Griffiths Chapter 8 – Conservation laws

8.1

The definition of work is the starting point of deriving the work energy theorem:

$$dW = \mathbf{F}_{\mathbf{net}} \cdot d\mathbf{l}$$

What is the second necessary relation in deriving the theorem?

A. $\mathbf{F}_{\mathbf{net}} = \sum_i \mathbf{F}_i$

B. $\mathbf{F}_{net} = \frac{d\mathbf{p}}{dt}$

C. $W = \int_{path} \mathbf{F}_{\mathbf{net}} \cdot d\mathbf{l}$

8.1a

The work energy theorem states:

$$W = \int_i^f \mathbf{F}_{\mathbf{net}} \cdot d\mathbf{l} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

This theorem is valid

A. only for conservative forces.
B. only for non-conservative forces.
C. only for forces which are constant in time
D. only for forces which can be expressed as potential energies
E. for all forces.

8.2

If energy density u_{EM} has units J/m^3, what units does the energy "flow" or flux density **S** have?

A. J/(s m^3)
B. W/m^3 (W = Watt)
C. J/(s m^5)
D. W/m^2
E. None of the above

Global conservation of charge
says
"Q_{total}" doesn't change with time.

Local conservation of electric charge is expressed mathematically by:

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \vec{\mathbf{J}}$$

where J is "current density"

In general, local conservation of "blah" looks like

$$\frac{\partial (\text{blah})}{\partial t} = -\nabla \cdot (\text{flow of blah})$$

Local conservation of electric charge is expressed mathematically by:

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \vec{\mathbf{J}}$$

where J is "current density"

J = ρ**v** has units of (charge/sec)/m^2

We are trying to come up with a "conservation of energy" expression:

$$\frac{\partial (\text{energy density})}{\partial t} = -\nabla \cdot (\textit{something})$$

What sort of beast is this "something" ?
- Is it a scalar, vector, something else?
- How would you interpret it, what words would you use to try to describe it?
- What are its UNITS?

A) J B) J/s C) J/m^2 D) J/(s m^2)
E) Other!

Local conservation of electric charged is expressed mathematically by:

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \vec{\mathbf{J}}$$

where J = ρv has units of (charge/sec)/m^2

We are trying to come up with a "conservation of energy" expression:

$$\frac{\partial (\text{energy density})}{\partial t} = -\nabla \cdot \vec{\mathbf{S}}$$

What exactly is "energy density" here? (Whose energy?)

A + and - charge are held a distance R apart and released.

The two particles accelerate toward each other as a result of the Coulomb attraction.

As the particles approach each other, the energy contained in the electric field surrounding the two charges...

A: increases B: decreases C: stays the same D: ??

First midterm:
Tomorrow evening, 7-9 PM
In G125

- You can bring the “purple crib sheet” (I’ll have copies there),
  and 1 page, ONE SIDE, with your own hand-written notes.
- You won’t need a calculator.

- No HW due Wed
- So, office hrs today/tomorrow by app’t (my office)
- The preflight (quiz) for today is still open.

Local conservation of electric charged is expressed mathematically by:

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \vec{\mathbf{J}}$$

where J = ρv has units of (charge/sec)/m^2

We are trying to come up with a "conservation of energy" expression:

$$\frac{\partial(\text{energy density})}{\partial t} = -\nabla \cdot \vec{\mathbf{S}}$$

What exactly is "energy density" here? (Whose energy?)

$$\frac{\partial}{\partial t} u_q = -\frac{\partial}{\partial t}\left(\frac{\varepsilon_0}{2} E^2 + \frac{1}{2\mu_0} B^2\right) - \nabla \cdot \vec{\mathbf{S}}$$

(Where **S**=**E** x **B** /μ_0)

How do you interpret this equation? In particular:
Does the – sign on the first term on the right seem OK?
A) Yup B) It's disconcerting, did we make a mistake? C) ??

$$\frac{\partial}{\partial t}\left(u_q + \frac{\varepsilon_0}{2} E^2 + \frac{1}{2\mu_0} B^2\right) = -\nabla \cdot \vec{\mathbf{S}}$$

$$\frac{\partial}{\partial t}(u_q + u_{EM}) = -\nabla \cdot \vec{\mathbf{S}}$$

A + and - charge are held a distance R apart and released.

The two particles accelerate toward each other as a result of the Coulomb attraction.

