Event 2 – light is detected; but the train (and the detector) have moved!

SR12



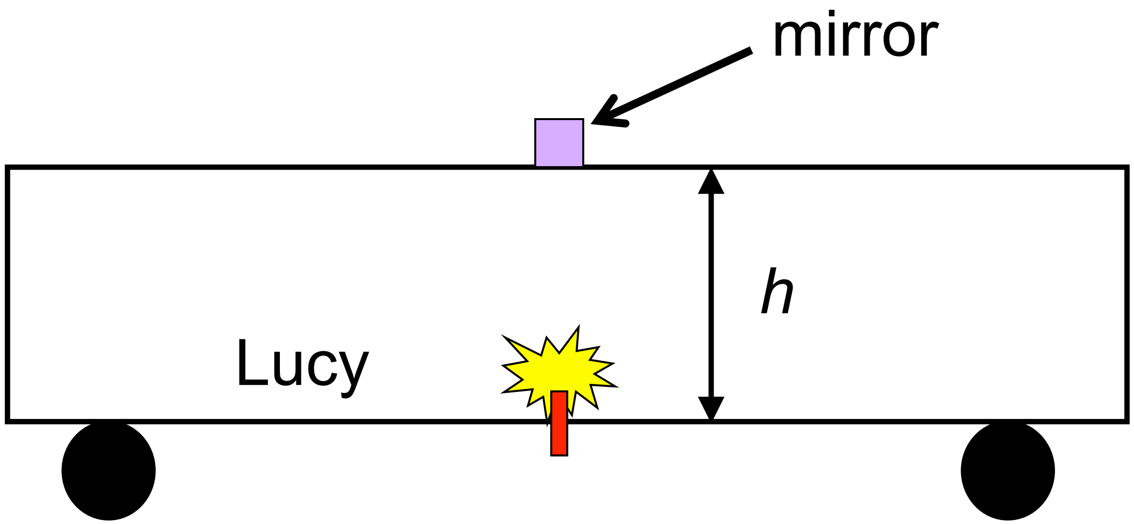
Ethel

If the time between events in Δt_E in Ethel's frame, the train has moved a distance $v\Delta t_E$. The distance between the events, in Ethel's frame, is

$$\sqrt{(v\Delta t_E)^2 + h^2}$$

Good old Pythagoras!

CT-SR13

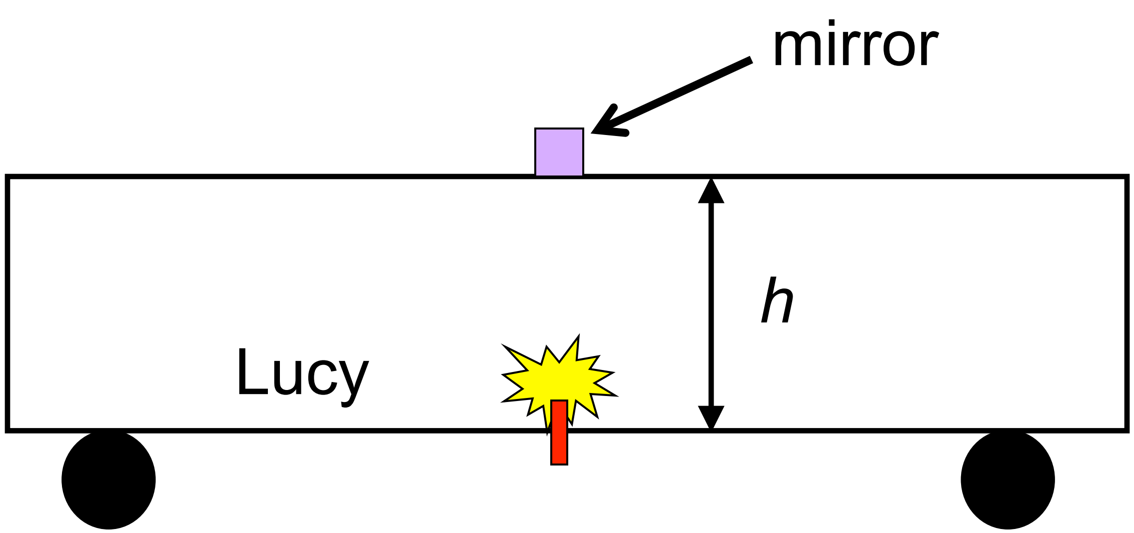


Event 1 – firecracker explodes
Event 2 – light reaches the mirror
Event 3 – light returns to Lucy
In Lucy's frame, how much time elapses between Event 1 and Event 3?

a) h/c b) c/h c) $2h/c$ d) $h/2c$

Are you still trying to figure out the concept test answers before moving on to the next slide?!

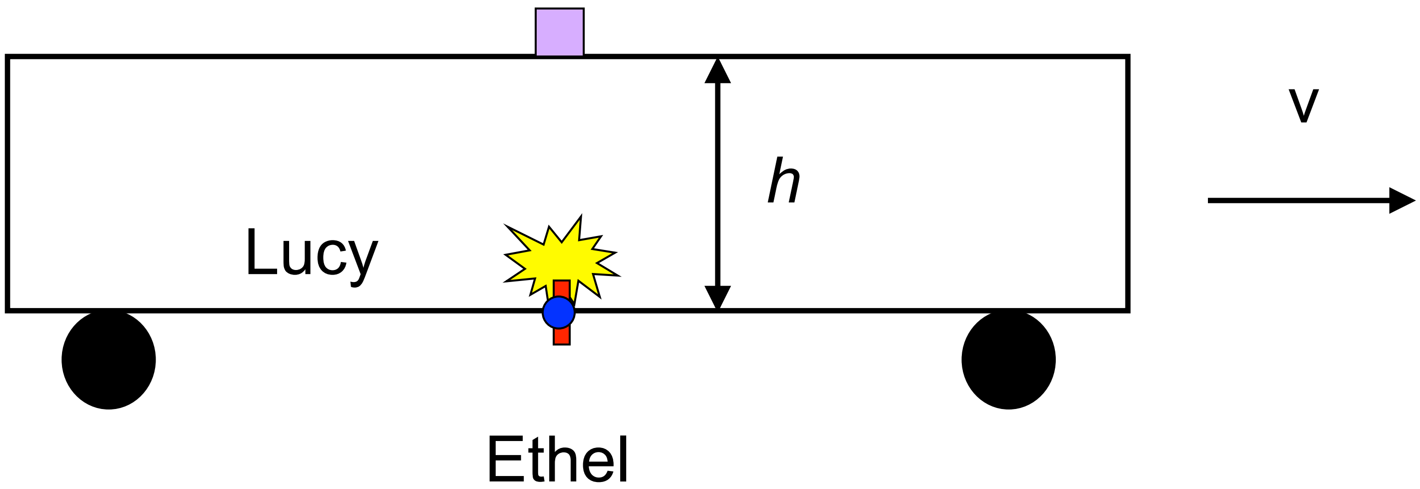
CT-SR13



Event 1 – firecracker explodes
Event 2 – light reaches the mirror
Event 3 – light returns to Lucy
In Lucy's frame, how much time elapses between Event 1 and Event 3?

