Have you studied the special theory of relativity?

A) Yes, pretty much just in Phys 2170
B) Yes, pretty much just in Phys 2130
C) Yes, in MORE than one class by now.
D) Not in a *class*, but I've read/learned some on my own
E) No, not really

12.1

Two major results of special relativity are Time Dilation and Lorentz Contraction. Please pick one of the choices below which best describes how well you feel you understand them.

A. No idea what these effects are
B. I remember having heard about these, but couldn't define them precisely right now.
C. I know what these effects are, (but I've forgotten how to derive them)
D. I know what these effects are, and I even sort of remember the derivation, but it would take me a while to sort it out
E. I'm confident I could derive these results right now

I have a stick of length L sitting in front of me.
In the reference frame of a passing train,(moving parallel to the stick) what is the measured length of the stick?

A) L B) γL C) L / γ

D) I'm sure it's either B or C, but I'm NOT sure which one!
E) I'm pretty sure it's "none of the above" or "it depends"

Everything should be made as simple as possible, but not simpler

-A. Einstein

I flash a lightbulb (event 1).
The light reaches a mirror, and
returns to me (event 2)

I measure the time $\Delta t = t_2 - t_1$ for the complete trip of the light.

A long train was passing (speed v) during this experiment.
In the reference frame of the train, what is the interval Δt' between those two events?

(As usual, $\gamma \equiv \frac{1}{\sqrt{1-v^2/c^2}}$)

A) $\Delta t' = \Delta t$ B) $\Delta t' = \gamma\,\Delta t$ C) $\Delta t' = \Delta t / \gamma$

D) I'm sure it's either B or C, but I'm NOT sure which one!
E) I'm pretty sure it's "none of the above"

Lorentz Transformations

We now have the tools to compare positions and times in different inertial reference frames.

In Galilean relativity

$$x' = x - vt$$
$$y' = y$$
$$z' = z$$
$$t' = t$$

Newton worked with these…

In Special relativity

$$x' = \gamma(x - vt) \qquad x = \gamma(x' + vt')$$
$$y' = y \qquad y = y'$$
$$z' = z \qquad z = z'$$
$$t' = \gamma(t - \frac{v}{c^2}x) \qquad t = \gamma(t' + \frac{v}{c^2}x')$$

but needs reworking of momentum and energy to work with these!

I flash a lightbulb (event 1).
The light reaches a mirror, and *returns to me* (event 2)

I measure the time $\Delta t = t_2 - t_1$ for the complete trip of the light.

A long train was passing (speed v) during this experiment.
In the reference frame of the train, what is the interval $\Delta t'$ between those two events?

(As usual, $\gamma \equiv \frac{1}{\sqrt{1-v^2/c^2}}$)

A) $\Delta t' = \Delta t$ B) $\Delta t' = \gamma\, \Delta t$ C) $\Delta t' = \Delta t / \gamma$

D) I'm sure it's either B or C, but I'm NOT sure which one!
E) I'm pretty sure it's "none of the above"

I have a stick of length L sitting in front of me.
In the reference frame of a passing train,(moving parallel to the stick) what is the measured length of the stick?

A) L B) γL C) L / γ

D) I'm sure it's either B or C, but I'm NOT sure which one!
E) I'm pretty sure it's "none of the above" or "it depends"

12.2

Ethel has carefully synchronized all the clocks in her frame of reference. In Lucy's frame, which is moving relative to Ethel's with speed v, Lucy has carefully synchronized all her own clocks. They agree to set the clocks so that noon occurs just when Lucy is at the same location as Ethel. Which of the following is true at noon?

A) ALL of the clocks are synchronized at noon, but then as time progresses the clocks in one frame get out of sync with those in the other frame.
B) Some of the clocks in both frames are synchronized at noon, but then get out of sync later.
C) It is impossible to synchronize any clocks in different frames without violating the postulates of special relativity.

12.4

Can one change the order of events in time by measuring them in a different inertial reference frame?

A. Always
B. Sometimes
C. Never

Lorentz Transformations

We now have the tools to compare positions and times in different inertial reference frames.