As the particles approach each other, the energy contained in the electric field surrounding the two charges...

A: increases   B: decreases   C: stays the same   D: ??

$$\frac{d}{dt}\iiint (u_q + u_{EM})\, d\tau = -\iiint \nabla \cdot \vec{\mathbf{S}}\, d\tau$$

(Where **S**=**E** x **B** /μ_0)

Would you interpret **S** as the

A) OUTFLOW of energy/area/time or
B) INFLOW of energy/area/time
C) OUTFLOW of energy/volume/time
D) INFLOW of energy/volume/time
E) ???

$$\frac{d}{dt}\iiint (u_q + u_{EM})\, d\tau = -\iiint \nabla \cdot \vec{\mathbf{S}}\, d\tau$$

(Where **S**=**E** x **B** /μ_0)

$$= -\oiint \vec{\mathbf{S}} \cdot d\vec{\mathbf{a}}$$

Would you interpret **S** as the
A) OUTFLOW of energy/area/time or
B) INFLOW of energy/area/time
C) OUTFLOW of energy/volume/time
D) INFLOW of energy/volume/time
E) ???

$$\frac{dW}{dt} = \pm \frac{dU_{EM}}{dt} - \oiint \vec{\mathbf{S}} \cdot d\vec{\mathbf{a}}$$

To make sense, should that term be
A) + or B) -?
C) ???

What are the SI metric units of the combination of fields $\mathbf{E} \times \mathbf{B} / \mu_0$?

A) J B) J/s C) J/m^2 D) J/(m^2 s)
E) Other!

8.2

If energy density u_{EM} has units J/m^3, what units should the energy "flow" or flux density **S** have?

A. J/(s m^3)
B. W/m^3
C. J/(s m^5)
D. W/m^2
E. None of the above

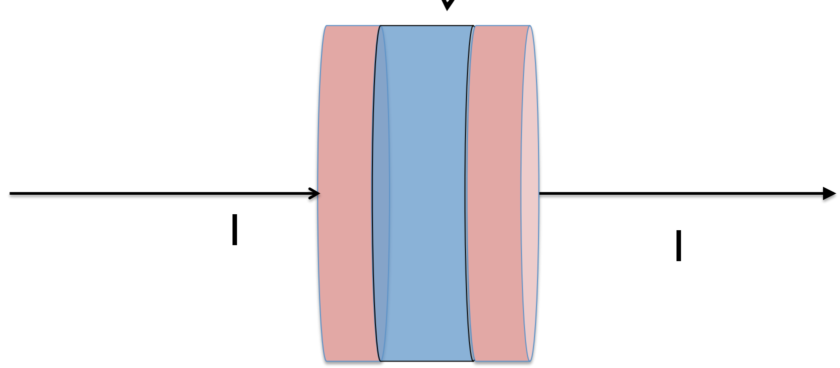
(W = Watt)

The fields can change the total energies of charged particles by:

A) Doing work on the particles
B) Changing the potential energies only
C) Changing the kinetic energies only
D) Applying forces only perpendicular to the particle motion.
E) None of the above.

Consider the cylindrical volume of space bounded by the capacitor plates. Compute **S = E** x **B** $/\mu_0$ at the outside (cylindrical, curved) surface of that volume.
Which WAY does it point?

A) Always inward B) Always outward
C) ???



Given a quantity with units of (Joules/m^3), you can convert it to a quantity with units of Joules/(m^2 * seconds) by multiplying by:

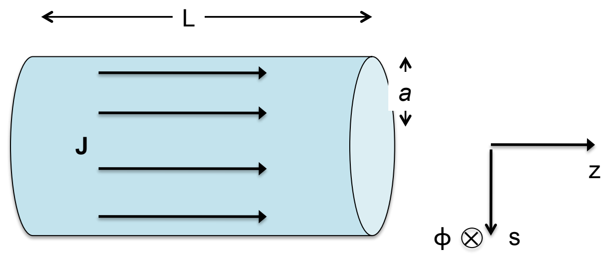
A) a length
B) a frequency
C) a speed
D) an acceleration
E) None of the above

Given the E and B vectors, and perhaps some constants like permeability and permittivity of free space, can you construct a VECTOR with units of Joules/(m^2 * seconds)?

A) We are working on it
B) We have one!

