HW #12, Physics 3320, Fall 2014, Due Wednesday, Dec 3 (start of class, right after T-giving)

1. A) In my lecture notes, I derived the *radiation resistance* of a wire joining two ends of a little dipole in electric dipole radiation. Derive that formula for yourself (and us). Then, let's think about applying it to your cell phone. Assuming we can use the "small electric dipole approximation" all the way up to the scale of wires in the phone, is the radiative contribution to the total resistance of the wire dominant to, comparable to, or tiny compared to the ordinary Ohmic resistance of the same wire? (You'll need to dig up the frequency at which your cell phone operates. Assume the length of the radiating antenna wire is roughly the size of your phone.) *(The assumptions here are not realistic for a real antenna, but this is the best we can do working with this section of Griffiths!)*

B) We won't work out the details of <u>magnetic</u> dipole radiation in class (Section 11.1.3) but the result in Eq 11.40 is fairly similar to 11.22, and easy enough to work with. (In that formula, m_0 is the magnetic dipole moment of the small current loop, see Eq 11.24) Work out the radiation resistance of a small magnetic dipole. For comparable sized dipoles (at the same frequency) one electric dipole and one magnetic dipole, which has the greater radiation resistance?
Is this consistent with Griffiths comment at the end of section 11.1.3?

2. A pirate radio station has been set up behind a residential home in East Boulder. At the top of a tall (height H) ham radio tower is a small electric dipole antenna, of length d, with its axis oriented vertically. They broadcast at a frequency ω, with total time averaged radiated power P. A neighbor living nearby is a friend of yours. Although they don't want to turn the station in to the authorities, your friend is worried that they are being "irradiated", with possible health effects. Knowing you're a physics major, they ask for your help:

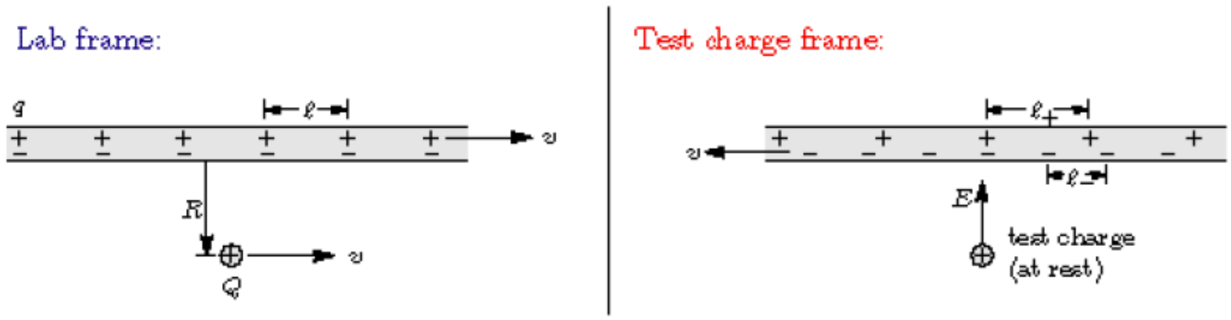
A) In terms of variable names given above, find a formula for the *intensity* of EM radiation at ground level a distance "s" away from the tower. What assumptions are you making in generating this formula? List them explicitly!

B) Let's identify the "worst case" – where should you measure the intensity for it to be as *large* as it can get at ground level? Use your formula and rewrite it at this spot. Simplify as much as possible.

C) Let's put in some plausible numbers. Suppose the pirate station is broadcasting at 100 MHz (FM), and putting out a power of 20 kW (That's still a big power draw for a private station, maybe enough to attract attention from the power company!) Their electric dipole antenna is 5 cm tall, and the tower is 50 m tall. According to an FCC information webpage, 100 mW/cm^2 is where bodies start to heat up (!), and in some circumstances intensity levels down to 1-10 mW/cm^2 are potentially harmful (e.g to eyeballs). What do you conclude – does your friend need to pull the plug on this thing? (Let's not get into ethical issues about pirate stations, and focus only on EM radiation exposures!)

