Phys3320 Spring 2011

Chapter 9 materials

A function, $f(x,t)$, satisfies this PDE:

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2}$$

Invent two different functions f(x,t) that solve this equation. Try to make one of them "boring" and the other "interesting" in some way.

9.6

The complex exponential, $e^{i2\pi}$ is equal to:

A) 0 B) *i* C) 1 D) *p*

E) Something else

$$e^{i2\pi} = 1$$

9.8

A function, f, satisfies the wave equation:

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$$

Which of the following functions work?

A) Sin(k(x – vt))

B) Exp(k(-x – vt))

C) a(x + vt)^3

D) All of these.

E) None of these.

9.8

A function, f, satisfies the wave equation:

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$$

Which of the following functions work?

A) Sin(k(x – vt))

B) Exp(k(-x – vt))

C) a(x + vt)^3

D) All of these.

E) None of these.

In fact, ANY function f (x +/- vt) is a good solution!!

Shape travels left (+) or right (-).

A "right moving" solution to the wave equation is:

$f_R(z,t) = A \cos(kz - \omega t + \delta)$

Which do you prefer for a "left moving" soln?

A) $f_L(z,t) = A \cos(kz + \omega t + \delta)$

B) $f_L(z,t) = A \cos(kz + \omega t - \delta)$

C) $f_L(z,t) = A \cos(-kz - \omega t + \delta)$

D) $f_L(z,t) = A \cos(-kz - \omega t - \delta)$

E) more than one of these!

(Assume k, ω, δ are positive quantities)

To think about; Is(are) the answer(s) really just "preference" (i.e. human convention) or are we forced into a choice?

A solution to the wave equation is:
$f(z,t) = A \cos(kz – \omega t + \delta)$

What is the speed of this wave?
Which way is it moving?
If δ is small (and >0), is this wave "delayed" or "advanced"?
What is the frequency?
The angular frequency?
The wavelength?
The wave number?

A solution to the wave equation is:
$f(z,t) = \mathrm{Re}[A\, e^{(kz – \omega t + \delta)}]$

What is the speed of this wave?
Which way is it moving?
If δ is small (and >0), is this wave "delayed" or "advanced"?
What is the frequency?
The angular frequency?
The wavelength?
The wave number?

$$\tilde{f}(\mathbf{r},t) = \tilde{A}e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$$

What is the speed of this wave?
Which way is it moving?
Why is there no δ?

What is the frequency?
The angular frequency?
The wavelength?
The wave number?

9.6

The electric field for a plane wave is given by:

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E_0}e^{i(\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}-\omega t)}$$

The vector **k** tells you:

A) The direction of the electric field vector.

B) The speed of the traveling wave.

C) The direction the plane wave moves.

D) A direction *perpendicular* to the direction the plane wave moves

E) None of these/MORE than one of these/???

9.6

The electric field for a plane wave is given by:

$$\mathbf{E}(\mathbf{r},t) = \vec{E}_0 e^{i(\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}-\omega t)}$$

Suppose $\mathbf{E_0}$ points in the +x direction.
Which direction is this wave moving?

A) The x direction.

B) The radial (**r**) direction

C) A direction *perpendicular* to both **k** and **x**

D) The **k** direction

E) None of these/MORE than one of these/???

The 1-D wave equation is $\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$

One particular "traveling wave" solution to this is
$f_1(z,t) = A_1 \cos(k_1 z - \omega_1 t + \delta_1)$
This wave has speed $v = \omega_1/k_1$ (do you see why?)

There are many *other* solutions, including $f_2(z,t)$ with the SAME functional form, but with higher frequency, $\omega_2 > \omega_1$.

What can you say about the *speed* of that new solution?
A) greater than v B) less than v C) equal to v
D) indeterminate!
By the way:
This wave travels rightward (do you see why?)
This wave has wavelength lambda= $2\pi/k_1$ (do you see why?)
This wave has period $2\pi/\omega_1$ (do you see why?)

The 3-D wave equation is $\nabla^2 f(x,y,z,t) = \frac{1}{v^2}\frac{\partial^2 f(x,y,z,t)}{\partial t^2}$

One particular "traveling wave" solution to this is often written

$\tilde{f}_1(x,y,z,t) = \tilde{A}\, e^{i(\vec{k}\cdot\vec{r}-\omega t)}$, where $\tilde{A} = A\, e^{i\delta}$.

This wave travels in the **k** direction (do you see why?)
This wave has wavelength lambda= 2π/|**k**| (do you see why?)
This wave has period 2π/ω (do you see why?)
This wave has speed v = ω/|**k**| (do you see why?)

What is the real form of this wave?

A) $A\cos(kx-\omega t)$ B) $A\cos(kx-\omega t+\delta)$

C) $A\cos(\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}-\omega t)$ D) $A\cos(\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}-\omega t+\delta)$

E) More than one of these/other/???

Maxwell's equations:

$\nabla\bullet\mathbf{E} = \rho/\varepsilon_0$ $\nabla\bullet\mathbf{B} = 0$

$\nabla\times\mathbf{E} = -\partial\mathbf{B}/\partial t$ $\nabla\times\mathbf{B} = \mu_0\mathbf{J} + \mu_0\varepsilon_0\partial\mathbf{E}/\partial t$

Maxwell's equations in vacuum:

$$\nabla \bullet \mathbf{E} = 0 \qquad \nabla \bullet \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t \qquad \nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \partial \mathbf{E}/\partial t$$

In vacuum, what is $\nabla \times (\nabla \times \mathbf{E})$?

$$\nabla \times (\nabla \times \mathbf{E}) = \nabla(\nabla \bullet \mathbf{E}) - \nabla^2 \mathbf{E}$$

$$\nabla \times (\nabla \times \mathbf{E}) = \nabla \times (-\partial \mathbf{B}/\partial t) = -\partial(\nabla \times \mathbf{B})/\partial t$$

In vacuum (!!!)....

$$-\nabla^2 \mathbf{E} = -\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{E} \qquad \mathbf{E} = \mathbf{E}_0 e^{i(\mathbf{k}\bullet\mathbf{r}-\omega t)}$$

"*It's of no use whatsoever* [...]
this is just an experiment that proves Maestro Maxwell was right - we just have these mysterious electromagnetic waves that we cannot see with the naked eye. But they are there." - Heinrich Hertz, 1888

Asked about the ramifications of his discoveries, Hertz replied, "*Nothing, I guess*."

Marconi's first wireless radio transmission over large distances (~6 km over water) was in 1897.

