# Potentials

10.1

Why can't we use a scalar potential to find the magnetic field, as we have done with the electric field, i.e., why can't we use

$$\mathbf{B} = -\nabla V_B(\mathbf{r})$$

A. Because the divergence of **B** is always zero
B. Because only either **E** or **B** can be described with a scalar potential, not both
C. Because **B** can have a non-zero curl
D. I don't know/remember
E. None of the above (but I know the right reason!)

If I tell you $\nabla \times \vec{F} = 0$, what can you conclude about **F**?

A) $\vec{F} = 0$

$B)$ $\vec{F} = \nabla f$ (for some f)

$C)$ $\vec{F} = \nabla \cdot \vec{g}$ (for some $\vec{g}$)

$D)$ $\vec{F} = \nabla \times \vec{g}$ (for some $\vec{g}$)

$E)$ Something else!

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If I tell you $\nabla f = 0$, what can you conclude about f?

A) $f = 0$

$B)$ $f = \nabla g$ (for some g)

$C)$ $f = \nabla \cdot \vec{g}$ (for some $\vec{g}$)

$D)$ $f = \nabla \times \vec{g}$ (for some $\vec{g}$)

$E)$ Something else!

$$\vec{B} = \nabla \times \vec{A}$$

$$\vec{E} = -\nabla V - \partial \vec{A} / \partial t$$

$$\vec{B} = \nabla \times \vec{A}_{old}$$

$$\vec{E} = -\nabla V_{old} - \partial \vec{A}_{old} / \partial t$$

If I change gauge, so $\vec{A}_{new} = \vec{A}_{old} + \nabla f$
I claim B is unaffected. But, what about E?

A) Looks like E is also unaffected
B) Looks like we changed E, but that's ok
C) Looks like we changed E, that doesn't seem acceptable
D) What are we doing?

$$\vec{A}_{new} = \vec{A}_{old} + \nabla f$$

$$V_{new} = V_{old} - \partial f / \partial t$$

Gauge transformation

$$\vec{B} = \nabla \times \vec{A}$$

$$\vec{E} = -\nabla V - \partial \vec{A} / \partial t$$

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The Lorentz gauge is defined by: $\nabla \cdot \vec{A} = -\frac{1}{c^2}\frac{\partial V}{\partial t}$

The main PDEs for the potentials are:

$$-\nabla^2 V + \frac{1}{c^2}\frac{\partial^2 V}{\partial t^2} = \frac{\rho}{\varepsilon_0}$$

$$-\nabla^2 \vec{A} + \frac{1}{c^2}\frac{\partial^2 \vec{A}}{\partial t^2} = \mu_0 \vec{J}$$

The solutions are:

$$V(\vec{r},t) = \frac{1}{4\pi\varepsilon_0}\iiint \frac{\rho(\vec{r}\,',t_R)}{|\vec{r}-\vec{r}\,'|}\,d^3\vec{r}\,' \quad \text{where } t_R = t - \frac{|\vec{r}-\vec{r}\,'|}{c}$$

$$\vec{A}(\vec{r},t) = \frac{\mu_0}{4\pi}\iiint \frac{\vec{J}(\vec{r}\,',t_R)}{|\vec{r}-\vec{r}\,'|}\,d^3\vec{r}\,'$$

How have we been "setting the gauge" so far in this class?

A) Requiring **A** and V go to 0 at infinity
B) Putting a condition on **A** like: $A=0$, or $\nabla\times A=0$
C) Putting a condition on $\nabla\cdot A$
D) Choosing the function "f" in the formula $A' = A+\nabla f$
E) Something else, none of these, MORE than one, not really sure…

In Coulomb's gauge (CG): $$V(\vec{r},t)=\frac{1}{4\pi\varepsilon_0}\iiint\frac{\rho(\vec{r}\,',t)}{|\vec{r}-\vec{r}\,'|}\,d^3\vec{r}\,'$$

In Lorentz' gauge:

$$V(\vec{r},t)=\frac{1}{4\pi\varepsilon_0}\iiint\frac{\rho(\vec{r}\,',t_R)}{|\vec{r}-\vec{r}\,'|}\,d^3\vec{r}\,' \qquad \text{where } t_R=t-\frac{|\vec{r}-\vec{r}\,'|}{c}$$

These look subtly different, which is correct?