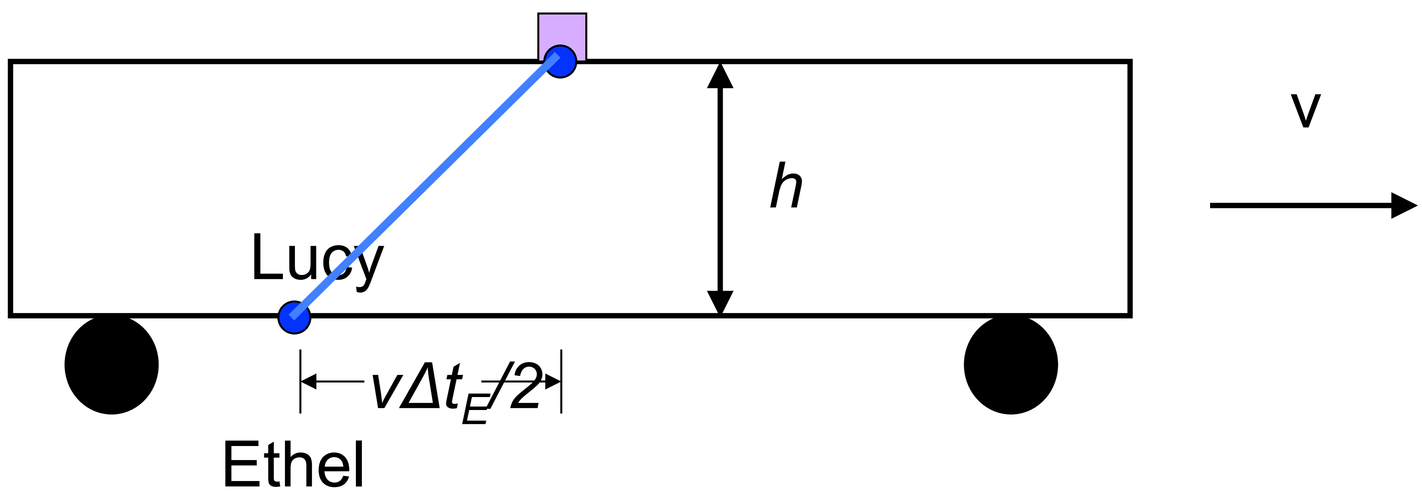
a) h/c b) c/h c) $2h/c$ d) $h/2c$

SR14a



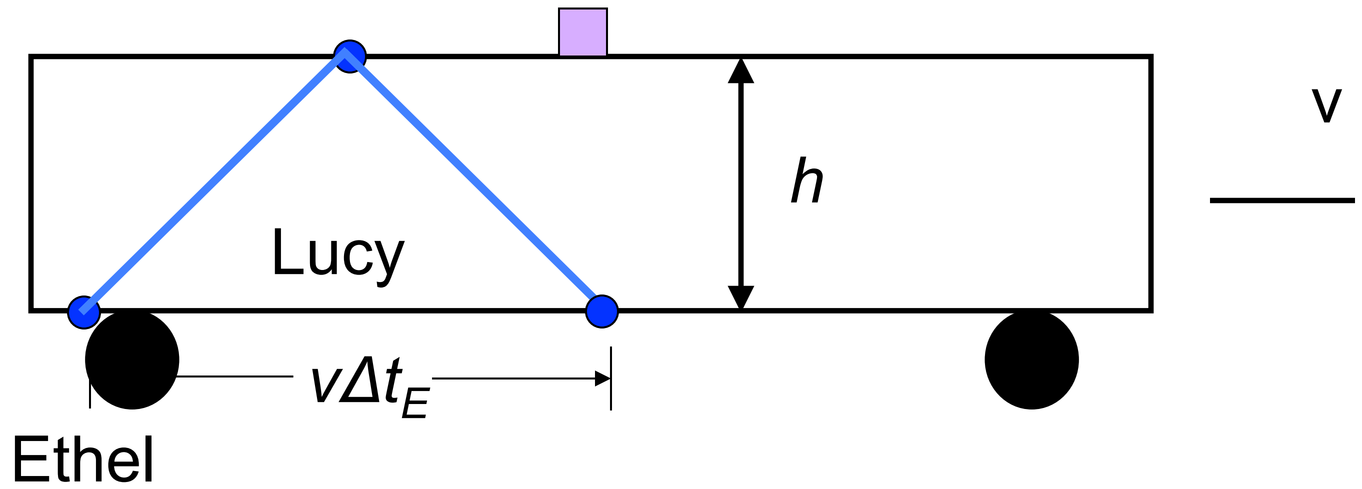
Event 1 – firecracker explodes

SR14b



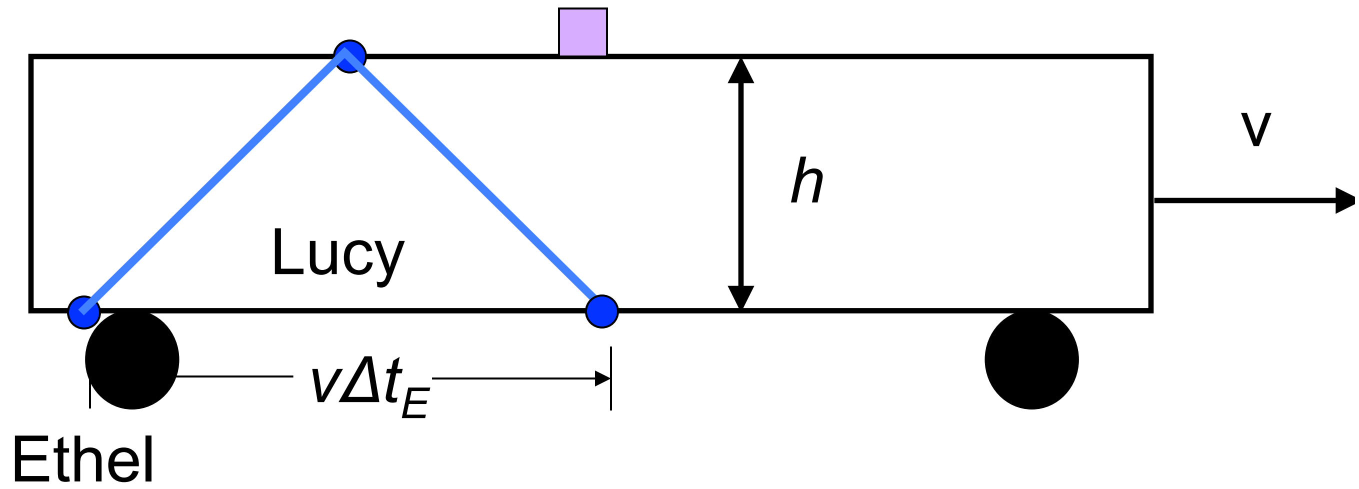
Event 1 – firecracker explodes
Event 2 – light reaches the mirror