9.10

The electric fields of two E/M waves in vacuum are both described by:

$$\vec{E} = E_0 Sin(kx - \omega t)\hat{y}$$

The "wave number" k of wave 1 is larger than that of wave 2, $k_1 > k_2$.

Which wave has the larger frequency f?

A) Wave 1 B) Wave 2 C) impossible to tell

9.11

The electric field of an E/M wave is described by

$$\vec{E} = E_0 Sin(kx - \omega t)\hat{y}$$

k = 0.1 m^{-1}, at x=1m and t=0, what is the direction of the B-field?

A) +x B) +y C) –x D) +z E) -z

You have this solution to Maxwell's equations in vacuum:

$$\tilde{\vec{E}}(x,y,z,t)=\tilde{\vec{E}}_0\exp\left[i\left(\vec{k}\bullet\vec{r}-\omega t\right)\right]$$

If this wave travels in the y direction, is polarized in the x direction, and has a complex phase of 0,
what is the x component of the physical wave?

A) $E_x = E_0\cos(kx-\omega t)$
B) $E_x = E_0\cos(ky-\omega t)$
C) $E_x = E_0\cos(kz-\omega t)$
D) $E_x = E_0\cos(k_x x+k_y y-\omega t)$
E) Other!!

To think about: What is the y component?
What would change if the complex phase of E_0 was 90^0? -90^0?

Think about the first of Maxwell's Equations (Gauss's Law) in vacuum: $\vec{\nabla}\bullet\vec{E}=0$

Try a complex exponential "linearly polarized plane wave":

$$\tilde{\vec{E}}(x,y,z,t)=\tilde{\vec{E}}_0\exp\left[i\left(\vec{k}\bullet\vec{r}-\omega t\right)\right]$$

Then, Gauss's Law becomes:

A) $i\vec{k}\bullet\vec{E}_0=0$
C) $i\vec{k}\times\vec{E}_0=0$
B) $\vec{k}\left|\vec{E}_0\right|=0$
D) $\left|i\vec{k}\right|\left|\vec{E}_0\right|=0$
E) None of these.

What does this mean, in words?

Given the wave solutions

$$\tilde{\vec{E}}(x,y,z,t)=\tilde{\vec{E}}_0\,e^{i(\vec{k}\bullet\vec{r}-\omega t)},\qquad \tilde{\vec{B}}(x,y,z,t)=\tilde{\vec{B}}_0\,e^{i(\vec{k}\bullet\vec{r}-\omega t)}$$

What does Faraday's law $\vec{\nabla}\times\vec{E}=-\partial B/\partial t$ tell us?

A) $\nabla\times\tilde{\vec{E}}_0=-\dfrac{\partial\tilde{\vec{B}}_0}{\partial t}$

B) $\nabla\times\tilde{\vec{E}}_0=i\omega\tilde{\vec{B}}_0$

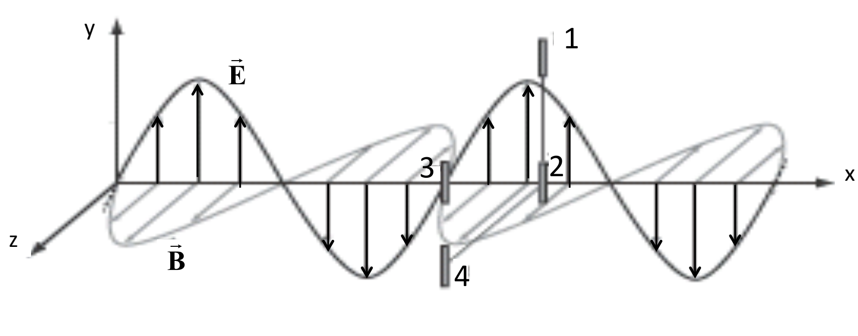
C) $k\tilde{\vec{E}}_0=\omega\,|\tilde{\vec{B}}_0|$

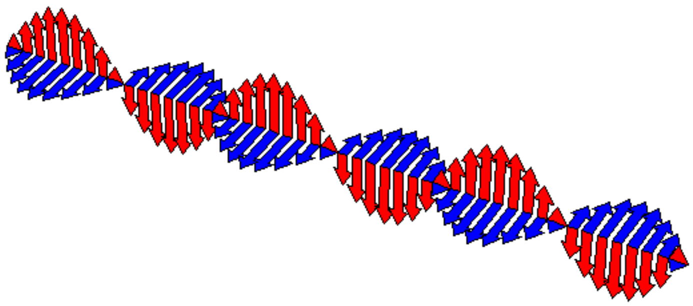
D) $\vec{k}\times\tilde{\vec{E}}_0=\omega\tilde{\vec{B}}_0$

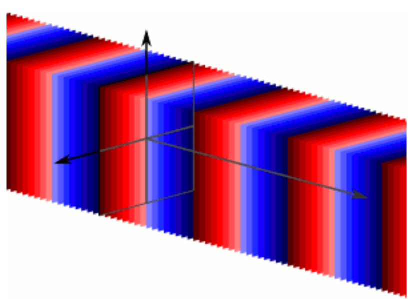
E) None of these!

An electromagnetic plane wave propagates to the right.
Four vertical antennas are labeled 1-4.
1,2, and 3 lie in the *x-y* plane.
1,2, and 4 have the same *x*-coordinate, but antenna 4 is located further out in the *z*-direction.
Rank the time-averaged signals received by each antenna.

A) 1=2=3>4 B) 3>2>1=4 C) 1=2=4>3
D) 1=2=3=4 E) 3>1=2=4







Griffiths considers a "wave on a 1D string", hitting a boundary between 2 strings of different propagation speeds. He gets a reflected and a transmitted wave.

$$\tilde{f} = \begin{cases} \tilde{A}_I e^{i(k_1 z - \omega t)} + \tilde{A}_R e^{i(-k_1 z - \omega t)} & \text{(z<0)} \\ \tilde{A}_T e^{i(k_2 z - \omega t)} & \text{(z>0)} \end{cases}$$

Boundary conditions (continuity) give the results:

$$\tilde{A}_R = \left(\frac{k_1 - k_2}{k_1 + k_2}\right)\tilde{A}_I, \qquad \tilde{A}_T = \left(\frac{2k_1}{k_1 + k_2}\right)\tilde{A}_I$$

Is the transmitted wave in phase with the incident wave?
A) Yes, always B) No, never C) Depends

"Wave on a 1D string", hitting a boundary between 2 strings of different speeds:

$$\tilde{f} = \begin{cases} \tilde{A}_I e^{i(k_1 z - \omega t)} + \tilde{A}_R e^{i(-k_1 z - \omega t)} & \text{(z<0)} \\ \tilde{A}_T e^{i(k_2 z - \omega t)} & \text{(z>0)} \end{cases}$$

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Is the transmitted wave in phase with the incident wave?
A) Yes, always B) No, never C) Depends

Why? How do you decide?

