Homework Set #10, Physics 3320, Fall 2013, Due Wednesday, Nov 5 (start of class)

1. In Griffiths (and in class), we derived equations for EM waves in conductors, assuming a "good conductor". Interestingly, it turns out that the formulas we got can be pushed a good deal further than you might naively expect, into regimes where e.g. σ is not so large ("poor conductors".) In this case, you will need to use Griffiths more careful results (9.126) for the real and imaginary parts of the k vector and work with those.

A) Based on the above comments, show that the skin depth for a "poor conductor" (*i.e.,* $\sigma \ll \varepsilon\omega$) is $d \approx ??\sqrt{\varepsilon/\mu}$, independent of frequency or wavelength. Work out what the "??" is in this equation. (Also, check the units of your answer <u>explicitly</u>, please!)

B) Show that the skin depth for a "good conductor" (*i.e.,* $\sigma \gg \varepsilon\omega$) is $d \approx \lambda/??$, where λ is the wavelength in the conductor. (Work out the ?? in that formula.) Find the skin depth for microwaves in Cu ($f = 2.5$ GHz). (Briefly, how do you interpret that answer physically?)

C) About how thick does aluminum foil have to be, to be optically opaque? (Comment briefly. We talked about this in class the day after the solar eclipse – this was how the viewers work!)

D) For biological tissues (like skin), ε and σ depend on frequency, you can't use their free space values. (μ on the other hand is close to its free space value) At microwave frequencies (say, about 2.5 GHz), their values are ~ $\varepsilon=47\,\varepsilon_0$, and $\sigma=2.2\ \Omega^{-1}\text{m}^{-1}$. Is this the "good conductor" or "poor conductor" case, or neither? Evaluate the skin depth for microwaves hitting your body.

E) If the EM wave from part D (e.g. from a radar station) hits your body, what fraction of the incident power do you absorb? *(Hint: think about "R" first, then you can get "T" easily!)*

F) Let's think about contacting submarines by radio... For low frequency radio waves (say, f=3 kHz) estimate the skin depth in the sea, and comment on the feasibility/issues of such radio communication. (What is the wavelength of this same radio wave in *free space*, by the way?) (*Hint: Treating seawater as like a human body is ok for this one- just use the given values from part D as needed. But, as mentioned there, in reality they'll be different at this very different* frequency *and slightly different material. Gold star if you can find more appropriate values, and give us the reference!)*

2. In Prof. Cumalat's lab in the high energy experimental group at CU, they are working on novel particle detectors made of (artificial) diamond. If the CERN accelerator beam deposits some free charge into the bulk of such a piece of diamond, make a rough estimate for how long it would take for the charge to flow to the detector surface (and thus presumably be carried away to ground). *(Hint: you will need to look some numbers up. Start with Griffiths' Table 7.1!)*

Fun stuff to think about, but not required for the homework: There are many very practical real world issues if you dig into this. Look at the LHC Wikipedia page to learn about the "bunching" of protons in the CERN beam. Do you think the possible buildup of charge inside the bulk of this detector is an issue for them? Next, look up the resistivity of diamond online – you will probably find it is 10-15 orders of magnitude larger than the CRC value quoted by Griffiths (!!) I don't know the source of this huge discrepancy. If you take the larger value, what's your <u>new</u> conclusion regarding diamond detectors at CERN? And, what additional information would we need to investigate? (Turns out they "clear charge" out of their detectors by applying a voltage across the detector to sweep any buildup out in time for the next bunch' arrival…)

3. We keep saying that you can always sum up plane waves to get real wave packets. Let's try it! Consider a localized wavepacket that satisfies the one-dimensional wave equation from a sum of sinusoidal waves using Fourier's integral method

$$f(x,t) = \int_{-\infty}^{\infty} A(k) e^{ik(x-ct)} dk.$$

- First, show that $f(x,t)$ satisfies the wave equation with wave speed c.
- Assume $A(k)$ is given by a Gaussian distribution centered at some positive wavevector k_0.

$$A(k) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(k-k_0)^2\sigma^2}{2}\right)$$

- Sketch this function. Roughly, what range of wavevectors Δk contribute significantly to the wave packet?

B) Calculate $f(x,t)$ from the above A(k).

Hint: There is a famous and handy Gaussian integral: $\int_{-\infty}^{\infty} exp\left(-\frac{q^2y^2}{2} + zy\right) dy = \sqrt{\frac{2\pi}{q^2}} exp\left(\frac{z^2}{2q^2}\right)$.

(This works for ANY z, even complex!)

- Describe $f(x,t)$ physically as best you can. How is the *x*-width Δx of the "wavepacket" related to the k-width Δk? Does this relationship between Δk and Δx remind you of anything from quantum mechanics (PHYS 2170 or 3220)?

Pick a visible wavelength and sketch or plot (with MMA) a Gaussian pulse that lasts 1 femtosecond. (When I say sketch, I'm thinking of Re[f(x,t=0)], and then think about what happens as time goes by) Try to get as many details reasonably correct as you can.

Laser physicists can indeed create pulses of visible light that last only a few femtoseconds or less. Graduate and undergraduate students in the labs of Professors Kapteyn, Murnane, Schibli, Raschke and others here at CU do this every day.

4. Spherical waves: I mentioned in class that later in the term we will consider electromagnetic radiation from a "pointlike" antenna, and will get what are called spherical waves, rather than plane waves. Here is a *simplified* mathematical formula for such a spherical wave (simplified meaning that it is accurate for large distances from the origin)

$$\vec{E}(r,\theta,\phi,t) = A\frac{\sin\theta}{r}\cos(kr-\omega t)\hat{\phi}$$

A) Why would we call this a "spherical wave"? Describe it in words or pictures as best you can. Far from the origin, how is it similar to/different from a polarized traveling plane wave?

- Show that this wave satisfies Gauss' law in free space. Then use Faraday's law to find the formula for the corresponding magnetic field. Since we are assuming we are far from the origin, you can (and should!) toss terms that appear which drop off FASTER than 1/r...

B) Discuss the physics of this B field. Does it agree with everything you already know from plane waves about how **E** and **B** are supposed to be related to each other (and to the direction of propagation)? *(consider magnitudes, and directions)*

- Verify that the formulas you just got (for **E** and **B**) satisfy the remaining two Maxwell equations in free space (remember that you are ignoring any terms that drop faster than 1/r!)

C) What is the direction of the Poynting vector? Does that make physical sense?

- Find the mathematical fall-off with distance from the origin, and argue that your result makes physical sense, given simple arguments about energy conservation/flow from a point source.