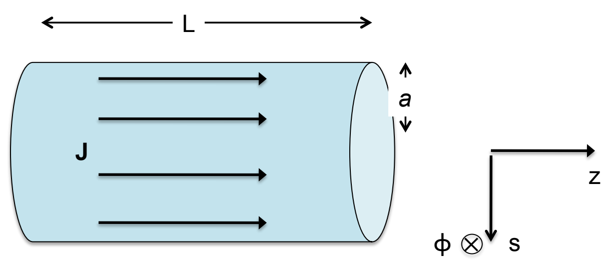
8.3

Consider a current I flowing through a cylindrical resistor of length L and radius *a* with voltage V applied. What is the E field inside the resistor?



8.3

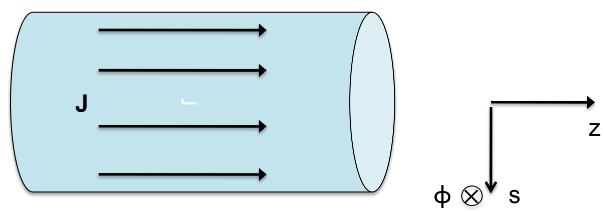
Consider a current I flowing through a cylindrical resistor of length L and radius *a* with voltage V applied. What is the E field inside the resistor?



A. (V/*a*) z-hat
B. (V/*a*) ϕ-hat
C. (V/*a*) s-hat
D. (Vs/*a*²) z-hat
E. None of the above

8.4

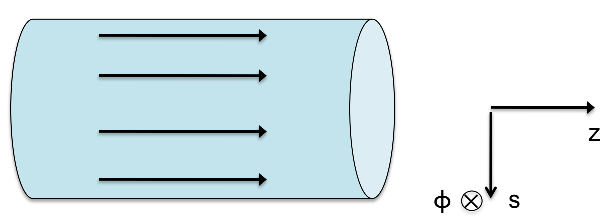
Consider a current I flowing through a cylindrical resistor of length L and radius a with voltage V applied. What is the B field inside the resistor?



A. $(I\mu_0/2\pi s)$ ϕ-hat
B. $(I\mu_0 s/2\pi a^2)$ ϕ-hat
C. $(I\mu_0/2\pi a)$ ϕ-hat
D. $-(I\mu_0/2\pi a)$ ϕ-hat
E. None of the above

8.5

Consider a current I flowing through a cylindrical resistor of length L and radius a with voltage V applied. What is the direction of the S vector on the outer curved surface of the resistor?



A. ±ϕ-hat
B. ± s-hat
C. ± z-hat
D. ???

And, is it + or -?

How was the exam last night?

A) Too easy - exams should be a lot harder than that!
B) Fine/fair
C) Little hard (here and there), but I managed
D) Out of line/too hard no fair!
E) (No comment/none of the above/other comment!)

How was your "time allocated to 3320" spent this last week (exam but no homework) compared to usual?

I spent .....

A) MORE time
B) About the SAME time
C) LESS time

prepping for the midterm than I usually spend reading/doing homework for this course

D) No comment/none of the above/other comment!

I have read Chapter 8 material on Conservation Laws, the Poynting Vector, and the Maxwell Stress Tensor:

A) True, all of it
B) True, up to the tensor stuff
C) Just some of it…
D) Reading? During an exam week?

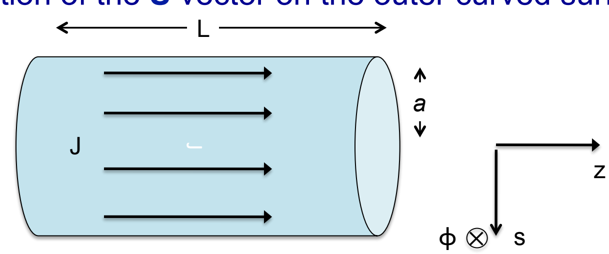
In general, this term are you

A) Reading Griffiths *ahead* of lecture (mostly)
B) Reading Griffiths, but *behind* lecture (mostly)
C) Doing some reading, but definitely not all
D) Not doing very much reading, (just what's absolutely needed for homework)
E) Other…! E.g, you're using my lecture notes rather than Griffiths, or you found a book you prefer, etc…

The energies stored in the electric and magnetic fields are:

A) individually conserved for both E and B, and cannot change.
B) conserved only if you sum the E and B energies together.
C) are not conserved at all.
D) ???

Consider a current I flowing through a cylindrical resistor of length L and radius a with voltage V applied.
What is the direction of the **S** vector on the outer curved surface of the resistor?



