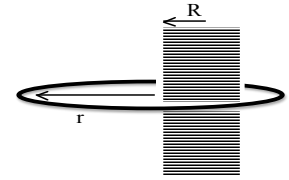**1. Mutual inductance**. We have an infinitely tall solenoid (radius R, n coils per unit length)
Wrapped around it is a single loop of wire (radius r, centered at the left edge of
the solenoid, shown)
a) Find the mutual inductance M between the loop and the solenoid.
- Would it be bigger, smaller, or the same, if r was twice as big?
b) Suppose now that the loop is doubled up, so (with the <u>same</u> radius r) it
wraps around the solenoid *twice*. Again, find the mutual inductance M.
- If the solenoid was *not* infinitely long, would the mutual inductance between the solenoid and the
single loop get larger or smaller? Briefly (qualitatively only) why?
c) Find the *self-inductance per unit length* of the infinite solenoid, all by itself.
- Explicitly check that the units of your answer work out correctly.

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2. More mutual inductance.
Two circular loops of wire are arranged so that they lie in the same plane and
are concentric. One loop, of radius *a,* is much smaller than the other, which
has radius *b*, i.e. $a \ll b$ .If the small loop is driven with a time varying current
$I_1(t)$, derive an *approximate* formula for the induced EMF in the large loop.
(Please state the approximations or assumptions you make in your derivation.)

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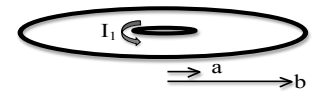
3. Self Inductance of coax: Consider a coaxial cable which is an (infinitely) long wire of radius a,
with given current flowing *uniformly* up it (so J(t) = I(t)/πa^2 goes uniformly through the wire)
At the outside edge (r=a), there is an infinitesimally thin insulator, and just outside that, an
infinitesimally thin sheath that returns ALL the current, basically a surface current going down the
opposite direction carrying a total -I(t) back down.
- Figure out the B field everywhere in space (direction and magnitude) in terms of givens, and
then find the self-inductance per length of this cable.
Notes: Assume I(t) changes slowly in time, so we can use our usual quasi-static ideas.
*There are several ways to proceed. You might start from Φ=LI, but this turns out to be tricky here, since
the current is not confined to a single path! It's best to think about the total magnetic energy, W, stored by
the magnetic field, and use our usual relation between energy and current: W = L I²/2.*
For you to think about: *You found L for, say, 1 meter of solenoid in Q1. Now that you know that the wire
making up the solenoid has its own intrinsic L (from this question), which of these two formulas
"dominates" when you are trying to figure out the inductance of a REAL solenoid, wrapped with real wire?*

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4. Consider a circuit with a *capacitor* and an *inductor* only. Suppose this LC circuit starts at
time *t=0* with charge Q_0 on the capacitor and zero initial current.
a) Write down the differential equation for charge on the capacitor as a function
of time and solve for *Q(t)*.
- Find the voltage across the capacitor, the voltage across the inductor, energy
stored in the capacitor, and energy stored in the inductor, as functions of time.
b) Describe in words the energy flow in the circuit. Show that the total
electromagnetic energy in the circuit is conserved.
c) Describe an analogous mechanical system (i.e. one that obeys the same differential equation) and
provide "translations" between all important variables

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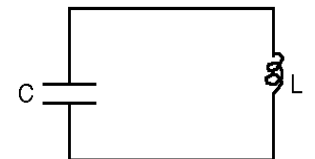
This last question is subtle! But, it really pulls together a lot of ideas about EMF and circuits.

5.) Consider the electrical circuit shown. There is an infinitely long solenoid (shaded, shown end-on in the figure). The current around that solenoid circulates CW and increasing with time, producing a flux (into the page) that is linear in time: $\Phi(t)=\alpha t$. Surrounding that solenoid are some ideal wires and two resistors, as shown, with ideal voltmeters (that means they have *very* high internal resistance, and thus negligible current flows through them) connected as shown. (The probes of voltmeter V_1 are directly across resistor R_1, and those of V_2 are across R_2.)
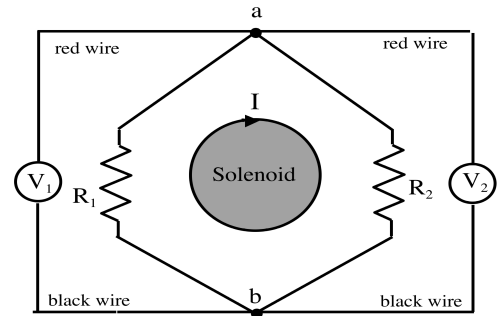

a) What do the two voltmeters read? (Including signs)
How can it be that the two readings are different? (Look carefully - shouldn't points *a* and *b* have a unique voltage drop between them? If not... why not?)

b) Now consider two of those circuits connected as shown in the figure below.
The solenoids are identical, both with the same cross sectional area (= 0.2 m²), and the magnitude of the magnetic field inside both is equal and increasing at a constant rate (0.02 T/s). (However, note the *direction* of the current is opposite in the two solenoids, so the B-fields are in fact increasing in opposite directions.) The resistors have the values shown.
 - Determine the current passing through each resistor. (Magnitude and direction)
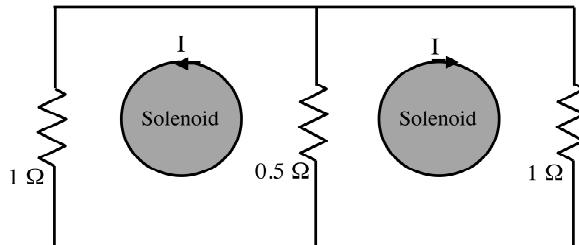
Extra Credit: In class, we saw a "jumping ring" demo, where we applied AC voltage (and thus AC current) to a solenoid, and a ring of copper wrapped around the solenoid launched into the air. Griffiths talks about this demo (Ex 7.6, see his figure!) The explanation I gave in class went like this: "*The changing flux caused by the alternating current in the solenoid induces a current in the copper ring. The current in the ring causes a magnetic field which opposes the field from the solenoid, so the ring is forced away from the solenoid.*" But now I'm changing my story! During some parts of the cycle, the induced current in the ring causes a magnetic field in the *same direction* as the field from the solenoid, thus pulling the ring back on to the solenoid!
(a) Show that in the simplest approximation, the time-averaged force on the ring is zero!

(b) Show that if there is a small *phase lag* in the response of the ring (due to, say, the self-inductance of the ring), so that its response is slightly time delayed with respect to the ideal case, then the time average force will be *nonzero*, directed away from the solenoid.
Hint: Note that we are not asking you to compute *the numerical force on the ring. That would be a significantly harder problem (you would need more information.) We are only asking for the* sign *of the answer. Think about the force between two nearby current loops – parallel currents attract, opposites repel, with a force proportional to current.*