In Galilean relativity

$$x' = x - vt$$
$$y' = y$$
$$z' = z$$
$$t' = t$$

In Special relativity

$$x' = \gamma(x - vt) \qquad x = \gamma(x' + vt')$$
$$y' = y \qquad y = y'$$
$$z' = z \qquad z = z'$$
$$t' = \gamma(t - \frac{v}{c^2}x) \qquad t = \gamma(t' + \frac{v}{c^2}x')$$

Newton worked with these…

but needs reworking of momentum and energy to work with these!

I'm in frame S, and Charlie is in Frame S'
(which moves with speed V in the +x direction.)

An object moves in the S' frame in the +y direction with speed v'_y.

Do I measure its y component of velocity to be $v_y = v_y'$?

A) Yes
B) No
C) ???

Lorentz transformation, written in 4-vector notation

$$\begin{pmatrix} x^{0'} \\ x^{1'} \\ x^{2'} \\ x^{3'} \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ \beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}$$

$$x'^{\mu} = \Lambda^{\mu}_{\upsilon}\, x^{\upsilon}$$

Any 4-component object which transforms like this when you change frames is a "Contravariant 4-vector"

Displacement is a defined quantity:

$$\Delta x^{\mu} \equiv \left(x_A^{\mu} - x_B^{\mu} \right)$$

<u>Is displacement a 4-vector?</u>

A) Yes
B) No
C) Ummm… don't know how to tell
D) None of these.

Be ready to explain your answer.

Which of the following equations is the correct way to write out the Lorentz product?

A) $a \bullet b = -a^0 b^0 + a^1 b^1 + a^2 b^2 + a^3 b^3$

B) $a \bullet b = +a_0 b^0 + a_1 b^1 + a_2 b^2 + a_3 b^3$

C) $a \bullet b = a_\mu b^\mu$

D) More than one (but not all three)

E) All three are equally correct

Velocity is a defined quantity: $\vec{u} \equiv \frac{\Delta \vec{r}}{\Delta t} = \begin{pmatrix} \frac{\Delta x}{\Delta t} & \frac{\Delta y}{\Delta t} & \frac{\Delta z}{\Delta t} \end{pmatrix}$

In another inertial frame, seen to be moving to the right, parallel to x, observers see:

$$\vec{u}' \equiv \frac{\Delta \vec{r}'}{\Delta t'} = \begin{pmatrix} \frac{\Delta x'}{\Delta t'} & \frac{\Delta y'}{\Delta t'} & \frac{\Delta z'}{\Delta t'} \end{pmatrix}$$

Is velocity a 4-vector?

A) Yes
B) No
C) Sometimes yes, sometimes no
D) None of these.

4-velocity?

Imagine this quantity: $u^\mu \equiv \begin{pmatrix} c \\ \frac{\Delta x}{\Delta t} \\ \frac{\Delta y}{\Delta t} \\ \frac{\Delta z}{\Delta t} \end{pmatrix}$

Is this quantity a 4-vector?

A) Yes, and I can say why.
B) No, and I can say why.
C) None of the above.

4-velocity?

Does it transform via the Lorentz Transformation?: $\bar{u}^\mu = \Lambda^\mu_\nu u^\nu \quad ??$

4-velocity?

Imagine this quantity: $u \equiv \begin{pmatrix} c \\ \dfrac{\Delta x}{\Delta t} \\ \dfrac{\Delta y}{\Delta t} \\ \dfrac{\Delta z}{\Delta t} \end{pmatrix}$

Is this quantity a 4-vector?

~~A) Yes, and I can say why.~~
B) No, and I can say why.
C) None of the above.

This object does not Lorentz Transform. NOT a a 4-vector.

4-velocity?