"Wave on a 1D string", hitting a boundary between 2 strings of different speeds:

$$\tilde{f} = \begin{cases} \tilde{A}_I e^{i(k_1 z - \omega t)} + \tilde{A}_R e^{i(-k_1 z - \omega t)} & (z<0) \\ \tilde{A}_T e^{i(k_2 z - \omega t)} & (z>0) \end{cases}$$

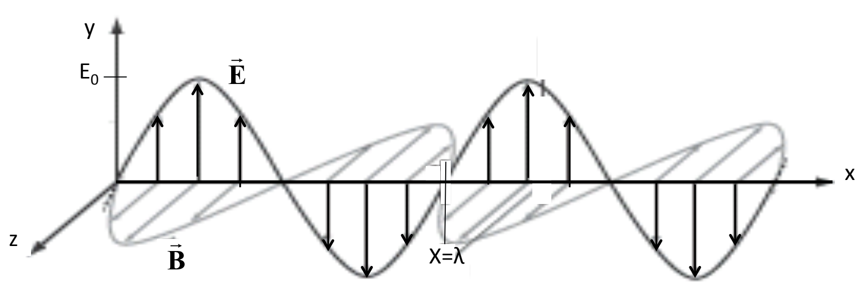
Boundary conditions (continuity) give the results:

$$\tilde{A}_R = \left(\frac{k_1 - k_2}{k_1 + k_2}\right)\tilde{A}_I, \qquad \tilde{A}_T = \left(\frac{2k_1}{k_1 + k_2}\right)\tilde{A}_I$$

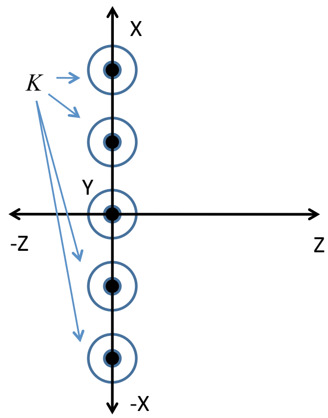
Is the reflected wave in phase with the incident wave?
A) Yes, always B) No, never C) Depends

Why? How do you decide?

Below is an idealized picture of a traveling EM plane wave.
It is a snapshot at t=0. (Wavelength λ is given, as is E_0.)
Write down a pair of mathematical formulae which describe this wave (in complex form) for all times. (One for **E** and **B**)

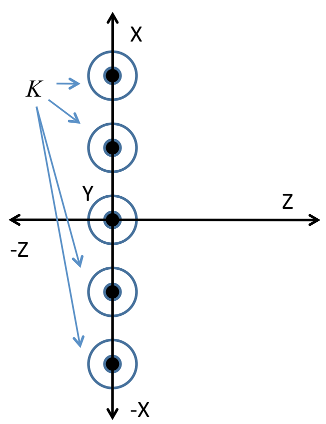


A charge neutral and infinite static current sheet, **K**, flows in the *x-y* plane, in the *y*-axis direction. Therefore, to the right of the *x-y* plane, according to what you know from Phys 3310, the **E** and **B** field directions are:



A) **E** along *z*-axis, **B** is zero
B) **B** along *z*-axis, **E** is zero
C) **B** along *y*, **E** along *z*
D) **B** along *x*, **E** along *y*
E) None of these

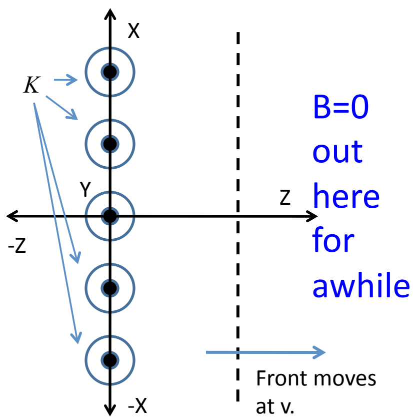
A charge neutral and infinite current sheet, **K**, is turned on at *t*=0, flows in the *x-y* plane, in the *y*-axis direction. Therefore, immediately afterwards, the **E** and **B** fields:



A) Remain zero (and for all time).
B) Immediately appear with their static values in all space.
C) Appear only near **K**
D) Appear only to the right of **K**
E) None of these

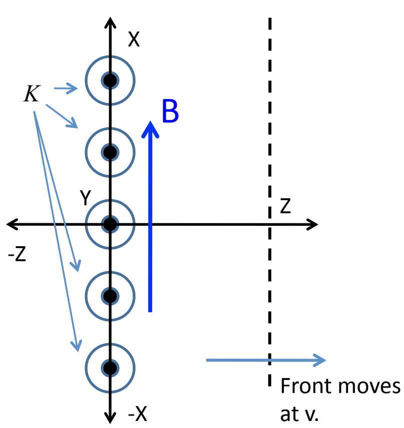
A charge neutral and infinite current sheet, **K**, is turned on at *t*=0, flows in the *x*-*y* plane, in the *y*-axis direction. Therefore, shortly afterwards, the **B** field near the sheet:

A) is in the *z*-direction
B) is in the *x*-direction
C) is in the *y*-direction
D) is actually zero close to **K**.
E) None of these



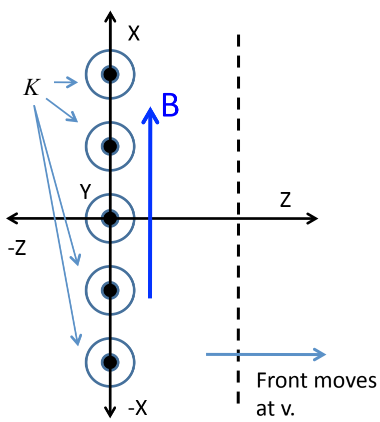
A charge neutral and infinite current sheet, **K**, is turned on at *t*=0, flows in the *x*-*y* plane, in the *y*-axis direction. Therefore, shortly afterwards, the **E** field very near the sheet:

A) is in the *z*-direction
B) is in the *x*-direction
C) is in the *y*-direction
D) is actually zero close to **K**.
E) None of these