SR14c



Event 1 – firecracker explodes
Event 2 – light reaches the mirror
Event 3 – light returns to Lucy

CT-SR15

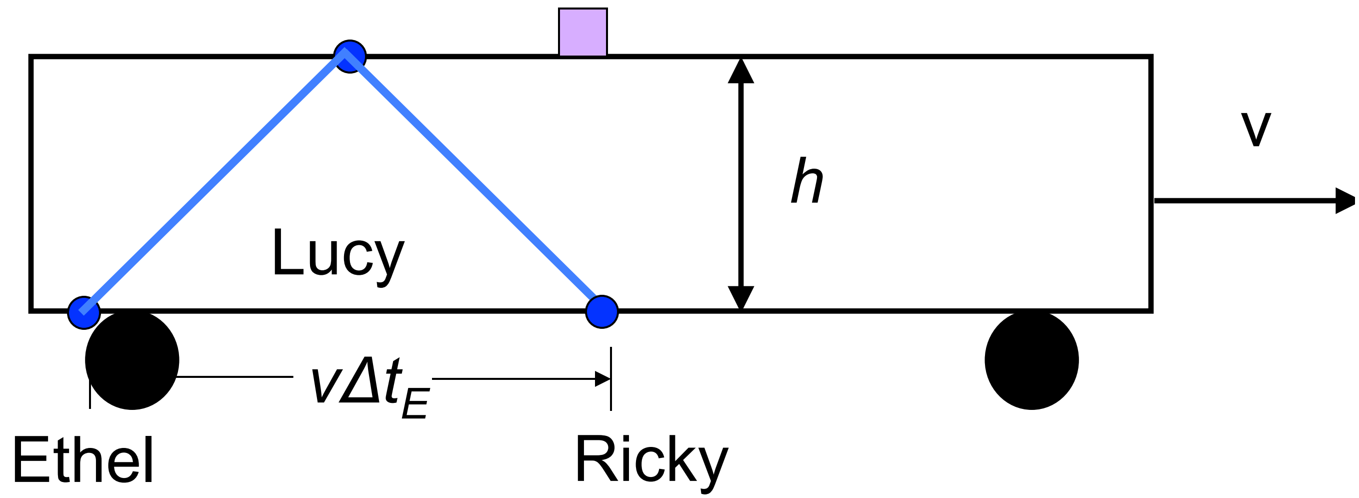


Event 1 – firecracker explodes
Event 2 – light reaches the mirror
Event 3 – light returns to Lucy

In Ethel's frame, how many clocks are required to determine the time between Event 1 and Event 3?

A) 0 B) 1 C) 2 D) 3 E) none of these

CT-SR15

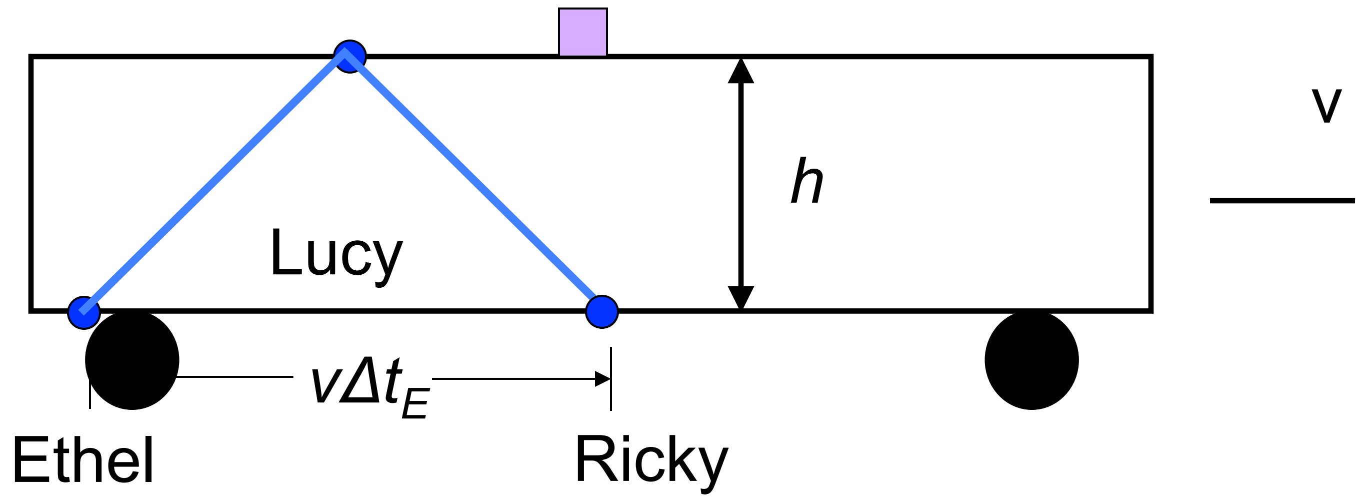


Event 1 – firecracker explodes
Event 2 – light reaches the mirror
Event 3 – light returns to Lucy

In Ethel's frame, how many clocks are required to determine the time between Event 1 and Event 3?

A) 0 B) 1 C) 2 D) 3 E) none of these

SR16



If the time between events in Δt_E in Ethel's frame, the train has moved a distance $v\Delta t_E$. The distance between the events, in Ethel's frame, is

$$2\sqrt{\left(v\,\Delta t_E/2\right)^2 + h^2}$$

Good old Pythagoras!

Connecting the two frames

In Ethel's frame,
distance between events =(speed of light) X (time between these events)

$$2\sqrt{\left(v\,\Delta t_E/2\right)^2+h^2}=c\Delta t_E$$

Algebra

$$\Delta t_E=\frac{2h}{c}\frac{1}{\sqrt{1-v^2/c^2}}=\Delta t_L\,\frac{1}{\sqrt{1-v^2/c^2}}$$

Recall: $2h = c\Delta t_L$ is the distance between the events in Lucy's frame.

If you just glazed over on that last slide…
Do that algebra!

CT-SR18

“Standard” form

Time between events (Ethel) = γ X time between events (Lucy)

$$\Delta t_E = \gamma \Delta t_L \qquad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

According to Ethel, the time between the events is

a) Greater than
b) Less than

the time between events according to Lucy.

CT-SR18

"Standard" form

Time between events (Ethel) = γ X time between events (Lucy)

$$\Delta t_E = \gamma \Delta t_L \qquad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

According to Ethel, the time between the events is

a) Greater than
b) Less than

the time between events according to Lucy.

This is true no matter how fast their relative speed is.

CT-SR19

General question: is there something special about these events in Lucy's frame?

a) No b) Yes

Be prepared to explain your answer.

CT-SR19

General question: is there something special about these events in Lucy's frame?

a) No b) Yes

Be prepared to explain your answer.

Answer: Yes! Both events occur at the *same place* in Lucy's frame.