A. ±ϕ-hat
B. ± s-hat
C. ± z-hat
D. ???

And, is it + or -?

The momentum density (momentum/m^3) of the electromagnetic field is:

A) Not yet defined in this class.
B) Not an individually conserved quantity.
C) Not related to a vector momentum current density.
D) All of the above
E) None of the above

The fields can change the total momentum of charged particles by:

A) Fields cannot change particle momentum
B) Applying a net force to the particles
C) Changing only the potential energy
D) Only if they do net work on the particles.
E) None of the above.

Given the E and B vectors, and perhaps some constants like permeability and permittivity of free space, can you construct a TENSOR that depends upon both E and B, and with units of Joules/m^3 ?

A) We are working on it
B) We have one!
C) JUST KIDDING!

8.5a

Consider two point charges, each moving with constant velocity v, charge 1 along the +x axis and charge 2 along the +y axis. They are equidistant from the origin.

What is the direction of the magnetic force on charge 1 from charge 2? (You'll need to sketch this! Don't do it in your head!)

A. +x
B. +y
C. +z
D. More than one of the above
E. None of the above

8.5a

Consider two point charges, each moving with constant velocity v, charge 1 along the +x axis and charge 2 along the +y axis. They are equidistant from the origin.

What is the direction of the magnetic force on charge 2 from charge 1? (You'll need to sketch this! Don't do it in your head!)

A. Equal to the answer of the previous question
B. Equal but opposite to the answer of the previous question
C. Something *different* than either of the above.

Conservation of energy looks like this: $\frac{\partial}{\partial t}(u_q + u_{EM}) = -\nabla \cdot \vec{\mathbf{S}}$

Where $u_{EM} = \frac{\varepsilon_0}{2}E^2 + \frac{1}{2\mu_0}B^2$ and $\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}} \times \vec{\mathbf{B}}$

With units of (energy/m^3) and energy/m^2/s respectively.

Now I'd like to find a "conservation of momentum" expression:

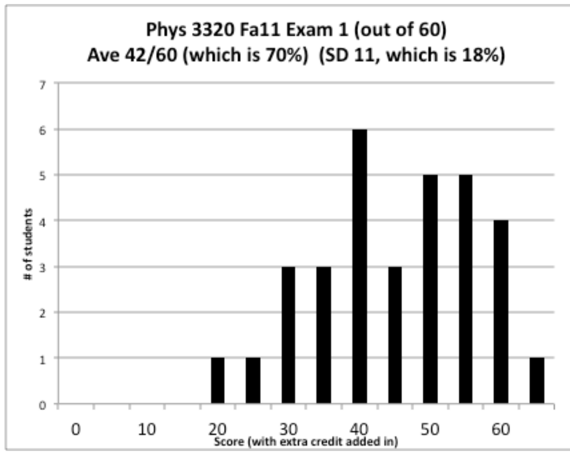
$$\frac{\partial}{\partial t}(\vec{p}_q / volume + \vec{p}_{EM} / volume) = \nabla \cdot (something, \text{called T})$$

We seek a local conservation law that relates the time change in momentum density (units of momentum/m^3), to the divergence of a current density, "T", with units of:

A) Newtons/m^2
B) kg*m/(m^2*second^2)
C) Joules/m^3
D) More than one of the above
E) None of the above

We seek a local conservation law that relates the time rate of change in momentum density (momentum/m^3), to the divergence of a current density, "T", with units of:

A) Newtons/m^2
B) kg*m/(m^2*second^2)
C) Joules/m^3
D) More than one of the above
E) None of the above



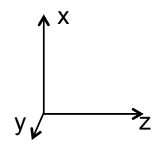
$$\frac{\partial}{\partial t}(\vec{p}_q / volume + \vec{p}_{EM} / volume) = \nabla \cdot (\mathrm{T})$$

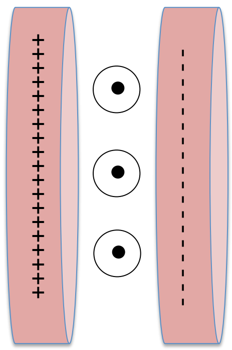
Working this out just as we did for energy, starting from dp/dt = F (instead of dW = F.dl)
We find the momentum density is given by

$$\vec{p}_{EM} / volume = \mu_0 \varepsilon_0 \vec{\mathbf{S}}$$

Momentum in the fields: $\vec{p}_{EM} / volume = \mu_0 \varepsilon_0 \vec{\mathbf{S}}$

Consider a charged capacitor placed in a uniform B field in the +y direction





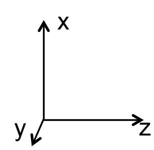
Which way does the stored field Momentum in this system point?