B=0
out
here
for
awhile

A charge neutral and infinite current sheet, **K**, is turned on at *t*=0, flows in the *x*-*y* plane, in the *y*-axis direction. Therefore, shortly afterwards, the **E** field near the wavefront:



A) is in the –*z* direction
B) is in the –*x* direction
C) is in the –*y* direction
D) is actually zero close to *the front*.
E) None of these

B=0 out here for awhile

In matter, we have

$$\begin{cases} \nabla\cdot\vec{D}=\rho_F \\ \nabla\cdot\vec{B}=0 \\ \vec{\nabla}\times\vec{E}=-\dfrac{\partial\vec{B}}{\partial t} \\ \vec{\nabla}\times\vec{B}=\vec{J}_F+\dfrac{\partial\vec{D}}{\partial t} \end{cases}$$

$$\begin{cases} \vec{D}=\varepsilon_0\vec{E}+\vec{P} \\ \vec{H}=\vec{B}/\mu_0-\vec{M} \end{cases}$$

If there are no free charges or currents, can we argue

$$\nabla\cdot\vec{E}=0\ ?$$

A) Yes, always
B) Yes, under certain conditions (what are they?)
C) No, in general this will NOT be true!
D) ??

9.10a

For linear materials, we found that the index of refraction *n* is given by:

$$n = \sqrt{(1+\chi_E)(1+\chi_M)}$$

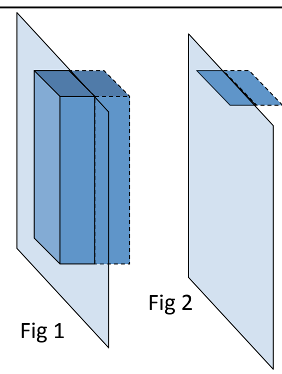
Can *n* be less than one?

A. Yes

B. No

In matter with no free charges or currents, we have:

$$\begin{cases} \text{i)}\ \nabla \cdot \vec{D} = 0 \\ \text{ii)}\ \nabla \cdot \vec{B} = 0 \\ \text{iii)}\ \vec{\nabla} \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t} \\ \text{iv)}\ \vec{\nabla} \times \vec{H} = +\dfrac{\partial \vec{D}}{\partial t} \end{cases}$$



To figure out formulas for boundary conditions, match the picture to the PDE(s) above.

A) Fig 1 goes with i and iii *(i.e. the ones involving D and E),*
Fig 2 goes with ii and iv *("" B and H)*

B) Fig 1 goes with i and ii *("" div),*
Fig 2 goes with iii and iv *("" curl)*

C) Fig 1 goes with ii only *("" B field),*
Fig 2 goes with iii only *("" E field)*

D) Something else! E) Frankly, I don't really understand this question.

i) $\nabla \cdot \vec{\mathrm{D}} = 0$

ii) $\nabla \cdot \vec{B} = 0$

iii) $\vec{\nabla} \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$

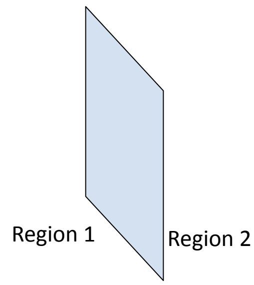
iv) $\vec{\nabla} \times \vec{H} = +\dfrac{\partial \vec{D}}{\partial t}$

1) $B_1^{\perp} = \mathrm{B}_2^{\perp}$

2) $\mathrm{E}_1^{\perp} = \mathrm{E}_2^{\perp}$

3) $\mathbf{B}_1^{//} = \mathbf{B}_2^{//}$

4) $\mathbf{E}_1^{//} = \mathbf{E}_2^{//}$



Match *the* Maxwell equation (i-iv) in matter (no free charges) with the corresponding *boundary condition* (1-4) it generates:

A) i ➔ 4, ii ➔ 3 iii ➔ 2 iv ➔ 1
B) i ➔ 2, ii ➔ 1 iii ➔ 4 iv ➔ 3
C) Wait, only SOME of the BC's on the right are correct!
D) Wait, NONE of the BC's on the right are correct!
E) Frankly, I don't really understand this question.

i) $\nabla \cdot \vec{\mathrm{D}} = 0$

ii) $\nabla \cdot \vec{B} = 0$

iii) $\vec{\nabla} \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$

iv) $\vec{\nabla} \times \vec{H} = +\dfrac{\partial \vec{D}}{\partial t}$

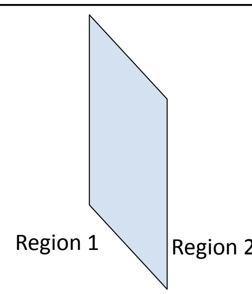
1) $B_1^{\perp} = \mathrm{B}_2^{\perp}$

2) $\varepsilon_1 \mathrm{E}_1^{\perp} = \varepsilon_2 \mathrm{E}_2^{\perp}$

3) $\mathbf{B}_1^{//} / \mu_1 = \mathbf{B}^{//} / \mu_2$

4) $\mathbf{E}_1^{//} = \mathbf{E}_2^{//}$

(For linear materials)



Match *the* Maxwell equation (i-iv) in matter (no free charges) with the corresponding *boundary condition* (1-4) it generates:

B) i ➔ 2, ii ➔ 1 iii ➔ 4 iv ➔ 3

An EM plane wave in free space comes from the left towards an interface.
Which statement is true?

A) Only certain frequencies are allowed.
B) You are free to choose the wave speed.
C) A compensating wave must travel towards the interface from the right too.
D) You may independently select the frequency and the k-vector.
E) None of the above.

An EM plane wave in free space comes from the left towards an interface. Which statement is true?

A) Only certain wave speeds are allowed.
B) You are free to choose **k**.
C) A reflected wave on the left and a transmitted wave on the right may travel away from the interface too.
D) All of the above.
E) None of the above.

