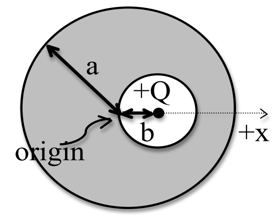
Homework Set 1, Physics 3320, Fall 2014
Due Wednesday, Sep 3 (start of class)

In general, each lettered part of a problem will be worth the same # of points.
In all problems, show your work and explain your thinking. *Correct answers for which we cannot follow the work are worth no credit.*

Note that office hours Monday are cancelled due to the holiday, but our homework help session (11th floor, 4-6) Tuesday will be held as usual.



1. Consider a solid, neutral, conducting metal sphere (radius a), centered at the origin, as shown.
It has a small spherical cavity inside it (an off-center hole) with radius b, centered on the +x axis, a distance "b" away from the origin.
At the center of the cavity, i.e. at (b,0,0), is a positive point charge +Q.

(a) Describe how charge is distributed on the metallic shell. Be specific: where does the charge reside and how is it distributed? Explicitly calculate any charge densities that are non-zero and discuss what kind of symmetry they have (if any).

(b) Determine the electric field **E**(x) *everywhere* along the x axis. You may use whatever method you think best, but be sure to explain your reasoning clearly.
(I'd like formulas. They may be different in different regions of space)
- Also, sketch the magnitude |E(x)| as a function of x.
Note all interesting features of the sketch, including e.g. the value of |E|, in terms of given symbols, at any "interesting" points…)
- Finally, sketch the E field lines everywhere in space, so we can see the "pattern".

(c) Find the voltage *V(x)* everywhere along the x-axis, using the usual assumption that $V(x\to\infty) = 0$. Then, sketch V(x), noting interesting features (like the value of V at interesting points, in terms of given symbols.)

2. Suppose an electric field in a region of space is given by

$$\mathbf{E} = \frac{B_0}{2\tau}\left(-y\hat{x} + x\hat{y}\right) \qquad (1)$$

where τ is a constant with units of time and B_0 is some given constant.

(a) What are the units of the constant B_0 ?

Sketch the E-field in the *x–y* plane (by hand), and then check yourself with *Mathematica* (Hint: VectorPlot[...] is a built in command! Use MMA help to learn the syntax, and include your *MMA* plot with your homework. I prefer everyone does this on their own – run and print MMA yourself, don't let a partner do this for you)

(Continued…)

(b) Calculate the closed line integral

$$\oint_L \mathbf{E} \cdot d\ell \qquad (2)$$

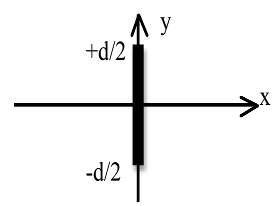
where L is a circle of radius R, parallel to the x–y plane, centered at $(0,0,z_0)$. Integrate counterclockwise as viewed from "above". (i.e viewed from +z)

(c) Calculate the closed line integral equation (2) again, this time where L is a rectangle (sides of length a and b, centered around the origin) oriented parallel to the x–y plane. (Again, integrate counterclockwise as viewed from "above") Clearly describe in words how the value of equation (2), here and in part (b), depends on B_0/τ and the geometry of L.

(d) Calculate $\nabla \times \mathbf{E}$ and describe the resulting vector field in words. Show that the closed line integral values in 3(b) and 3(c) are equal to the corresponding surface integrals of $\nabla \times \mathbf{E}$. Relate this to Stokes' theorem!

(e) Determine the scalar potential that gives equation (1) (or explain why no such potential exists.) Finally - describe how you could use such a static electric field to make a lot of money and save the world. Does the electric field in this problem violate any of Maxwell's equation for a static situation?
Does this "mathematical exercise" strike you as a physically unrealistic problem? We'll come back to this field soon, there's more to it than meets the eye!

3. Consider a thin rod of length d, negligible diameter, and uniform charge per length λ.
The rod is centered on the origin , oriented as shown in the diagram.



a) Compute the voltage at position x on the x-axis. (As usual $V(r\to\infty) = 0$)

b) From the voltage, compute the E-field at position x on the x-axis. Simplify the result as much as possible.

c) Write down expressions for the E-field in the limits x>>d/2 and 0 < x << d/2. (Note that the limit x >> d/2 does not mean x = ∞. At x =∞ , the E-field is zero. We want an expression for how E approaches zero, as x goes to infinity.)

d) Argue that the limits you wrote down in part (c) make sense, and that you could have figured out both limits directly without first computing the exact expression. *(Note: I'm guessing the case x>>d/2 should be "obvious" to you. For x<<d/2, think about it physically. Can you work out the E field in that limit without nasty integrals?)*

4. Magnetic Resonance Imagers:
Example 5.6 in Griffiths gives the magnetic field along the axis of a single circular loop of wire of radius a, carrying a current I. *In this problem, consider a solenoid, which is effectively many such circular loops stacked up to make a cylinder.*
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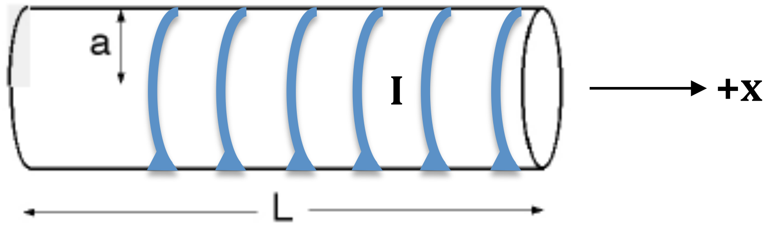

(a) Let's design a cylindrical solenoid for a magnetic resonance imager capable of imaging an adult human. Assume the solenoid has radius *a*, length *L*, is wrapped with *n* turns per unit length of wire, and the wire carries current *I*. Derive the functional form of the field, **B**(x) along the center line of the device, by integrating the contributions of a set of infinitesimal current loops. (I would choose x=0 to be at the very middle of the solenoid)

Explicitly show that your result approaches the "known" result (e.g. Example 5.9 of Griffiths, or last week's last problem) if the solenoid becomes infinite.

(b) Choosing appropriate (physically sensible) values for the length *L* and radius *a* (discuss your choices!), use *Mathematica to* make a simple plot of the magnetic field vs. position (just along the central, x-axis)
Please make your horizontal axis be the "dimensionless variable" x/L,
and your vertical axis be the "dimensionless magnetic field" $B/(\mu_0 n I)$.
Verify graphically that for $L \gg a$ the field near the center of the solenoid is nearly constant and is nearly equal to the field inside an infinite solenoid
(On the plot you turn in, briefly discuss any interesting features.)

Extra Credit c) *(worth as much as one lettered part above).*
Show that your solution for the B field satisfies Ampere's Law around the following Amperian loop: along the central (x) axis from $-\infty$ to $+\infty$, then perpendicular to the axis for an infinite distance, then parallel to the *x*–axis from $+\infty$ to $-\infty$ but an infinite distance from the axis, and finally, close the loop at $-\infty$.
Give a convincing argument that the contributions from the paths at infinity vanish.

***Extra Credit** d) (worth as much as one lettered part above).*
The field near the center of CU's new research MRI is 3 T. Explain why it would be infeasible for an MRI magnet to use ordinary copper wire.
(This means making some reasonable guesstimates about wiring and geometry, looking up basic information you might need... You will need some basics from freshman physics, which is all contained in Griffiths 7.1.1 We aren't looking for a very detailed or particularly accurate calculation, but an order-of-magnitude estimate to show why this would not work in practice. Can you guess what the technological solution is to this problem?)