A) CG is unphysical and incorrect, it violates relativity
B) CG result is only correct for time independent problems, LG is what you want for time dependent problems
C) They only LOOK different, but in fact they give the same result for V when you work them out
D) Both are equally "correct", it's just a gauge choice. You can use *either* one in *any* situation
E) Something else is going on, none of the above articulates my opinion very well here!

## How do you interpret

$$t_r \equiv t - \frac{|\vec{r} - \vec{r}'|}{c}$$

A) is the actual time of observation at point *r.*
B) is the time light needs to travel from *r*' to *r*.
C) is a time in the future when light emitted from point *r* at time *t* arrives at point *r*' .
D) is a time in the past such that light emitted from point *r*' arrives at *r* at time *t.*
E) None of these.

## The 'retarded time',:

$$t_r \equiv t - \frac{|\vec{r} - \vec{r}'|}{c}$$

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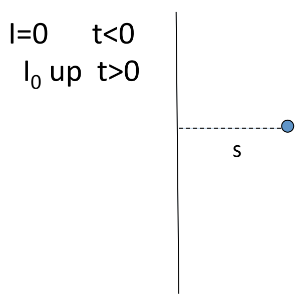
At what time, $t$, does an observer at $s$ first know the current was turned on?




A) t=0
B) t=c s
C) t=s/c
D) Other!!
E) Not sure about this?

At what time $t$, after $s/c$, does an observer at $s$ see current from the entire wire?

A) Immediately
B) Never
C) Something else

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A) Immediately
B) Never
C) Something else

Only see contributions from regions inside the light travel distance: $\sqrt{s^2+z^2} \leq ct$

Or $|z| \leq \sqrt{(ct)^2 - s^2}$

$$I(t) = \begin{cases} 0 & t \leq 0 \\ I_0 & t > 0 \end{cases}$$

$$\mathbf{A}(s,t) = \frac{\mu_0}{4\pi} \int_{-\infty}^{\infty} \frac{\mathbf{I}(z,t_R)}{\sqrt{s^2+z^2}} dz \qquad t_R \equiv t - \frac{\sqrt{s^2+z^2}}{c}$$

What is $I(z,t_R)$?

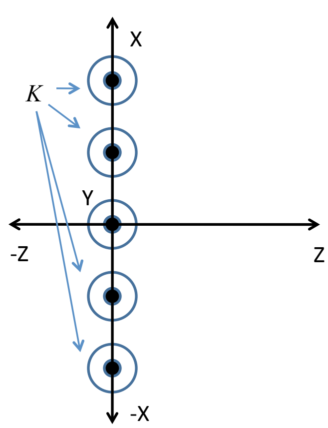
$$I(t) = \begin{cases} 0 & t \le 0 \\ I_0 & t>0 \end{cases}$$

$$\mathbf{A}(s,t) = \frac{\mu_0}{4\pi} \int_{-??}^{??} \frac{I_0 \hat{z}}{\sqrt{s^2 + z^2}} dz$$

What should the limits of integration be?

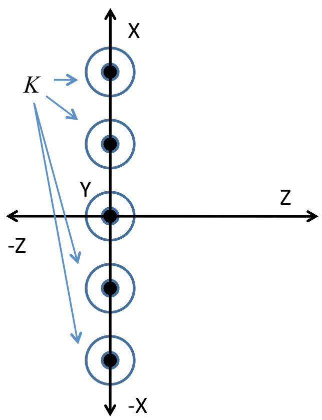
A) -∞ to +∞
B) -ct to +ct
C) $-\sqrt{c^2t^2 + s^2}$ to $+\sqrt{c^2t^2 + s^2}$
D) $-\sqrt{c^2t^2 - s^2}$ to $+\sqrt{c^2t^2 - s^2}$
E) Something else, not sure, ...