An EM plane wave comes from the left towards an interface.
Reflected and transmitted waves leave.
Therefore, the total E-field is like:

$$\vec{E}_1 = \tilde{\vec{E}}_I \, e^{\left[i\left(\vec{k}_I \cdot \vec{r} - \omega_I t\right)\right]} + \tilde{\vec{E}}_R \, e^{\left[i\left(\vec{k}_R \cdot \vec{r} - \omega_R t\right)\right]} \qquad \text{(region 1)}$$

$$\vec{E}_2 = \tilde{\vec{E}}_T \, e^{\left[i\left(\vec{k}_T \cdot \vec{r} - \omega_T t\right)\right]} \qquad \text{(region 2)}$$

For our reflected and transmitted waves, how many unknowns have we introduced?

$$\mathbf{E}_R = \tilde{E}_{0R} e^{i(k_R z - \omega_R t)} \hat{n}_R$$

$$\mathbf{E}_T = \tilde{E}_{0T} e^{i(k_T z - \omega_T t)} \hat{n}_T$$

A. 2
B. 4
C. 8
D. 12
E. None of the above

An EM plane wave comes from the left towards an interface.
Reflected and transmitted waves leave.
Therefore, the total E-field is like:

$$\vec{E}_1 = \vec{E}_I \, e^{\left[i\left(\vec{k}_I \cdot \vec{r} - \omega_I t\right)\right]} + \vec{E}_R \, e^{\left[i\left(\vec{k}_R \cdot \vec{r} - \omega_R t\right)\right]} \qquad \text{(region 1)}$$

$$\vec{E}_2 = \vec{E}_T \, e^{\left[i\left(\vec{k}_T \cdot \vec{r} - \omega_T t\right)\right]} \qquad \text{(region 2)}$$

A) True.
B) False.
C) Don't understand and have questions
D) Too confused for questions...

An EM plane wave comes from the left towards an interface.
Reflected and transmitted waves leave. At the interface we expect to match the waves with something like: $\mathbf{E}_1^{//} = \mathbf{E}_2^{//}$

$$\left(\text{blah}\right)e^{\left[i\left(\vec{k}_I \cdot \vec{r} - \omega_I t\right)\right]} + \left(\text{blah}\right)e^{\left[i\left(\vec{k}_R \cdot \vec{r} - \omega_R t\right)\right]} = \left(\text{blah}\right)e^{\left[i\left(\vec{k}_T \cdot \vec{r} - \omega_T t\right)\right]}$$

$$\left(\text{blah}\right)e^{-i\omega_I t} + \left(\text{blah}\right)e^{-i\omega_R t} = \left(\text{blah}\right)e^{-i\omega_T t}$$

$$\omega_I = \omega_R = \omega_T$$

An EM plane wave comes from the left towards an interface. Reflected and transmitted waves leave. At the interface we expect to match the waves with something like: $\mathbf{E}_1^{//} = \mathbf{E}_2^{//}$

$$(\text{blah})e^{\left[i\left(\vec{k}_I \bullet \vec{r} - \omega_I t\right)\right]} + (\text{blah})e^{\left[i\left(\vec{k}_R \bullet \vec{r} - \omega_I t\right)\right]} = (\text{blah})e^{\left[i\left(\vec{k}_T \bullet \vec{r} - \omega_I t\right)\right]}$$

$$(\text{blah})e^{i\vec{k}_I \bullet \vec{r}} + (\text{blah})e^{i\vec{k}_R \bullet \vec{r}} = (\text{blah})e^{i\vec{k}_T \bullet \vec{r}}$$

$$\vec{k}_I \bullet \vec{r} = \vec{k}_R \bullet \vec{r} = \vec{k}_T \bullet \vec{r}$$

For our reflected and transmitted waves, how many unknowns have we introduced?

$$\mathbf{E}_R = \tilde{E}_{0R} e^{i(-k_I z - \omega_I t)} \hat{n}_I$$

$$\mathbf{E}_T = \tilde{E}_{0T} e^{i(k_T z - \omega_I t)} \hat{n}_I$$

A. 2
B. 4
C. 8
D. 12
E. None of the above

$$\begin{cases} 1)\ B_1^{\perp} = \mathrm{B}_2^{\perp} \\ 2)\ \varepsilon_1 \mathrm{E}_1^{\perp} = \varepsilon_2 \mathrm{E}_2^{\perp} \\ 3)\ \mathbf{B}_1^{//} / \mu_1 = \mathbf{B}^{//} / \mu_2 \\ 4)\ \mathbf{E}_1^{//} = \mathbf{E}_2^{//} \end{cases}$$

(For linear materials)

9.9

In matter without any free charge density, I can conclude that

A. Divergence of **D** and **E** are zero
B. Divergence of **D** is zero and divergence of **E** may be zero
C. Divergence of **D** and **E** are not zero
D. Divergence of **D** may be zero and divergence **E** is zero
E. Not enough information to tell

9.13c

In the case where medium 1 had a very slow wave velocity and medium 2 had a much higher wave velocity, $\mu_2 v_2 >> \mu_1 v_1$. Assuming that the permeabilities (μ's) are essentially equal to the permeability of the vacuum, what is the relation between ε_1 and ε_2?

A. $\varepsilon_1 \cong \varepsilon_2$
B. $\varepsilon_1 > \varepsilon_2$
C. $\varepsilon_1 < \varepsilon_2$
D. Not enough information to tell

9.13c

Suppose medium 1 has a very slow wave velocity and medium 2 had a much higher wave velocity, $v_2 >> v_1$.
Assuming that the permeabilities (μ's) are essentially equal to the permeability of the vacuum, what is the relation between ε_1 and ε_2?

A. $\varepsilon_1 \cong \varepsilon_2$
B. $\varepsilon_1 > \varepsilon_2$
C. $\varepsilon_1 < \varepsilon_2$
D. Not enough information to tell

9.12

$$\mathbf{E}_R = \tilde{E}_{0R} e^{i(k_R z - \omega_R t)} \hat{n}_R$$
$$\mathbf{E}_T = \tilde{E}_{0T} e^{i(k_T z - \omega_T t)} \hat{n}_T$$

For our reflected and transmitted waves, we found that $\omega_R = \omega_T = \omega_I$. What can we now conclude about the wavelengths of the transmitted and reflected waves?

A. $\lambda_R = \lambda_T = \lambda_I$
B. $\lambda_R = \lambda_T \neq \lambda_I$
C. $\lambda_R \neq \lambda_T = \lambda_I$
D. $\lambda_R = \lambda_I \neq \lambda_T$
E. Need more information

$$\begin{cases} 1)\ B_1^{\perp} = \mathrm{B}_2^{\perp} \\ 2)\ \varepsilon_1 \mathrm{E}_1^{\perp} = \varepsilon_2 \mathrm{E}_2^{\perp} \\ 3)\ \mathbf{B}_1^{//}/\mu_1 = \mathbf{B}^{//}/\mu_2 \\ 4)\ \mathbf{E}_1^{//} = \mathbf{E}_2^{//} \end{cases}$$

(For linear materials)

For light at normal incidence, we found:

$$\mathrm{R} = \frac{(\mathrm{n}_1 - n_2)^2}{(\mathrm{n}_1 + n_2)^2}, \quad T = \frac{4 n_1 n_2}{(\mathrm{n}_1 + n_2)^2}$$

What gives a large transmission of light at normal incidence?