SR20

Proper time

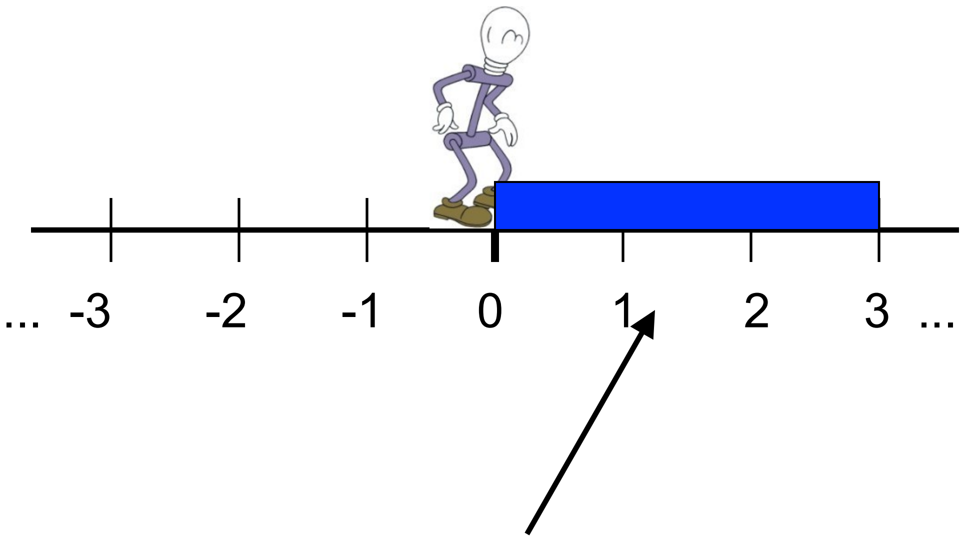
If two events occur at the SAME LOCATION, then the time between them can be MEASURED BY A SINGLE OBSERVER WITH A SINGLE CLOCK (This is the "Lucy time" in our example.) We call the time measured between these types of events the Proper Time, Δt_0

Example: any given clock never moves with respect to itself. It keeps proper time in its own frame.

Any observer moving with respect to this clock sees it run slowly (i.e., time intervals are longer). This is time dilation. $\Delta t = \gamma \Delta t_0$

SR21

Length of an object



This length, measured in the stick's rest frame, is its proper length.

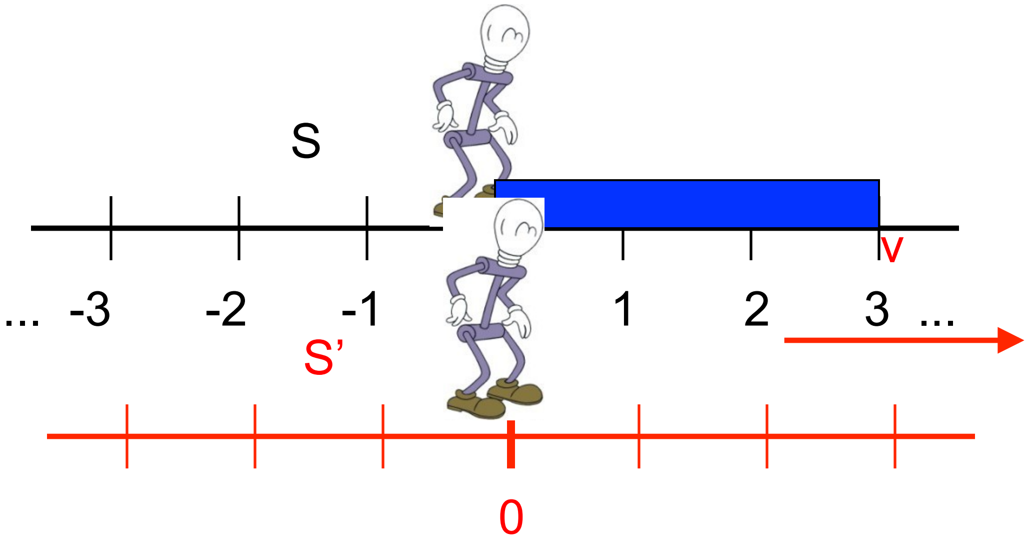
This stick is 3m long. I measure both ends at *the same time* in my frame of reference.

Or not. It doesn't matter, because the stick isn't going anywhere.

But as we know, "at the same time" is relative – it depends on how you're moving.

SR22a

Length of an object



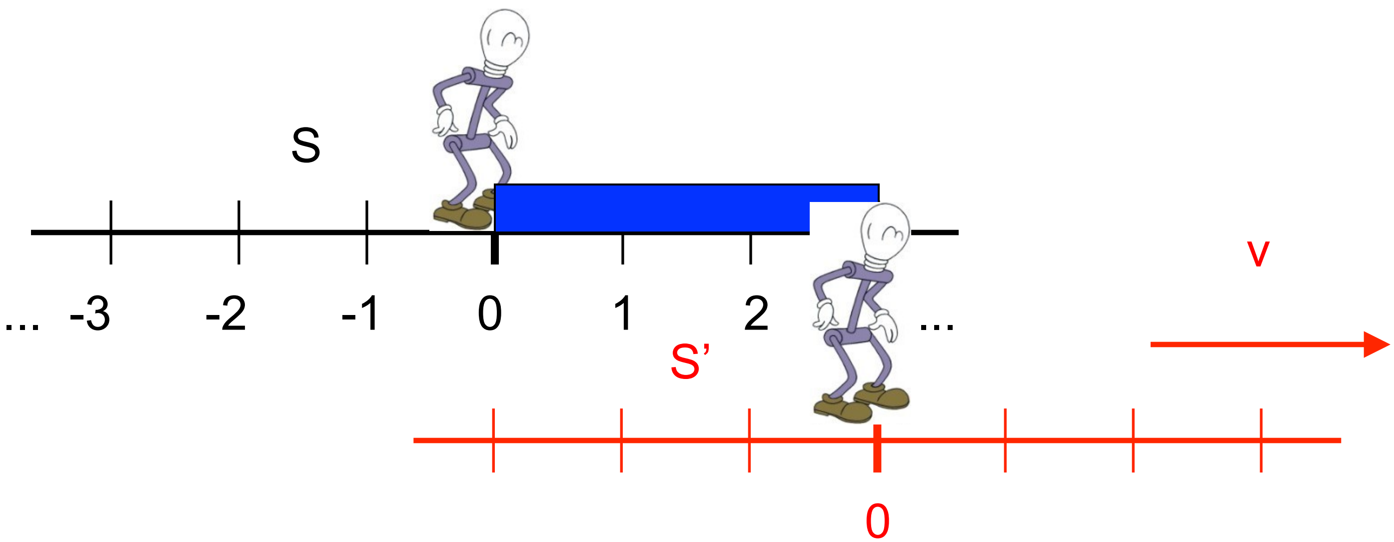
Another observer comes whizzing by at speed v. This observer measures the length of the stick, *and keeps track of time*.

