HW #13, Physics 3320, Fa 2014, Due Wed, Dec 10. Last Homework!
Note that you won't get this back before the final – but I will post solutions on D2L right away!
The adjective "inertial" should be assumed in front of the word "frame" in all problems this week!

1. A few years ago there was a brief fuss in the media about "Faster than Light neutrinos". Let's investigate those claims! Define the LHC lab in Switzerland to be located at $x_1=0$ in the earth's rest frame, and the Italian Gran Sasso neutrino detection facility is located at $x_2=+730$ km. (Ignore any motion of the earth in this problem.)

A) Suppose a beam of photons travels from LHC (starting at $x_1=t_1=0$) heading to Gran Sasso in a straight line (through vacuum). At what time t_2 will the photons arrive at the detector in Gran Sasso (in the earth's rest frame)? (Neglect general relativity ☺)

- Consider the photon arrival time as measured in a frame that is moving past LHC towards Gran Sasso at high speed v. (But of course, not >c!) This frame has been synchronized so $x_1'=t_1'=0$ when the photons leave LHC on their way towards Gran Sasso (i.e. the origins of the two frames ($x_1=x_1'=0$) coincide at $t_1=t_1'=0$). In the limit that v approaches c (from below), what happens to the photon arrival time (t_2') as measured in the moving frame? Show that t_2' is NEVER less than *or* equal to 0 in a frame that could contain a physical observer (one that is moving at less than c). Interpret this result in words (think about "causality").

B) Now SUPPOSE that a beam of neutrinos could travel from LHC (starting at $x_1=t_1=0$) heading to Gran Sasso in a straight line with a speed $v = c(1+2.5\times10^{-5})$. You read that right, let's suppose for the sake of argument that neutrinos somehow travel FASTER than c by 25 ppm.

- At what time t_2 will these neutrinos arrive at the detector in Gran Sasso (in the earth's rest frame)? How much sooner do they arrive than the photons of part (a)?
- Now consider the neutrino arrival time, as measured in a frame that is moving past LHC towards Gran Sasso at high speed v. (But of course, not >c!) Again, synchronized so $x_1'=t_1'=0$ when the neutrinos leave LHC on their way towards Gran Sasso (i.e. the origins of the two frames coincide at $t_1=t_1'=0$). What speed (with respect to the earth frame) does a frame need, in order for a local observer to measure the neutrinos hitting the detector at $t_2'=0$ (i.e., at EXACTLY the same time as they left in that frame?) What is γ for this frame? (Is it physically possible to have an "observer" with such a γ?)
- If this frame were moving just a little FASTER than the above speed (but still less then c, always!) what would be the SIGN of t_2', the time in that frame at which the neutrinos arrive? Is there any physical principle forbidding you from being an observer in this moving inertial frame? (In such an inertial frame, I claim the neutrinos are measured to arrive at the detector before they are emitted from the source. What do you think of this result?)

C) Sketch the world line of a neutrino emitted from LHC and ending at Gran Sasso. On the same diagram, with a dashed line, indicate the world line of a photon making the same trip.

- Given such (claimed) data, is the space-time interval between emission and detection for individual neutrinos space-like, time-like, or light-like? Above, you considered a frame which flipped the time ordering. In that frame, again tell us about the character of the space-time interval (space-like, time-like, or light-like).
- Suppose, upon detection of a neutrino event at Gran Sasso, the detector sends back a light signal to LHC to indicate they "got it". Add the world line of that return signal to your diagram. Given the claim that the neutrinos traveled faster than c, some people in the media claimed this meant we had "backwards time travel". What do you think they mean by that? E.g,. does that mean that this "return signal" arrives before the original signal was sent? (Use your diagram to answer this question unambiguously.)

HW CONTINUED:

2. A) Use Griffiths 12.108 (.109 in 4th ed) to show that both **E·B** and E^2-c^2B^2 are Lorentz invariants. We found earlier this term that **E** and **B** are mutually perpendicular for traveling EM waves. Given that this is true in *some* frame, can there be any *other* reference frame in which you would find **E** and **B** *not* perpendicular for traveling EM waves?

B) Use the results of part A to answer these questions:
- Suppose E>cB in some frame. Show that there is *no possible* frame in which E=0.
- If E=0 in some frame, do these relations mean that E=0 in every other inertial frame?
- If B=0 (but E is nonzero) in some frame, can you always (ever?) find another frame in which E=0 (but B is nonzero)?