A) When $v_1 >> v_2$
B) When $v_2 >> v_1$
C) When v is very *different* in the two media
D) When v is nearly the *same* in the two media
E) None of these/other/I'm confused/…

9.8a

For an electric plane wave given by $\tilde{\vec{\mathbf{E}}} = \tilde{\vec{\mathbf{E}}}_{\mathbf{0}} e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$

The contribution from the E field to the (real) energy density u_{EM} is

A. $\frac{1}{2}\varepsilon_0(\tilde{\vec{\mathbf{E}}})^2$

B. $\frac{1}{2}\varepsilon_0 \mathrm{Re}\{(\tilde{\vec{\mathbf{E}}})^2\}$

C. $\frac{1}{2}\varepsilon_0(\mathrm{Re}\{\tilde{\vec{\mathbf{E}}}\})^2$

D. $\frac{1}{2}\varepsilon_0 \tilde{\vec{\mathbf{E}}}^* \cdot \tilde{\vec{\mathbf{E}}} = \frac{1}{2}\varepsilon_0 \left|\tilde{\vec{\mathbf{E}}}\right|^2$

E. More than one of the above is true

9.8a

For an electric plane wave given by $\tilde{\vec{\mathbf{E}}} = \tilde{\vec{\mathbf{E}}}_{\mathbf{0}} e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$

The contribution from the E field to the (real) energy density u_{EM} is

A.

B.

C. $\frac{1}{2}\varepsilon_0(\mathrm{Re}\{\tilde{\vec{\mathbf{E}}}\})^2$

D.

E. More than one of the above is true

9.8a

For an electric plane wave given by $\mathbf{E} = \tilde{\mathbf{E}}_{\mathbf{0}} e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} = E_0 e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t+\delta)}\hat{n}$

The Poynting vector is given by $\vec{S} = \frac{1}{\mu}\vec{E}\times\vec{B}$

(And, $v^2 = \frac{1}{\varepsilon\mu}$)

$$\text{So, } \mathrm{S} = \varepsilon v E_0^2 \cos^2(\mathbf{k}\cdot\mathbf{r} - \omega t + \delta) = \varepsilon v[\mathrm{Re}(\tilde{\vec{E}})]^2$$

$$\mathrm{I} = \langle \mathrm{S} \rangle = \frac{1}{2}\varepsilon v E_0^2 \quad = \frac{1}{2}\varepsilon v\left|\tilde{\vec{E}}\right|^2$$

(Surprise?! It's *not* ~[Re(E)]^2 !)

9.14

In the case where medium 1 had a very slow wave velocity and medium 2 had a much higher wave velocity, we found that R➔1 and T➔0. In the opposite case, where the wave velocity in medium 1 is much higher than that in 2, we expect

A. R➔1, T➔0
B. R➔0, T➔1
C. R➔1/2, T➔1/2
D. Not enough information to tell

When plane EM waves are incident on a planar interface, what defines the "plane of incidence"?

$$\begin{cases} 1)\ B_1^{\perp} = \mathrm{B}_2^{\perp} \\ 2)\ \varepsilon_1 \mathrm{E}_1^{\perp} = \varepsilon_2 \mathrm{E}_2^{\perp} \\ 3)\ \mathbf{B}_1^{//}/\mu_1 = \mathbf{B}^{//}/\mu_2 \\ 4)\ \mathbf{E}_1^{//} = \mathbf{E}_2^{//} \end{cases}$$

(For linear materials)

$$\Rightarrow$$

$$\tilde{E}_{0R} = \left(\frac{\alpha-\beta}{\alpha+\beta}\right)\tilde{E}_{0I}$$

$$\tilde{E}_{0T} = \left(\frac{2}{\alpha+\beta}\right)\tilde{E}_{0I}$$

When plane EM waves are incident on a planar interface, what defines the plane of incidence?

A. The planar interface
B. $\mathbf{k}_I$
C. The normal of the planar interface
D. $\mathbf{k}_T$
E. None of the above

$$\begin{cases} 1)\ B_1^{\perp} = B_2^{\perp} \\ 2)\ \varepsilon_1 E_1^{\perp} = \varepsilon_2 E_2^{\perp} \\ 3)\ \mathbf{B}_1^{//}/\mu_1 = \mathbf{B}^{//}/\mu_2 \\ 4)\ \mathbf{E}_1^{//} = \mathbf{E}_2^{//} \end{cases} \Rightarrow \quad \tilde{E}_{0R} = \left(\frac{\alpha-\beta}{\alpha+\beta}\right)\tilde{E}_{0I}, \quad \tilde{E}_{0T} = \left(\frac{2}{\alpha+\beta}\right)\tilde{E}_{0I}$$

$$where \quad \alpha \equiv \cos\theta_T / \cos\theta_I, \quad \beta \equiv \mu_1 v_1 / \mu_2 v_2 \approx n_2 / n_1$$

(For linear materials)

What is the relative phase angle between the incident wave and the transmitted wave?

A. 0
B. 90
C. 180
D. Can be more than one of the above
E. Depends on the incident angle and dielectric properties

For reflected and transmitted waves at oblique incidence and electric polarization parallel to (in) the plane of incidence,
Fresnel's equations are:

$$\tilde{E}_{0R} = \left(\frac{\frac{\cos\theta_T}{\cos\theta_I} - \frac{\mu_1 v_1}{\mu_2 v_2}}{\frac{\cos\theta_T}{\cos\theta_I} + \frac{\mu_1 v_1}{\mu_2 v_2}} \right) \tilde{E}_{0I}, \qquad \tilde{E}_{0T} = \left(\frac{2}{\frac{\cos\theta_T}{\cos\theta_I} + \frac{\mu_1 v_1}{\mu_2 v_2}} \right) \tilde{E}_{0I}$$

What is the relative phase angle between the incident wave and the reflected wave?