Does it transform via the Lorentz Transformation?: $\bar{u}^{\mu} = \Lambda^{\mu}_{\nu} u^{\nu} \quad ??$

Do the direct comparison

$x' = \gamma(x - vt)$ $\qquad$ $\bar{u}_x = (u_x - v) \Big/ \left(1 - \dfrac{v u_x}{c^2}\right)$

$y' = y$ $\qquad$ $\bar{u}_y = u_y \Big/ \gamma\left(1 - \dfrac{v u_x}{c^2}\right)$

$z' = z$ $\qquad$ $\bar{u}_z = u_z \Big/ \gamma\left(1 - \dfrac{v u_x}{c^2}\right)$

$t' = \gamma(t - \dfrac{v}{c^2} x)$ $\qquad$ $c = c$

4-velocity?

Imagine this quantity: $\eta^{\mu} \equiv \frac{1}{\Delta\tau}\begin{pmatrix} c\Delta t \\ \Delta x \\ \Delta y \\ \Delta z \end{pmatrix}$

Proper time

Is this quantity a 4-vector?

A) Yes, and I can say why.
B) No, and I can say why.
C) None of the above.

Can we define?

$$c\Delta\tau \equiv \sqrt{-(\Delta x)_{\mu}(\Delta x)^{\mu}}$$

Proper time

$$= \sqrt{c^2(\Delta t)^2 - \left[(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2\right]}$$

Is this a reasonable definition?

A) Yes, and I can say why.
B) No, and I can say why.
C) None of the above.

Displacement is a defined quantity: $\Delta x^{\mu} \equiv \left(x_A^{\mu} - x_B^{\mu}\right)$

Interval is a defined quantity: $I \equiv \Delta x_{\mu} \Delta x^{\mu}$

Written in terms of the spatial distance, d, between events and the times separation between events, t, the interval is:

A) Working on it.
B) Think I've got it.
C) None of the above.

In my frame (S) I measure two events which occur at the same place, but different times t_1 and t_2
(they are NOT simultaneous)

Might Charlie (in frame S') measure those SAME two events to occur simultaneously in his frame?

A) Possibly, if he's in the right frame!
B) Not a chance
C) Definitely need more info!
D) ???

12.6

The 4-velocity η^{μ} is defined as

$$\eta^{\mu} = \frac{1}{\sqrt{1-u^2/c^2}}\begin{pmatrix} c \\ \mathbf{u} \end{pmatrix}$$

What is the invariant length squared of the 4-velocity, $\eta_{\mu}\eta^{\mu}$?

A. c^2
B. $-c^2$
C. $-c^2 + u^2$
D. $c^2 - u^2$
E. None of the above

12.5

Two events have a timelike separation. In a "1+1"-D space time (Minkowski) diagram
(*x* horizontal, *ct* vertical),
the magnitude of the slope of a line connecting the two events is

A. Greater than 1
B. Equal to 1
C. Less than 1

For isolated systems, the total 4-momentum is CONSERVED (this is an experimental fact).

Is 4-momentum invariant ?

A) Yes, and I can say why.
B) No, and I can say why.
C) None of the above.

12.7a

Are energy and rest mass Lorentz invariants?

A. Both energy and mass are invariants
B. Only energy is an invariant
C. Only rest mass is an invariant
D. Neither energy or mass are invariants

12.7b

Are energy and rest mass conserved quantities?

A. Both energy and mass are conserved
B. Only energy is an conserved
C. Only rest mass is conserved
D. Neither energy or mass are conserved

Minkowski 4-force

We define the 4-force via the 4-momentum:

$$\frac{\Delta p^{\mu}}{\Delta \tau} = K^{\mu}$$

Proper time

Is K, so defined, a 4-vector?

A) Yes, and I can say why.
B) No, and I can say why.
C) None of the above.

Minkowski 4-force

To match the behavior of non-relativistic classical mechanics, we might tentatively assign which of the following values to K:

A) $K^0 = \vec{F} \bullet \vec{u} \quad K^{1,2,3} = \vec{F}$

B) $K^0 = \vec{F} \bullet \vec{u} / \gamma c \quad K^{1,2,3} = \vec{F} / \gamma$

C) $K^0 = \gamma \vec{F} \bullet \vec{u} / c \quad K^{1,2,3} = \gamma \vec{F}$

D) Something else