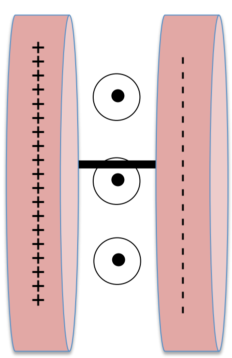
A) +/- x
B) +/- y
C) +/- z
D) Zero!
E) Other/???

Momentum in the fields:

$$\vec{p}_{EM} / volume = \mu_0 \varepsilon_0 \vec{\mathbf{S}}$$

Now “short out” this capacitor with a small wire.
As the current flows, (while the capacitor is discharging)…





which way does the magnetic force push the wire (and thus, the system)?

A) +/- x
B) +/- y
C) +/- z
D) Zero!
E) Other/???

Is your answer consistent with “conservation of momentum”?

8.6

What units should a momentum density have?

A. N s/m^3
B. J s/m^3
C. kg/(s m^2)
D. More than one of the above
E. None of the above

$$\frac{\partial}{\partial t}(\vec{p}_q / volume + \vec{p}_{EM} / volume) = \nabla \cdot (\mathrm{T})$$

$$\vec{p}_{EM} / volume = \mu_0 \varepsilon_0 \vec{\mathbf{S}}$$

But what kind of beast is T? (Vector, scalar, other?)

8.7

What units should a momentum flux density have?

A. N/m^3
B. N/m^2
C. kg/(s m)
D. More than one of the above
E. None of the above

The Poynting vector is $\vec{\mathbf{S}} = (S_x, S_y, S_z)$

$\overleftrightarrow{T}$ is the Maxwell stress tensor. It is a matrix:

$$\begin{bmatrix} T_{xx} & T_{xy} & T_{xz} \\ T_{yx} & T_{yy} & T_{yz} \\ T_{zx} & T_{zy} & T_{zz} \end{bmatrix}$$

$$T_{ij} = \varepsilon_0 (E_i E_j - \tfrac{1}{2}\delta_{ij} E^2) + \frac{1}{\mu_0}(B_i B_j - \tfrac{1}{2}\delta_{ij} B^2)$$

8.8

The Maxwell stress tensor is given by:

$$T_{ij} = \varepsilon_0 (E_i E_j - \tfrac{1}{2}\delta_{ij} E^2) + \frac{1}{\mu_0}(B_i B_j - \tfrac{1}{2}\delta_{ij} B^2)$$

What is the E field part of the T_{zx} term?

A. $\varepsilon_0(E_z E_x - ½(E_x^2 + E_z^2))$
B. $\varepsilon_0(E_z E_x - ½E_y^2)$
C. $\varepsilon_0(E_z E_x - ½(E_x^2 + E_y^2 + E_z^2))$
D. $\varepsilon_0(E_z E_x)$
E. None of the above

8.8a

The Maxwell stress tensor is given by:

$$T_{ij} = \varepsilon_0 (E_i E_j - \tfrac{1}{2}\delta_{ij} E^2) + \frac{1}{\mu_0}(B_i B_j - \tfrac{1}{2}\delta_{ij} B^2)$$

What is the E field part of the T_{zz} term?

A. $\varepsilon_0(E_z^2 - (E_x^2 + E_y^2))/2$
B. $\varepsilon_0(E_z^2 - ½(E_x^2 + E_y^2)$
C. $-\varepsilon_0(E_x^2 + E_y^2)$
D. $\varepsilon_0(E_z^2)$
E. None of the above

What is $\overleftrightarrow{T} \cdot d\vec{A}$?

$$\begin{bmatrix} T_{xx} & T_{xy} & T_{xz} \\ T_{yx} & T_{yy} & T_{yz} \\ T_{zx} & T_{zy} & T_{zz} \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} T_{xx}a_x + T_{xy}a_y + T_{xz}a_z \\ T_{yx}a_x + T_{yy}a_y + T_{yz}a_z \\ T_{zx}a_x + T_{zy}a_y + T_{zz}a_z \end{bmatrix}$$

in general $(\overleftrightarrow{\mathrm{T}} \cdot \vec{\mathrm{a}})_i = \sum_{j=x,y,z} T_{ij} a_j$

Similarly, $(\vec{\nabla} \cdot \overleftrightarrow{\mathrm{T}})_j = \sum_{i=x,y,z} \frac{\partial}{\partial x_i} T_{ij}$

$$\frac{\partial}{\partial t}(\vec{p}_q / volume + \vec{p}_{EM} / volume) = \nabla \cdot (\mathrm{T})$$

If we integrate both sides over volume,
what is the first term on left side?