A. 0
B. 90
C. 180
D. Can be more than one of the above
E. Depends on the incident angle and dielectric properties

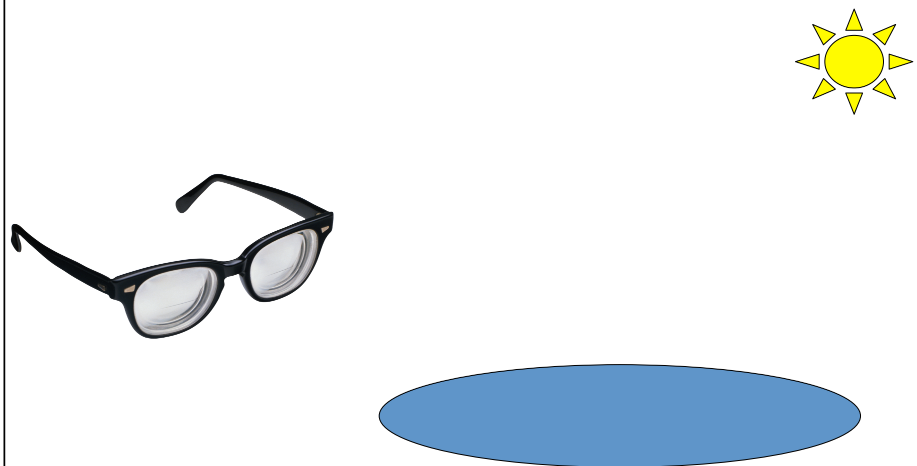
$$\begin{cases} 1)\ B_1^{\perp} = B_2^{\perp} \\ 2)\ \varepsilon_1 E_1^{\perp} = \varepsilon_2 E_2^{\perp} \\ 3)\ \mathbf{B}_1^{//}/\mu_1 = \mathbf{B}^{//}/\mu_2 \\ 4)\ \mathbf{E}_1^{//} = \mathbf{E}_2^{//} \end{cases} \Rightarrow \quad \tilde{E}_{0R} = \left(\frac{\alpha-\beta}{\alpha+\beta}\right)\tilde{E}_{0I}, \quad \tilde{E}_{0T} = \left(\frac{2}{\alpha+\beta}\right)\tilde{E}_{0I}$$

(For linear materials)

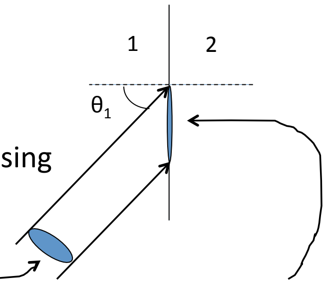
where
$\alpha \equiv \cos\theta_T / \cos\theta_I$
$\beta \equiv \mu_1 v_1 / \mu_2 v_2 \approx n_2 / n_1$

What is the relative phase angle between the incident wave and the transmitted wave?

A. 0
B. 90
C. 180
D. Can be more than one of the above
E. Depends on the incident angle and dielectric properties



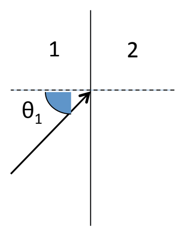
To reduce glare off the lake, should the polarization axis of these sunglasses be
A) Vertical B) Horizontal C) Makes no difference D) ????



The power/m² passing
Here is <S>

What is the power/m² striking the boundary wall?

A) Still <S> B) <S> cosθ_1 C) <S> /cosθ_1
D) Something else! (sinθ_1 or cos²θ_1 or cosθ_2, or …)



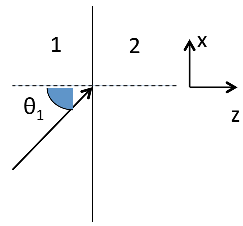
Our general solution for the transmitted wave is

$$\vec{\tilde{E}}_T(\vec{r},t) = \vec{\tilde{E}}_{0T} e^{i(\vec{k}_2 \cdot \vec{r} - \omega t)}$$

Snell's law tells us $n_1 \sin\theta_1 = n_2 \sin\theta_2$

If $n_2<n_1$, there is a critical angle, $\sin\theta_{1,C} = \eta_2 / n_1$
beyond which there is no real solution for θ_2.

How should we interpret this lack of solution *physically?*



$$\tilde{\vec{E}}_T(\vec{r},t) = \tilde{\vec{E}}_{0T} e^{i(\vec{k}_2 \cdot \vec{r} - \omega t)}$$

If we are pigheaded, we can proceed with $\cos\theta_2$ *imaginary,* so

$$\vec{k}_2 \cdot \vec{r} = (real) * x + (imaginary) * z$$

Can we interpret this?

The square root of a + b*i* is

A) $\sqrt{a} + i\sqrt{b}$

B) $\sqrt{a^2 + b^2}$

C) $\sqrt{a^2 - b^2} + i\sqrt{2ab}$

D. It is not defined

E. Something else (this is harder than it looks)

We have a traveling wave solution $\vec{\tilde{E}}(\vec{r},t) = \vec{\tilde{E}}_0 e^{i(\tilde{k}\, z - \omega t)}$
where the (complex) wave vector satisfies

$$\tilde{k}^2 = \omega^2 \mu\varepsilon + i\,(\omega\mu\sigma)$$

True (A) or False (B): This traveling wave is “transverse”.

(Or C) I have no good idea what that means)

The magnetic field amplitude in a metal associated with a linearly polarized electric EM wave is

$$\tilde{B}_0 = \left(\frac{k_R + ik_{\mathrm{Im}}}{\omega} \right) \tilde{E}_0$$

True (A) or False (B):The B field is in phase with the E field.

(C) It depends!

The magnetic field amplitude in a highly conductive metal (σ>>εω) associated with a linearly polarized electric EM wave is

$$\tilde{B}_0 = \sqrt{\frac{\mu\sigma}{\omega}}\frac{1+i}{\sqrt{2}}\tilde{E}_0$$
$$= \sqrt{\frac{\sigma}{\varepsilon_0\omega}}\frac{1+i}{\sqrt{2}}\frac{\tilde{E}_0}{c}$$

True (A) or False (B): The B field is in phase with the E field.

(C) It depends!

For a good conductor, $\tilde{B}_0 = (\ldots)e^{i\pi/4}\tilde{E}_0$

Is the B field

A) Leading B) Lagging C) Matching D) It depends!

the E field.

