HW #6, Physics 3320, Fall 2014 Due Wednesday, Oct 8 (start of class)

1.a) Consider three different physical AC voltage sources described by these 3 *complex* formulas:

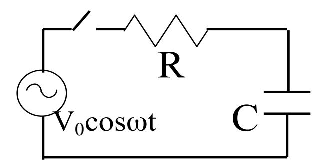
i. $V_1(t) = A \exp(i[\omega t + \pi/6])$

ii. $V_2(t) = A \exp(i\omega t)$

iii. $V_3(t) = A \exp(i[\omega t - \pi/6])$

- Is voltage source 1 *ahead of, behind,* or *in phase with* voltage source 3? (Briefly, explain.)
- Illustrate your answer by showing $V_1(t)$, $V_2(t)$, and $V_3(t)$ for $t = 0$ in the complex plane. Explain how your "phasor plot" is consistent with your answer above.
- Illustrate your answer again by graphing the real parts of $V_1(t)$, $V_2(t)$, and $V_3(t)$ on one single set of *y vs. t* axes. Explain how your graph is consistent with your answer above.
- Consider one more formula, $V_4(t) = A \exp(-i\omega t)$. Does this differ in any physical way from $V_2(t)$?

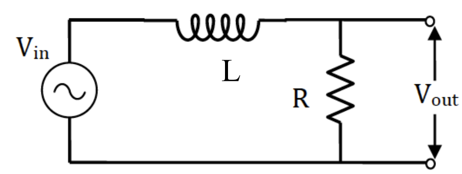
b) Last week, I showed you a driven RC circuit shown with an AC source $V = V_0 \cos(\omega t)$, closing the switch at t=0, where



$$I(t) = \frac{V_0}{\sqrt{R^2 + 1/C^2\omega^2}} \cos(\omega t + \varphi) + \frac{V_0}{R} \frac{1}{1 + R^2C^2\omega^2} e^{-t/RC}, \text{ with } \varphi = \tan^{-1}(1/RC\omega)$$

I have one last question about that circuit: What is happening right at $t=0^+$?
Does the formula agree with what you *expect* it should be?
(Hints: $I(t=0^+)$ is not 0!! Why not? What should it be? To work out the details, you will need to figure out cos(ϕ), given $\varphi = \tan^{-1}(1/RC\omega)$ *I suggest you draw yourself a little right triangle that satisfies this equation and * read off* what the cosine is from your picture! Meanwhile, what is exp[-t/RC] when t is basically 0? No need to get fancy here!!)*

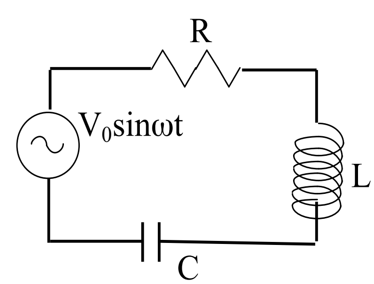
2.) This figure shows a different (series) LR circuit driven by an AC voltage source:
$V_{in}(t) = V_0 \cos(\omega t) = \text{Re}[\tilde{V}_{in}(t)]$



This circuit can be viewed as a filter that changes V_{in} into V_{out}.
(a) Using the phasor method, solve for the "true" current I_{true} through the circuit: $I_{true}(t) = \text{Re}[\ I(t)\ ]$.

(b) Solve for the complex ratio $\tilde{V}_{out}/\tilde{V}_{in}$ (give the magnitude and phase of this ratio) as a function of frequency.
- Sketch the magnitude of $\tilde{V}_{out}/\tilde{V}_{in}$ vs. frequency.
- Check that your answer makes sense in the limits $\omega \to 0$ and $\omega \to \infty$.
- How would you describe this filter? (Is it a high-pass filter? Low-pass? Band-pass?)