Event 1 – Origin of S' passes left end of stick.

r43

SR22b

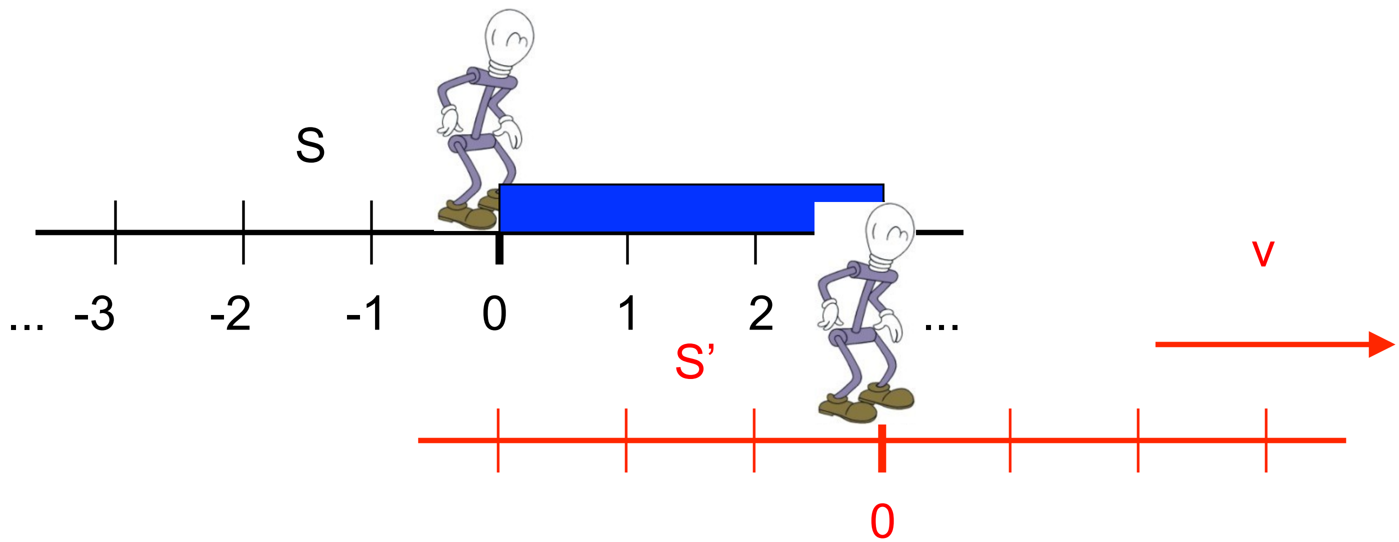
Length of an object



Event 1 – Origin of S' passes left end of stick.
Event 2 – Origin of S' passes right end of stick.

CT-SR23

Length of an object

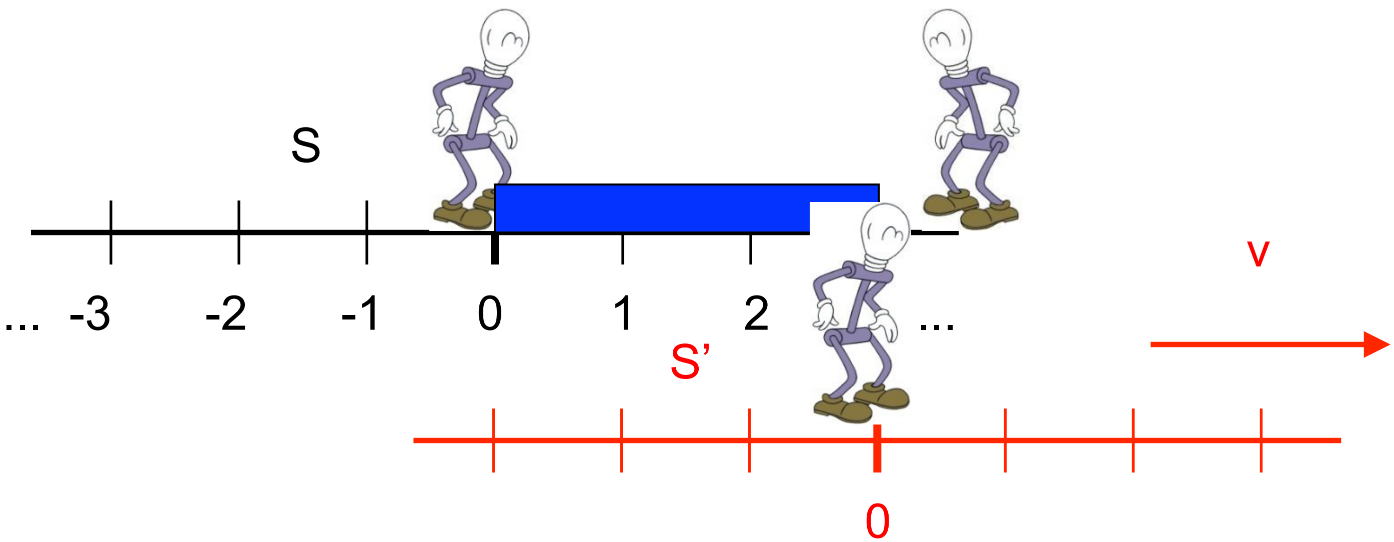


Event 1 – Origin of S' passes left end of stick.
Event 2 – Origin of S' passes right end of stick.

How many observers are needed in S to measure the time between events? A) 0 B) 1 C) 2 D) Something else

CT-SR24

Length of an object

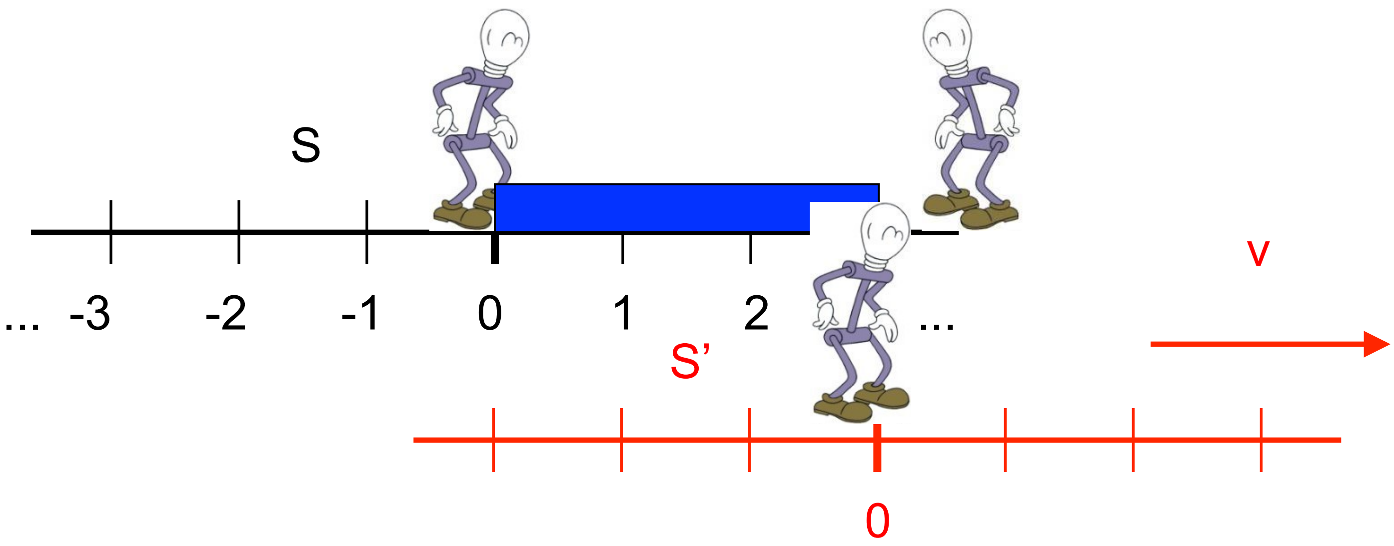


Event 1 – Origin of S' passes left end of stick.
Event 2 – Origin of S' passes right end of stick.