9.21

Maxwell's equations in matter are:

$$\nabla \cdot \mathbf{D} = \rho_f \qquad \nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \nabla \times \mathbf{H} = \mathbf{J_f} + \frac{\partial \mathbf{D}}{\partial \mathbf{t}}$$

At the boundary between a linear dielectric insulator and a metal conductor, what will be the boundary condition for $E^{\perp}$

A. $\varepsilon_1 E_1^{\perp} = \varepsilon_c E_c^{\perp}$
B. $\varepsilon_1 E_1^{\perp} = \varepsilon_c E_c^{\perp} - \sigma_f$
C. $E_1^{\perp} = E_c^{\perp} - \sigma_f / \varepsilon_c$
D. $E_1^{\perp} = E_c^{\perp}$
E. None of the above

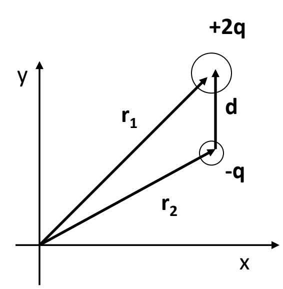
MD6 - 3

For a collection of point charges, the dipole moment is defined as

$$\vec{p} = \sum_i q_i \, \vec{r}_i$$

Consider the two charges, +2q and –q, shown.
Which statement is true?

A) The dipole moment is independent of the origin.
B) The dipole moment depends on the position of the origin.
C) The dipole moment is zero.
D) The dipole moment is undefined.



In a long-ago homework, you figured out why the "jumping ring" demo works.

We had a solenoid with a sinusoidal current, and a ring around it which got an *induced* current (from Faraday's law). Those two currents repelled and the ring jumped.
Except,… sometimes they attracted!!

Averaged over one cycle, what was the average repulsion?

What made it so that, on average, we DID get a net repulsion?

If $E_x(x,y,z,t)=E_0 \exp[i (kz-\omega t)]$, and you have a free charge q which responds to this E field, (F=qE)
what can you say about the relative phase of v(t) and E(t) at any point in space?

A) They are in phase
B) They are 90° out of phase
C) They are 180° out of phase
D) I don't really know what this means
E) Something else…

The Fresnel equation (for normal incidence) is $\tilde{E}_{0R} = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)\tilde{E}_{0I}$

If region 2 is a conductor, n_2 = (c/ω) k_2 is *complex!*

But, recall that for a "good conductor", $\omega/k_R \ll c$.
What do you conclude?

A) $\tilde{E}_{0R} \approx \tilde{E}_{0I}$ B) $\tilde{E}_{0R} \approx -\tilde{E}_{0I}$ C) $\tilde{E}_{0R} \approx 0$

D) Something more complicated (literally, complex!)
E) ???

From last class: If you *model* a dielectric as "charges on springs" (with damping) a traveling EM wave will drive those charges, resulting in polarized molecules.
If x(t)=displacement of charge q, and N = molecules/volume, what is the volume polarization, P?

A) $P = x(t)$ B) $P = qx(t)$ C) $P = Nx(t)$
D) $P = Nq\,x(t)$ E) Something else!?

$$\nabla^2 \vec{E} = \varepsilon\mu \frac{\partial^2 \vec{E}}{\partial t^2},$$

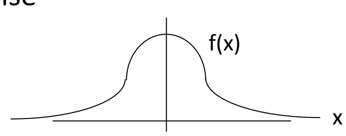
$$\tilde{\vec{E}} = \tilde{\vec{E}}_0 e^{i(kz - \omega t)}$$

The index of refraction is $n = ck/\omega = c\sqrt{\varepsilon\mu}$

1) What does it mean if n is complex?
2) What does it mean if n depends on ω?

Fourier tells us that we can write a "pulse" by summing up sinusoidal functions:

$$f(x) = \int_{-\infty}^{\infty} a(k) e^{ikx}\, dk$$



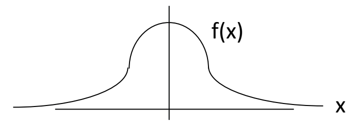
If you were to compute $\int_{-\infty}^{\infty} a(k) e^{i(k(x - vt))}\, dk$ (with v a known constant) what would that give?

A) $f(x)$ B) $f(vt)$ C) $f(x - vt)$

D) Something complicated!

E) ???

Fourier tells us that we can write a "pulse" by summing up sinusoidal functions:



$$f(x) = \int_{-\infty}^{\infty} a(k) e^{ikx} \, dk$$

If you were to compute $\int_{-\infty}^{\infty} a(k) e^{i(k(x - v(k)t))} \, dk$

what would that give?

A) $f(x)$ B) $f(vt)$ C) $f(x - vt)$

D) Something complicated!

E) ???

Summary of last class: dispersion in a dilute dielectric:

Step1: Incoming E_{ext} polarizes electrons, so use Newton's law (with $qE_{ext} e^{-i\omega t}$ "driver") to solve for x(t) of the electron.

Step 2: Compute the polarization density (p=qx, then P=Np)(summing over different possible electron states)

Step 3: Use $D=\varepsilon_0 E+P$ (always), and $D=\varepsilon E$ (if linear), to extract ε. Done! We have ε (and thus n) for this material.

In which step did we assume the material was "dilute"?

A) Step 1 B) Step 2 C) Step 3 D) More than 1 step

E) Never! The result is quite general for this "charged-spring" model

For our atomic model of permittivity we found ε to be

$$\tilde{\varepsilon} = \varepsilon_0 \left(1 + \frac{Nq^2}{\varepsilon_0 m} \sum_i \frac{f_i}{(\omega_i^2 - \omega^2) - i\gamma_i \omega} \right)$$

We also know $\frac{n}{c} = \frac{\tilde{k}}{\omega} = \sqrt{\tilde{\varepsilon}\mu}$

i) Find (and simplify) a formula for n, assuming the term adding to "1" above is *small.*

ii) In that limit, find k_R and k_{Im}.
What does each one tell you, physically?

iii) Sketch both of these as functions of ω (assuming that only one term in that sum "dominates")

(Under what condition on ω *should* one term in that sum dominate?)

9.23

For our atomic model of permittivity we found the absorption coefficient to be:

$$\alpha = \frac{Nq^2\omega^2}{mc\varepsilon_0} \sum_i \frac{f_i \gamma_i}{(\omega_i^2 - \omega^2)^2 + \gamma_i^2 \omega^2}$$

In the limit as ω goes to infinity, α

A. Goes to zero
B. Approaches a constant
C. Goes to infinity
D. I don't know!

9.24

For our atomic model of permittivity we found the absorption coefficient to be:

$$\alpha = \frac{Nq^2\omega^2}{mc\varepsilon_0}\sum_i \frac{f_i\gamma_i}{(\omega_i^2-\omega^2)^2+\gamma_i^2\omega^2}$$

In the limit of no damping, absorption

A. Goes to zero
B. Approaches a constant
C. Goes to infinity
D. Other, (it’s not so simple).
E. I don’t know!