Just $\mathbf{F}_{mech}$!

$$\frac{\partial}{\partial t}(\vec{p}_q / volume + \vec{p}_{EM} / volume) = \nabla \cdot (\mathrm{T})$$

If we integrate both sides over volume, using $\vec{p}_{EM} / volume = \mu_0 \varepsilon_0 \vec{\mathbf{S}}$
The left side is thus

$$\mathbf{F}_{mech} + \varepsilon_0 \iiint \frac{\partial(\mathbf{E} \times \mathbf{B})}{\partial t} d\tau$$

In static situations, this is still just the net physical force on our collection of charges. (That seems useful!)

$$\frac{\partial}{\partial t}(\vec{p}_q / volume + \vec{p}_{EM} / volume) = \nabla \cdot (\mathrm{T})$$

If we integrate both sides over volume, what is the right side?

$$\oiint \vec{T} \cdot d\vec{A}$$

So, in static situations

$$\vec{F}_{mech} = \oint\!\!\oint \overleftrightarrow{T} \cdot d\vec{A}$$

Recall, T had units “force/area”.

Hence, T is called a “stress tensor”,
This formula looks like F = stress (or pressure) * area

8.11

Given a general Maxwell Stress tensor with all elements non-zero, what is the net force on a small isolated area element
*d***a** = (*dx dy)* **z** ?

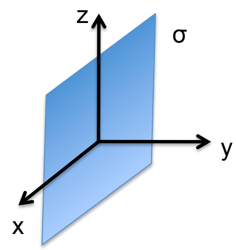
$$\vec{F} = \iint \overleftrightarrow{T} \cdot d\vec{A}$$

$$\begin{matrix} T_{xx} & T_{xy} & T_{xz} \\ T_{yx} & T_{yy} & T_{yz} \\ T_{zx} & T_{zy} & T_{zz} \end{matrix}$$

A. $T_{xz}\, dx\, dy$ **z**
B. $T_{yz}\, dx\, dy$ **z**
C. $T_{xz}\, dx\, dy$ **z**
D. (T_{xz} **x** + T_{yz} **y** + T_{zz} **z)** dx dy
E. Something else!

8.10

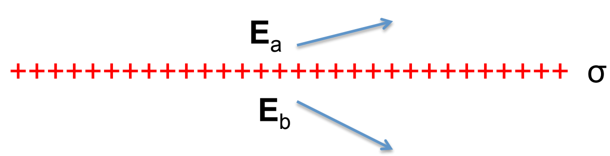
An infinite plane of surface charge σ lies in the *xz* plane. In the region *y* > 0, which element(s) of the stress tensor is(are) non-zero?



A. Tyy
B. Txx, Tyy, Tzz
C. Txy,Tyx,Tyy,Tyz,Tzy
D. Txy,Tyx,Tyy,Tyz,Tzy,Txx,Tyy
E. None of the above

8.9

Suppose we have a plane of surface charge σ with electric field $\mathbf{E}_a$ above the plane and $\mathbf{E}_b$ below the plane.



What is the net force per area on the plane?

A. $(\mathbf{E}_a + \mathbf{E}_b)\ \sigma$
B. $(\mathbf{E}_a - \mathbf{E}_b)\ \sigma$
C. $(\mathbf{E}_a + \mathbf{E}_b)\ \sigma/2$
D. $(\mathbf{E}_a - \mathbf{E}_b)\ \sigma/2$
E. None of the above

Conservation of angular momentum:

$$\vec{l}_{EM} \,/\, volume = \vec{r} \times \vec{p}_{EM} \,/\, volume$$
$$= \mu_0 \varepsilon_0 \vec{r} \times \vec{\mathbf{S}}$$
$$= \varepsilon_0 \vec{r} \times (\vec{E} \times \vec{B})$$