3. In a classical model, an electron in a hydrogen atom would circulate in a nice simple orbit, whose radius would be given by Newton's law, F=ma (so, the attractive Coulomb force from the nucleon on the electron = mv^2/r.) The electron's total energy would be given by the sum of kinetic and Coulomb potential energy. Use the Newtonian force equation you just wrote down to eliminate "v" from the kinetic energy term, so you have a simple formula for E(r) for the electron. So far, nothing fancy - this should all be Physics 1120 level work. Now you can take into account radiation, using the Larmor formula (Griffiths Eq 11.70. I discussed but did not derive it in my lecture notes, but you can simply use it directly) By conservation of energy, the Larmor formula tells you the rate of loss of energy of the electron. Thus you can set up an ordinary differential equation for r(t). Solve it, and then find the time at which the electron would hit the nucleus. Comment on this remarkable result!
Hint: The formula you get for r(t) should be of the form $r(t) = (r_0^3 - 3at)^{1/3}$ *, where a is a constant that you will figure out, built out of fundamental constants of nature that you can look up.*

4. Magnetism from basic relativity. *(The only relativity you need for this problem is the most basic Phys 2170 formula for length contraction by γ, and the result is oh so cool)*



Shown above is a model of a wire with a current flowing to the right. To avoid minus signs we take the current to consist of a flow of positive charge carriers, each with charge $+q$, separated by an average distance of ℓ. The wire is electrically neutral in the lab frame, so there must also be a bunch of negative charges, at rest, separated by the same average distance ℓ in this frame. *(Be aware that charge is Lorenz invariant: a charge Q has the same value in every inertial frame.)*

A) Using Gauss' law, what is the E-field outside this wire in the lab frame? Suppose there is a test charge $+Q$ outside the wire, a distance R from the center of the wire, moving right (For simplicity, let's say the velocity matches that of the moving charges in the wire, i.e. v, as shown in the figure.)

- Given your answer for the E-field, what is the electrostatic force on this charge, in this frame?
- Using Ampere's (and the Lorentz force) law – what is the $|F_{magnetic}|$ on the moving test charge Q?
- Put it together, what is the direction of the net force on the test charge, and what "causes" it?

B) Now consider how all this looks in the reference frame of the test charge, where it's at rest.

- In THIS frame, what is the magnetic force on the test charge Q? In this frame, it's the negative charges in the wire that are moving to the left. Note: because they're moving, the average distance between them is length-contracted! Meanwhile the positive charges are now at rest, so the average distance between them is now longer than ℓ.
- What is the average distance (ℓ_+) between the positive charge carriers in this frame? Both of these effects give the wire a non-zero charge density.
- Compute the charge density (charge per length) in this frame, with the correct overall sign.
- Use Gauss's Law to compute the electrostatic force on the test charge.
- In THIS frame, what is the magnitude and direction of $F_{\text{on test charge}}$, and what "causes" it?

C) For normal currents, $\beta = v/c$ is about 10^{-13}. (In other words, drift velocities are small!) Given this, show that the forces you computed in parts (a) and (b) are the same size.
Hint: *expand your formula in a Taylor series.*

HW CONTINUES ON NEXT PAGE, one more question this week!

5. More special relativity review:

A) Space probe #1 passes very close to earth at a time that both we (on earth) and the onboard computer on Probe1 decide to call t=0 in our respective frames. The probe moves at a constant speed of 0.6c away from earth. When the clock aboard Probe1 reads t=60 sec, it sends a light signal straight back to earth.

- At what time was the signal sent, according to the earth's rest frame?
- At what time in the earth's rest frame do we receive the signal?
- At what time in *Probe1's* rest frame does the signal reach earth?

(Can you do any "sensemaking" at the end, to check that everything is consistent here?)

B) Space probe #2 passes very close to earth at t=1 sec (earth time), chasing Probe1. Probe2 is moving at 0.3c (as viewed by us). Probe2 launches a proton beam (which moves at v=0.31c *relative to Probe2*) directed at Probe1. Does this proton beam strike Probe1? Please answer twice, once *ignoring* relativity theory, and then again using Einstein!

***Hint**: You can do part A just thinking about length contraction and time dilation from Phys 2170! But, in general, the formal path is to use Lorentz transformation equations, Griffiths Eq. 12.27. I recommend you try it both ways to get used to the Lorentz transformations. (In the end, Lorentz transformation formulas are much **easier**, you just put the numbers in, it requires much less mental juggling of things like "is it now dilated or contracted from this perspective", ...)*

Part B requires the basic "velocity addition" formula, Griffiths Example 12.6 (if you don't remember it from Phys 2170)