Which frame measures the Proper Time between the events? A) S B) S' C) neither

CT-SR24

Length of an object



Event 1 – Origin of S' passes left end of stick.
Event 2 – Origin of S' passes right end of stick.

Which frame measures the Proper Time between the events? A) S B) S' C) neither

CT-SR25

Connecting the measurements

In frame S:

length of stick = L (this is the proper length)

time between measurements = Δt

speed of frame S' is $v = L/\Delta t$

In frame S':

length of stick = L' (this is what we're looking for)

time between measurements = $\Delta t'$

speed of frame S is $v = L'/\Delta t'$

Q: a) $\Delta t = \gamma \Delta t'$ or b) $\Delta t' = \gamma \Delta t$

r49

CT-SR25

Connecting the measurements

In frame S:

length of stick = L (this is the proper length)

time between measurements = Δt

speed of frame S' is $v = L/\Delta t$

In frame S':

length of stick = L' (this is what we're looking for)

time between measurements = $\Delta t'$

speed of frame S is $v = L'/\Delta t'$

Q: a) $\Delta t = \gamma\Delta t'$ or b) $\Delta t' = \gamma\Delta t$

Follow the proper time!

r50

SR26

Now to the lengths measured…

Speeds are the same (both refer to the relative speed).

And so

$$v = \frac{L}{\Delta t} = \frac{L'}{\Delta t'} = \frac{\gamma L'}{\Delta t}$$

$$L' = \frac{L}{\gamma}$$

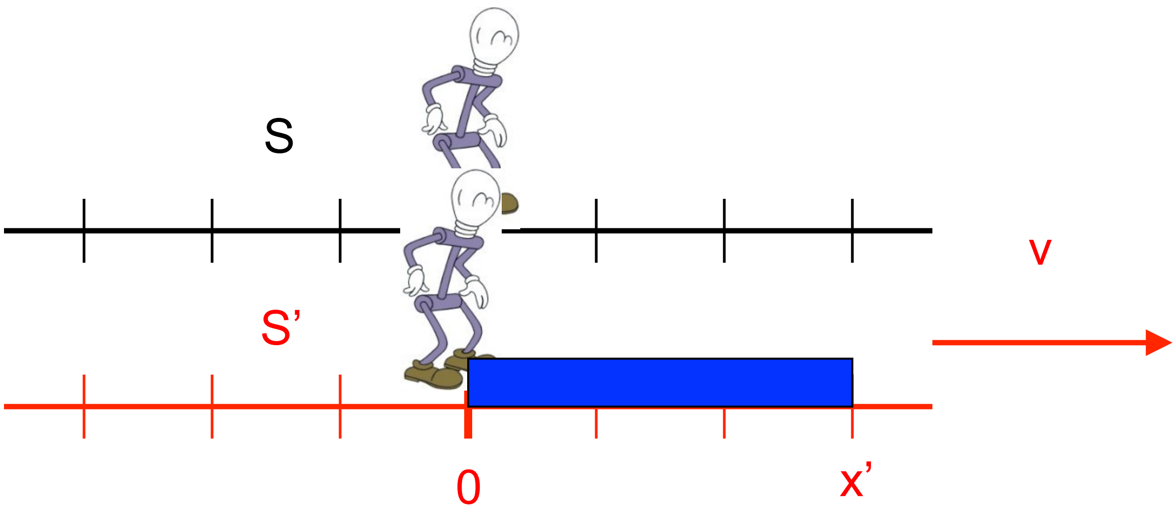
Length measured in moving frame

Length in stick's rest frame (proper length)

Length contraction is a consequence of time dilation (and vice-versa). This is also known as Lorentz Contraction

SR27

The Lorentz transformation



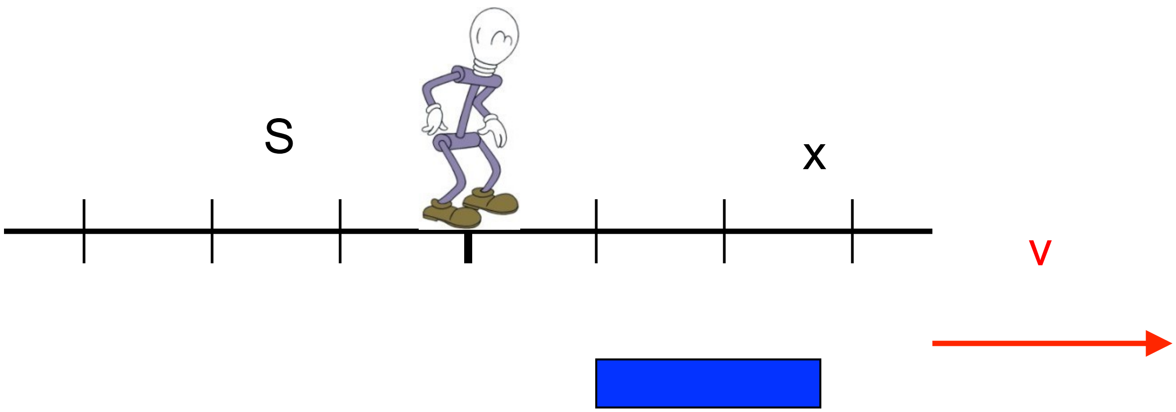
A stick is at rest in S'. Its endpoints are the events (position, c*time) = (0,0) and (x',0) in S'.
S' is moving to the right with respect to frame S.

