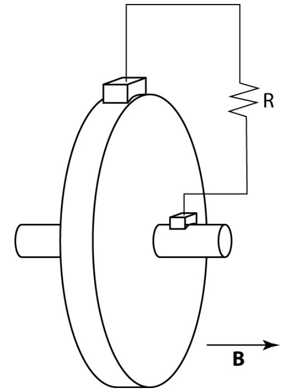
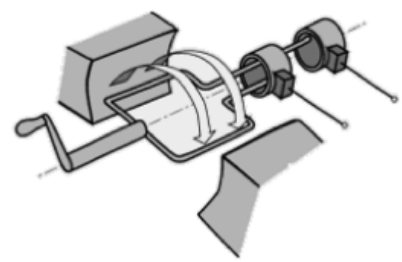
1.) Motional EMF



(a) Homopolar generator: Michael Faraday came up with a relatively simple DC generator called a homopolar generator (featured on our webpage this week.) A conducting wheel of diameter D rotates with angular velocity ω in a uniform B-field oriented along the wheel axis. Sliding contacts make an electrical connection between the center of the wheel and the edge, as shown, and an EMF is induced across a load resistance *R*.

- Show that the power dissipated in the resistor is $P = ??\,\omega^2 B^2 D^4 / R$, (*where the "??" is some numerical constant out front. Is it 1, 0.5, π,...?)*

- How fast would a 1 m diameter generator in a 0.2 Tesla magnetic field have to rotate to produce an EMF of 120 V? *(answer in Hz, please)*



(b) A more typical AC generator: A square loop with side *a* is mounted on a horizontal axis and rotates with a steady frequency *f* (rotations/sec.) A uniform magnetic field ***B*** points left to right between the two pole faces. The figure shows the configuration at time t=0. *(No flux at this instant!)*

(i) If the output is connected to a load resistance *R*, calculate the instantaneous and average power dissipated in the resistor.

(ii) Compare your results to the mechanical power needed to turn the loop. *(Hint: recall from mechanics that the mechanical power to turn a loop is given by power= torque*angular velocity, in direct analogy to power = force * velocity)*

(iii) If the rotation rate is 60 Hz, the loop has area 0.02m^2, and the B-field is 0.2T, about how many turns of wire would you need to produce a standard 120 V (RMS) output? Note: *RMS means "Root mean square", do you remember the connection between V_{max} and V_{RMS}?*

Just FYI, *a variant is to hold the loop fixed (the stator) with an electromagnet coil (the rotor) rotated around the stator. This configuration is called an alternator.*

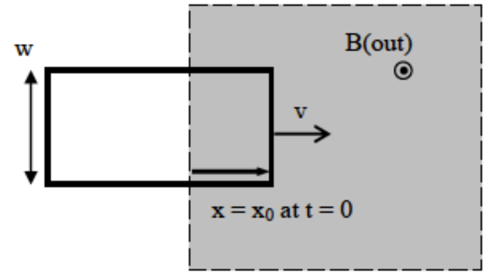
(c) Eddy current brake: An electromagnetic "eddy current brake" consists of a solid spinning wheel of conductivity σ and thickness *d*. A uniform field B_0 is applied perpendicular to the surface of the wheel over a small area *A* located a distance *s* from the axis.

- Show that the torque on this disk is given (very approximately) by $\tau = \sigma\omega B^2 s^2 A d$.

Extra credit #1: *Estimate how high B_0 should be for this kind of brake to be functional as a car brake, given a magnet size of 20 cm^2 per brake. ("Estimate" here means just that – make some reasonable guesses about the various parameters you need. Think about real cars in the real world! Order of magnitude is what we're after, don't fuss about factors of 2 or 3) Do you foresee any problem as the car slows down?*

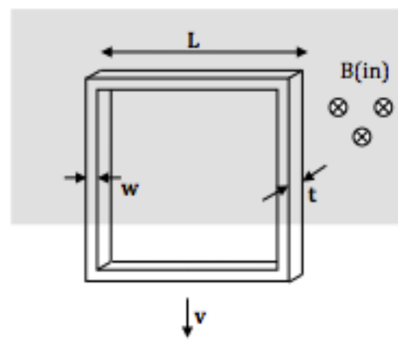
2.) Moving loop in a time-varying field

A rectangular loop of metal wire, of width w, moving with constant speed v, is entering a region of uniform B-field. The B-field is out of the page and is increasing at a constant rate $\mathbf{B} = B_0 + \alpha t$, where B_0 and α are positive constants. At $t = 0$, the right edge of the loop is a distance x_0 into the field, as shown. Note that the EMF around the loop has two different causes: the motion of the loop *and* the changing of the B-field.



