Homework Set 2, Physics 3320, Fall 2014
Due Wednesday, Sept. 10 (start of class)

1. (*Griffiths CH 5 review*)
Suppose the magnetic vector potential in a region of space is given by:

$$\vec{\mathbf{A}} = A_0 \exp\left(-\frac{(x^2+y^2)}{a^2}\right)\hat{z} \qquad (1)$$

(**a**) What are the units of the given constants a and A_0?
- Determine the B-field from this vector potential, $\vec{\mathbf{B}} = \vec{\nabla} \times \vec{\mathbf{A}}$. (Note: *Eq (1) is in Cartesian coordinates - what coordinate system would make your calculations easier? Use that!)*
- Determine the current density, **J**, from the B- field. *(use an appropriate Maxwell's eqn!)*

(**b**) Separately sketch the vector field **A,** the magnetic field **B**, and the current density **J** you found above**,** using any representation you feel conveys the most useful information.
- Briefly, for EACH of the three plots, also use English words to describe what they look like. (If your representation "hides" information, state what that is.)
- Use *Mathematica* (Perhaps VectorPlot3D might be handy) to plot at least one of these three fields - preferably the one(s) you feel is (are) the *hardest* to visualize.

(**c**) Integrate the current density to show that the total current flowing through *any* infinite plane parallel to the *x-y* plane is zero. Then, give a simple argument (without doing any formal integral) why you might have known before calculating that this must be the case.

(**d**) Calculate the divergence of the current density **J**. What does the value of the divergence imply, in terms of Griffiths: Equation (5.29)?
Does this set of fields strike you as an unphysical math exercise, or can you imagine some physical system or device, which might be (approximately) represented by this problem?

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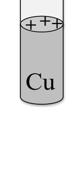
2. (**a**) We frequently use the relation for current density $\mathbf{J} = nq\mathbf{v}$, where n is the number density (number per volume) of charge carriers, q is the charge of each carrier, and **v** is the average velocity of the charge carriers: the "drift" velocity. This is not the DEFINITION of J, it is a derived expression - briefly but clearly (in your own words) summarize the chain of arguments in its derivation. (A little sketch helps! You might go back to Griffiths 5.1.3)

(**b**) In most metals, there is roughly one conduction electron per atom. Consider a copper wire (1 mm diameter) carrying a current of 10 A (these numbers are typical of home wiring) Compute the drift velocity of electrons in this wire. (Is it surprising? Comment, briefly)

(**c**) If I stretch a given piece of copper wire, making it 0.1% longer, how much would this (roughly) change the resistance from end to end? (What assumptions are you making?)

(**d**) Suppose I connect two 1-mm diameter wires end to end, made of *different* materials, copper to gold. When 10 A flows through the system, a thin layer of charge appears at the boundary. Estimate the total charge that has accumulated at the boundary. (I took a guess that it will come out + in the figure. Did I get it right? What determines this sign? Note: Griffiths Table 7.1 may be helpful here, and of course our usual boundary conditions, back in Griffiths Chapter 2.3.5)



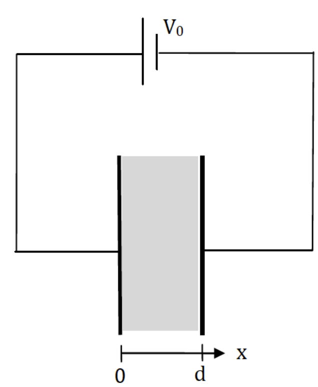
3. The region between two concentric metal spherical shells (radius *a* and *b*, respectively) is filled with a weakly conducting material of conductivity σ. Assume the outer shell is electrically grounded, and a battery maintains a potential difference of $|V| = V_0$ between the two shells. *In this problem, don't confuse conductivity, σ, with surface charge density! Also, ignore any dielectric properties of the weakly conducting material*!

(**a**) What total current, *I*, flows between the shells?
Also, what is the total resistance, *R*, of the weakly conducting material between the shells?

(**b**) Revisit Griffiths section 2.5.4, and re-express your final answers (for *I* and *R*) in part a, in terms of the total capacitance, C, for this same arrangement of two concentric shells. Now, suppose the battery maintaining $|V|=V_0$ between the concentric spheres is suddenly disconnected at $t = 0$. (At t=0, ΔV between inner and outer spheres is thus V_0, but there is no battery to *maintain this*) *Describe qualitatively what you expect happens over time*. Then, calculate the voltage, and the current that flows, between the two shells as a function of time, i.e., find *V(t)* and *I(t)*. *Does your calculation agree with your qualitative prediction?*

(**c**) For the situation of part b, calculate the original total energy stored in the spherical capacitor (Use Griffiths Eq. 2.55). By integrating Power (= V*I), explicitly confirm that the heat delivered to the resistance is equal to the energy lost!

(**Extra Credit**. Worth the same as any other lettered problem part)
Going back to the original setup: Adapt your equation for the resistance, to a situation where a conducting sphere of radius *a* is embedded in a VERY large uniform volume with conductivity σ out to infinity (and held at a potential of V_0 with respect to infinity)
- What would be the resistance for this arrangement (for current flowing out to infinity?)
Now use the above in the following real-world application: take a single spherical conductor of radius a_1, and lower it by a conducting wire into a large, deep, body of water. Do the same with a second conducting sphere of radius a_2, located a significant distance away. (What does "significant" mean here?) Set up a fixed potential difference of $|V|=V_0$ between the two spheres with a car battery.
- Describe what electrical quantit(ies) you would measure to determine the resistivity, ρ, of the water medium in which these two spheres are immersed. (Include a formula telling how you would deduce ρ from what you measured) Putting in plausible numbers, do you think this experiment could work in practice (e.g. to measure the resistivity of sea-water)?



4. Consider a parallel-plate capacitor attached to a battery of constant voltage V_0 as shown in the diagram.
The plates are separated by a distance *d*. The plates are square with area L^2, where $L >> d$, so that the edge effects are negligible (the diagram exaggerates the dimension *d*). The space between the plates is filled with a weakly conducting material that has a *non-*

constant conductivity, a conductivity that depends on position *x* between the plates as $\sigma(x) = \sigma_0 + \sigma' \cdot x$, where σ_0 and $\sigma' = d\sigma/dx > 0$ are constants.
(As before, ignore the dielectric properties of the material)

(**a**) Consider the following quantities: current *I*, current density $\mathbf{J}$, electric field $\mathbf{E}$, and voltage *V*. Which of these quantities is uniform (i.e. independent of position) in the space between the plates? Explain.

(**b**) Solve for the electric field between the plates in terms of the battery voltage V_0 and the other known quantities.
- Make a qualitative sketch of the magnitude of the field vs. position *x*.
- Also, show that your expression makes sense by considering the limit where $\sigma' \to 0$, i.e. the limit of constant conductivity.

(**c**) Perhaps slightly surprising to me (but related conceptually to the last part of Q2d above), I claim the steady-state charge density in the region between the plates is *non-zero*! Show this, by deriving an expression for the charge density ρ (*watch out: ρ here is now charge density, NOT resistivity!)* between the plates in terms of the known quantities in the problem, and check that your answer makes sense by considering the limit where $\sigma' \to 0$, i.e. the limit of **constant** conductivity.
- Explain your results briefly (include a physical justification for the *sign* of the charge density).

(**d**) Derive an expression for the total resistance R of the material between the plates. Show that your expression makes sense by considering the limit where $\sigma' \to 0$, i.e. the limit of constant conductivity.