Event 1 – left of stick passes origin of S. Its coordinates are (0,0) in S and (0,0) in S'.

r52

CT-SR28

The Lorentz transformation



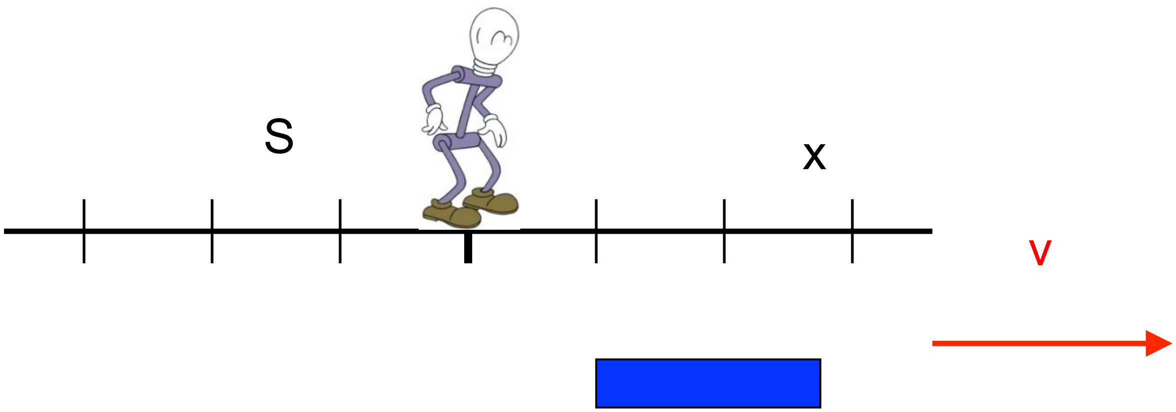
As viewed from S, the stick's length is x'/γ. Time t passes. According to S, where is the *right* end of the stick?

a) $x = vt$
b) $x = -vt$
c) $x = vt + x'/\gamma$
d) $x = -vt + x'/\gamma$
e) $x = vt - x'/\gamma$

r53

CT-SR28

The Lorentz transformation



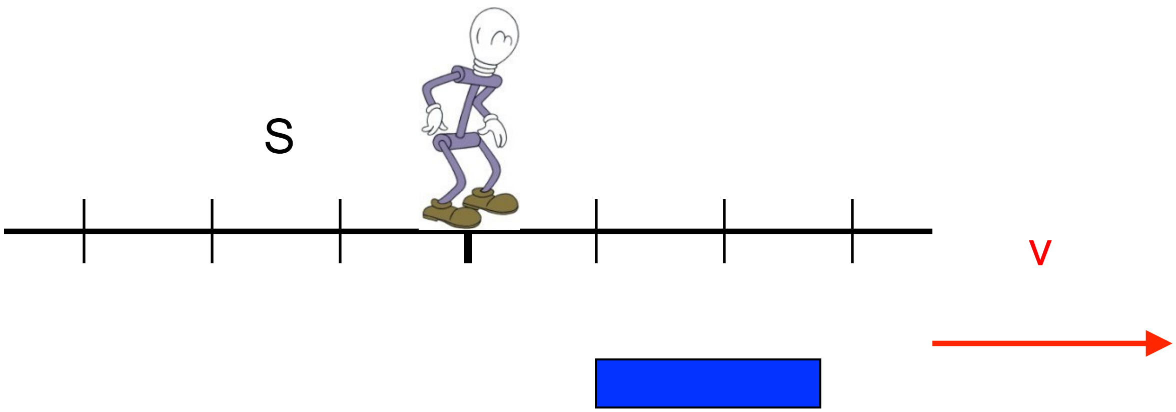
As viewed from S, the stick's length is x'/γ. Time t passes. According to S, where is the *right* end of the stick?

a) $x = vt$ b) $x = -vt$ c) $x = vt + x'/\gamma$

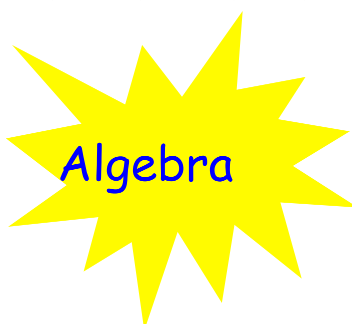
d) $x = -vt + x'/\gamma$ e) $x = vt - x'/\gamma$

SR29

The Lorentz transformation



$x = vt + x'/\gamma$. This relates the coordinates of an event in one frame to its coordinates in the other.



$$x' = \gamma(x - vt)$$

Transformations – summary!

If S' is moving with speed v in the positive x direction relative to S, then the coordinates of the same event in the two frames is related by:

In Galilean relativity

$$x' = x - vt$$
$$y' = y$$
$$z' = z$$
$$t' = t$$

In Special relativity

$$x' = \gamma(x - vt)$$
$$y' = y$$
$$z' = z$$
$$t' = \gamma(t - \frac{v}{c^2}x)$$

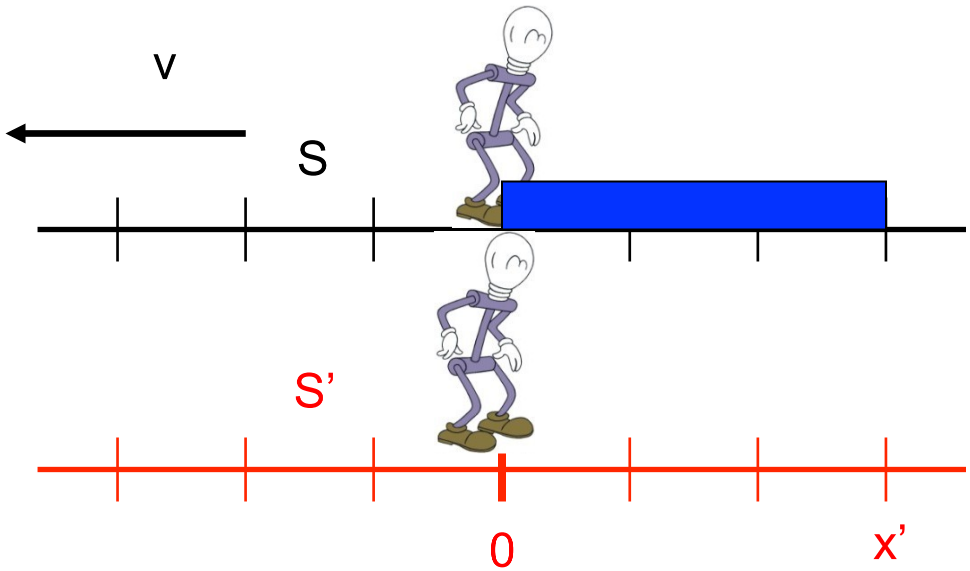
In a minute…

Remark: this assumes (0,0) is the same event in both frames.

r56

SR30

The Lorentz transformation 2

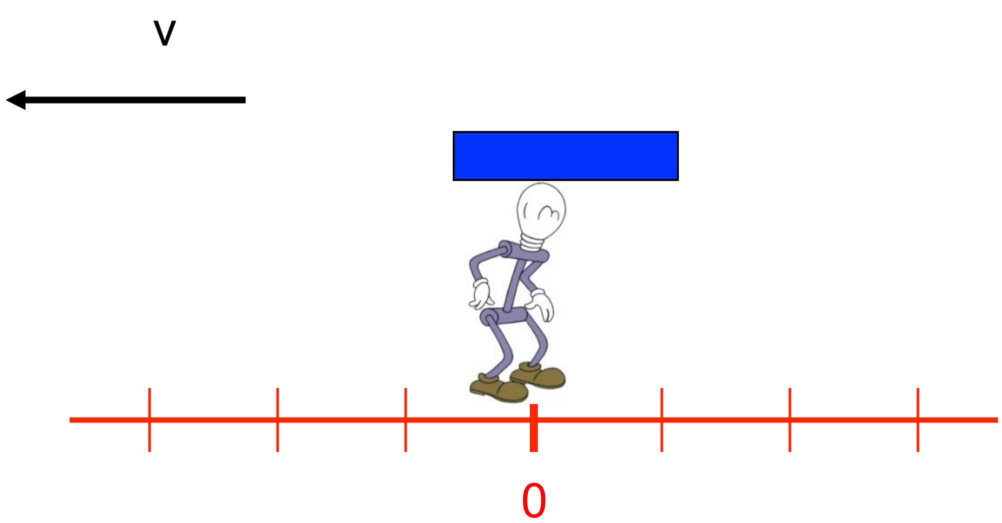


A stick is at rest in S. Its endpoints are the events (position, c*time) = (0,0) and (x,0) in S.
S is moving to the left with respect to frame S'.