(a) Derive an expression for the magnitude of the EMF around the loop as a function of time, while the loop is entering the field.
(b) Is the induced current in the loop clockwise, counterclockwise (or impossible to determine without knowing the values of v and a?) Explain.
-Also, explicitly check that your answer to part a) makes sense by i) checking units and ii) considering the two cases v = 0 and $\alpha = 0$. Explain how these limits give answers you might expect.

3.) A square metal loop is released from rest and falls straight down. The loop is between the poles of a magnet with uniform B field, and initially, the top of the loop is inside the field and the bottom of the loop is outside the field. The metal has mass density ρ_m and electrical resistivity ρ_e. The loop has edge length L, and is made of a rectangular wire with *very small* transverse dimensions w and t.



(a) What is the EMF around the loop in terms of the downward speed v of the loop?
- Assume the loop reaches terminal velocity before it passes entirely outside the field, and derive an expression for the terminal speed of the loop.
- Do some qualitative sensemaking/sanity checks: Units? Does the functional dependence on the various variables seem reasonable? *(Nothing quantitative, just e.g. arguing that v_{term} should be bigger if g is bigger - it's pulled harder by gravity!)*

(b) Show that (when traveling at v_{term}) the rate at which thermal energy is generated in the metal ($P_{thermal}$) is equal to the rate at which gravity does work on the loop (P_{grav}). Briefly, why *must* they be equal?

(c) At $t = 0$, the loop starts at rest. Use $F_{net} = ma$ to write down a differential equation for the speed v of the loop. Then, solve for the speed v as a function of time. You should find that the speed approaches the terminal speed exponentially – sketch v(t). What the time constant for this exponential motion? If the metal is aluminum, and B is, say, 0.2 T, what is the numerical value of the time constant? *(Hint: I claim the values of L, w, t don't matter. Show us why not!)*

(Continued…)

4.) A conducting disk with radius a, height $h \ll a$, and conductivity σ is immersed in a time varying but spatially uniform magnetic field parallel to its axis:
$\vec{\boldsymbol{B}} = B_0 \sin(\omega t)\hat{z}$.

- Ignoring the effects of any induced magnetic fields (!) find the induced electric field $\vec{\boldsymbol{E}}(\vec{\boldsymbol{r}}, t)$ and current density $\vec{\boldsymbol{J}}(\vec{\boldsymbol{r}}, t)$ in the disk. Sketch the current distribution.
- Compare $\vec{\boldsymbol{E}}(\vec{\boldsymbol{r}}, t)$ to the electric field in presented in problem (2) on Homework 1 when $\omega t \ll 1$.

Extra credit: Induction stoves (The rest all counts as one additional EC problem:)
- If the power dissipated in a resistor is $P = I \cdot V$, show that the power dissipated *per unit volume* is $\vec{J} \cdot \vec{E}$. Calculate the total power dissipated in the disk at time t, and the average power dissipated per cycle of the field.

- If this disk was roughly the size of the solid base of a typical frying pan, and the frequency was 20 kHz, use the above (*if you couldn't work it out, you can still just use the result:* $\frac{\text{power}}{\text{volume}} = \vec{J} \cdot \vec{E}$) to find what approximate scale for B_0 you would need to rapidly and significantly heat up the pan (say, 1000 watts of power). Does this seem feasible?

- Finally, use the current distribution from the original Q4 above to determine the induced magnetic field at the center of the pan. Show that the induced magnetic field is *not* small compared to the applied field.

*Note: For real-world induction stoves, the fact that the induced magnetic field is NOT small compared to the applied field suggests this entire calculation of **E** and **B** (which totally ignored the induced field!) is not quantitatively correct. We need to learn some more physics to improve this so-called "quasi-static" calculation.*