A charge neutral and infinite static current sheet, **K**, flows in the *x*-*y* plane, in the *y*-axis direction. Therefore, to the right of the *x*-*y* plane, according to what you know from Phys 3310, the **E** and **B** field directions are:



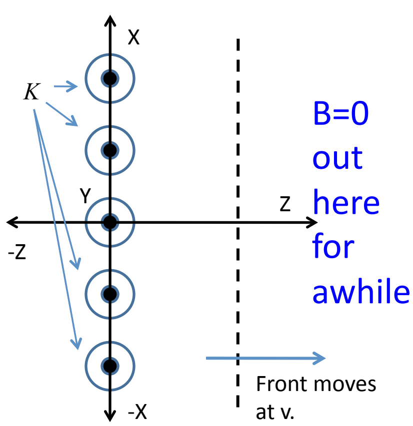

A) **E** along *z*-axis, **B** is zero
B) **B** along *z*-axis, **E** is zero
C) **B** along *y*, **E** along *z*
D) **B** along *x*, **E** along *y*
E) None of these

A charge neutral and infinite current sheet, **K**, is turned on at *t*=0, flows in the *x*-*y* plane, in the *y*-axis direction. Therefore, immediately afterwards, the **E** and **B** fields:




A) Remain zero (and for all time).
B) Immediately appear with their static values in all space.
C) Appear only near **K**
D) Appear only to the right of **K**
E) None of these

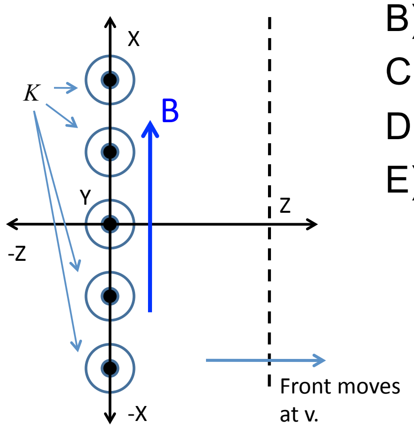
A charge neutral and infinite current sheet, **K**, is turned on at *t*=0, flows in the *x*-*y* plane, in the *y*-axis direction. Therefore, shortly afterwards, the **B** field near the sheet:




A) is in the *z*-direction
B) is in the *x*-direction
C) is in the *y*-direction
D) is actually zero close to **K**.
E) None of these

A charge neutral and infinite current sheet, **K**, is turned on at *t*=0, flows in the *x*-*y* plane, in the *y*-axis direction. Therefore, shortly afterwards, the **E** field very near the sheet:

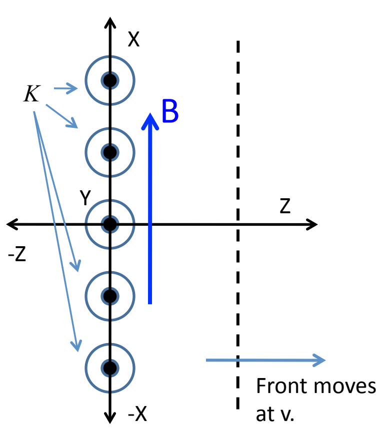
A) is in the -*z*-direction
B) is in the -*x*-direction
C) is in the -*y*-direction
D) is actually zero close to **K**.
E) None of these




B=0
out
here
for
awhile

A charge neutral and infinite current sheet, **K**, is turned on at *t*=0, flows in the *x*-*y* plane, in the *y*-axis direction. Therefore, shortly afterwards, the **E** field near the wavefront:

A) is in the –*z* direction
B) is in the –*x* direction
C) is in the –*y* direction
D) is actually zero close to *the front*.
E) None of these




B=0 out here for awhile