Event 1 – left of stick passes origin of S'. Its coordinates are (0,0) in S and (0,0) in S'.

CT-SR31

The Lorentz transformation 2



As viewed from S', the stick's length is x/γ. Time t' passes. According to S', where is the *right* end of the stick?

a) $x' = vt'$ b) $x' = -vt'$ c) $x' = vt' + x/\gamma$

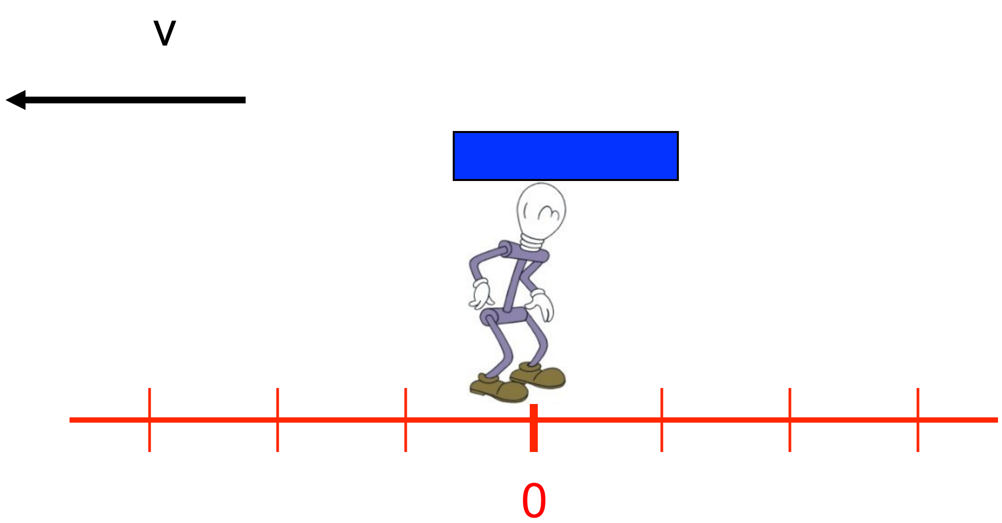
d) $x' = -vt' + x/\gamma$ e) $x' = vt' - x/\gamma$

You still with me? Did you work out that previous question?

r59

CT-SR31

The Lorentz transformation 2



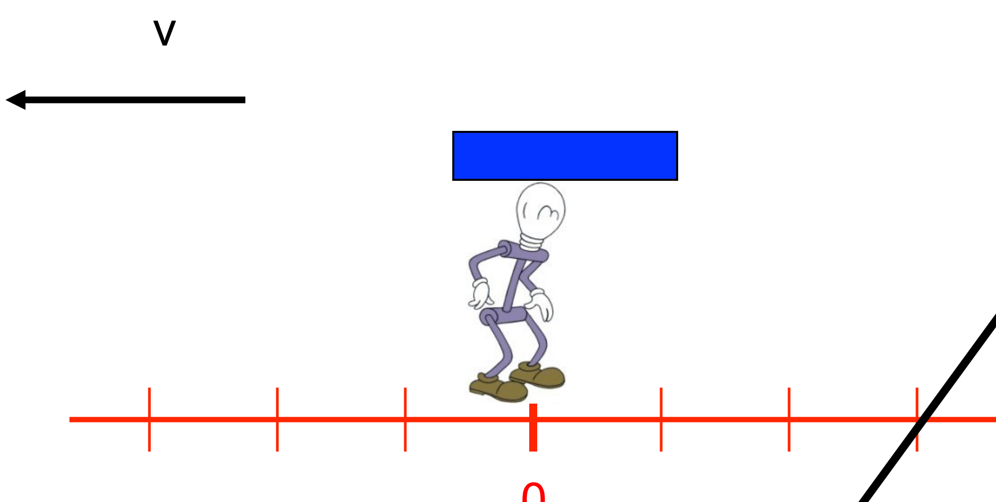
As viewed from S', the stick's length is x/γ. Time t' passes. According to S', where is the *right* end of the stick?

a) $x' = vt'$ b) $x' = -vt'$ c) $x' = vt' + x/\gamma$
d) $x' = -vt' + x/\gamma$ e) $x' = vt' - x/\gamma$

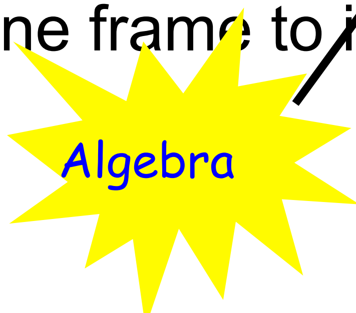
r60

SR32

The Lorentz transformation 2



x’ = -vt’ + x/γ. This relates the coordinates of an event in one frame to its coordinates in the other.



$$t' = \frac{x}{\gamma v} - \frac{x'}{v}$$

$$= \frac{x}{\gamma v} - \frac{\gamma(x - vt)}{v}$$

$$t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

Transformations – summary (again!)

If S’ is moving with speed v in the positive x direction relative to S, then the coordinates of the same event in the two frames is related by:

In Galilean relativity

$$x' = x - vt$$
$$y' = y$$
$$z' = z$$
$$t' = t$$

In Special relativity

$$x' = \gamma(x - vt) \qquad x = \gamma(x' + vt')$$
$$y' = y \qquad y = y'$$
$$z' = z \qquad z = z'$$
$$t' = \gamma(t - \frac{v}{c^2}x) \qquad t = \gamma(t' + \frac{v}{c^2}x')$$

Remark: this assumes (0,0) is the same event in both frames and of course motion is in x direction.

Transformations – summary (again!!)

We now have the tools to compare positions and times in different inertial reference frames. NOW we can talk about how velocities, etc. compare.:

In Galilean relativity

$$x' = x - vt$$
$$y' = y$$
$$z' = z$$
$$t' = t$$

In Special relativity

$$x' = \gamma(x - vt) \qquad x = \gamma(x' + vt')$$
$$y' = y \qquad y = y'$$
$$z' = z \qquad z = z'$$
$$t' = \gamma(t - \frac{v}{c^2}x) \qquad t = \gamma(t' + \frac{v}{c^2}x')$$

Newton worked with these…

but needs reworking of momentum and energy to work with these!

CT- SR34

To think about:

Can one change the order of events in time by viewing them from a different inertial reference frame?

A. Always
B. Sometimes
C. Never