AND TO WRAP UP – Some EXTRA CREDIT QUESTION options!
You can do up to TWO of these questions for credit, if you like… (Please don't turn in more than two, though)

EC #1) ***(4 pts)*** *Some cool math, a practical approach to "velocity addition"*
It is common in nuclear physics to talk about "rapidity" of a particle, defined as an angle:
$\phi=\cosh^{-1}\gamma$ (here γ is the usual relativistic gamma factor, and that's an inverse hyperbolic cosh)

A) Prove that the usual relativistic β (=v/c) is given by $\beta=\tanh\phi$, and then show $\beta\gamma = \sinh\phi$. With these, rewrite the Lorentz transformations in matrix form (i.e. Griffiths Eq 12.24) entirely in terms of the rapidity angle. The result you get might remind you of a rather different *kind* of transformation, please comment! *(Hint: Take a look back at Griffiths section 1.1.5!)*

B) Suppose that observer B has rapidity ϕ_1 as measured by observer A, and C has rapidity ϕ_2 as observed by B (both velocities are on the x-axis) Show that the rapidity of C as measured by A is just $\phi_1+\phi_2$, i.e. rapidities "add" (unlike velocities, which do not "properly" add in relativity!)
Hint: There is a hyperbolic identity you might find useful: $\tanh(a+b)=\dfrac{\tanh a+\tanh b}{1+\tanh a\tanh b}$.

EC #2) **(4 pts)** *Revisiting last week's E&B in different frames story – this time using Griffiths' formalism for transforming fields themselves.*
You are sitting in frame S, watching a long uniform line of charge (the z-axis) move by in the +z direction with constant speed v. In this frame S, the charge density is λ (C/m).

- Use Gauss' law and Ampere's law to compute E and B in frame S. (*This is Phys 3310 stuff!! Note that current $I = \lambda v$, as always. Although it might be tempting to write your results in cylindrical coordinates, for this problem it's best to write it explicitly in x, y, and z coordinates.)*

- Now, let's rederive this in a slightly convoluted but informative way. Move to a frame S' moving along with the charges. In this frame, *there is no current*, and thus no B field! Easy… (right?) So, first compute the pure electric field E' in this frame, expressed entirely in terms of primed coordinates. (Note that the charge density is NOT still λ, what is it?)
- Next, use Griffiths 12.108 (.109 in 3rd Ed) to transform BACK to the original frame S. (Actually, 12.108 was for boosts in the x direction. Ours is in the z direction. You'll need to think about how to rewrite 12.108 for this case. Do so first, and write it down!) Find both E and B in the S frame, even though you started with *just* pure electric E' in frame S'. Don't forget to rewrite any and all primed variables back in terms of the unprimed coordinate system (That requires Lorentz transformations

too, and don't forget, in the z direction, not the usual x direction!) Show that in the end, you get the exact *same results* for both E and B as you found in part A.

That is, starting from JUST pure electric fields, you can use nothing more than Lorentz and field transformations to derive the magnetic field in another frame, without knowing anything about Ampere's law (!) Magnetism is thus in some sense the same thing as electricity, just viewed by different observers. We saw this last week too - magnetism is a "relativistic effect"... This is an essential unification of electromagnetism!

EC #3 (4 pts) *Direct application of invariants to a practical and common particle physics situation:* One way to create exotic heavy particles at accelerators is to simply smash particles together. Consider the energy needed to produce the famous "J/ψ" particle by the reaction in which a positron and electron annihilate to produce it: $e^{+}+e^{-} \rightarrow J/\psi$. The mass of the J/ψ particle is 3.1 GeV/c^2, so you might naively think it takes about 3.1 GeV (??) Lets see!
- First, move to the "center of momentum frame". In this frame, electron and positron (which are equal in mass) crash together head on. In this frame, what kinetic energy do the e^+ and e^- need, to *just barely* produce J/ψ's? (This is the minimum, or "threshold" energy) Is your answer reasonable, is it what you might naively expect?
- Now consider the "lab frame", in which a target electron is at rest, and you smash the positron into it. What kinetic energy does the positron need in this frame, to just produce a J/ψ ?
Use your answers above to explain why particle physicists go to the trouble and expense of building colliding-beam experiments instead of "fixed target" experiments.

And last but not least, EC #4 (4 pts) *Understanding "conserved" vs "invariant"!*
Complete the following two sentences:
A conserved quantity is the same _______________.
An invariant quantity is the same _______________.
Can you have a conserved thing that is not invariant? How about the other way around? Can you have something that is both? Neither? Briefly, explain/comment.
This specific list might help: which of the following quantities are conserved and which are Lorentz-invariant: Proper time, rest mass, energy, 3-momentum, 4-momentum, charge.
- Suppose you have measured many processes involving particles with some initial momentum and energy, and some final momentum and energy. You find that the final and initial energies and momenta are related in that they are CONSERVED. Show that energy and momentum (use the relativistic versions!!) are conserved in ALL inertial frames if they are conserved in ONE inertial frame.
- Can relativistic momentum be conserved in all reference frames *without* conservation of relativistic energy?