3.) Consider the RLC series circuit shown:

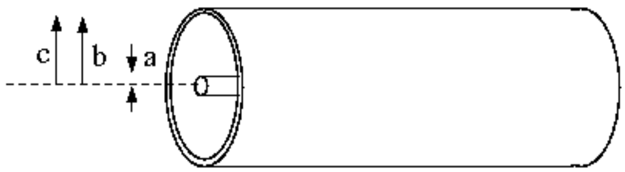


Assume the circuit is *underdamped*, (i.e. R is small)
(a) Just for a change, let's assume the AC power supply provides a voltage $V_0 \sin \omega t$. (Note, that's sin, not cos)
After letting all the transients from turning on the power die away (!), what is the current "through" the capacitor as a function of time?
- Make a VERY rough sketch of the magnitude of the current as a function

of frequency, showing (and briefly explaining) only the main features of your sketch (e.g., what are its limiting behaviours, and what are any "interesting features")

(b) Let's use some realistic values appropriate to an inexpensive oscillator, say a 1 Ohm resistor, a 30 pF (picoFarad) Capacitor, and a 100 nH (nanoHenry) inductor). This circuit has a natural "resonant" frequency – what is it in this case? (Does that number give you a clue as to what this circuit might be useful for?) Use MMA to plot the magnitude of current as a function of frequency. (Be careful to set the scale of frequency to run past the resonance) Does the graph match the expectations of your "sketch" in the previous part? Briely, comment.

4.) Consider (once again!) our standard coax cable: an "infinite" length wire of radius *a* surrounded by a thin conducting cylinder, coaxial with the wire, with inner radius *b* and outer radius *c*.



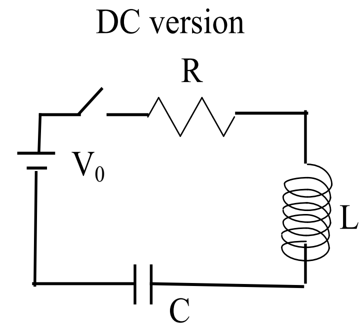
As usual, we assume $a<<b$ and $c-b << b$ (thin shell and wire). In past weeks, we investigated this (very common, very useful) object. In HW 4 we found the self-inductance per length of this coax. In HW5 we found the induced **E** field for a particular time dependent current $I(t) = I_0 \cos(\omega t)$, which flows along the wire (and flows back the opposite direction on the outer cylinder. In that case, we assumed it was "quasistatic", so we could use Ampere's law to find B, and Faraday's law to find E.

This week's task: First, find the displacement current density $\mathbf{J}_d$ in the "coax region" ($a < s < b$) for the electric field **E** that you found in the last HW. (Just ignore the region s<a)
- Then, integrate it to get the total displacement current I_d. (Again, just for a<s<b. This gets a wee bit ugly - hang in there. Please check the *units* of your final expression as a check of the integration)
- Using physically reasonable numbers for a real coax (say a = 1 mm and b = 1 cm), determine the frequency ω for which I_d finally equals 1% of I_0. Briefly, comment. (E.g, What sort of frequency is this, and do we need to worry about the "displacement current story" for *lower* or *higher* frequencies than this? What does this tell you about our usual "quasi-static" approximation, where we neglect displacement currents by using the simple version of Ampere's law?)

5.) Go to Griffiths section 7.2.4 (Energy), and work through the derivation that starts below Eq 7.29, and continues to its culmination at Eq 7.34. Imagine that you have been asked to give a guest lecture in an E&M II class, and your task is to do this derivation for the students. Work out all the details for yourself, really try to follow and understand this derivation. Write it up for yourself as though you were preparing your own lecture notes – explaining steps, pointing out spots you are confused about, making sense, thinking about WHY you're doing what you're doing. This doesn't mean copying Griffiths' solution (!) - it means figuring it out, and writing up for yourself how you think about it!

Extra Credit: Look back at Question #3, and suppose that, instead of an AC power supply, we had used a battery, like this:

If the circuit starts out "dead", and then the switch is closed at time t = 0, describe qualitatively what happens just after t=0, and then as t gets very large. Then compute a formula for the current through the capacitor as a function of time.



And one last question: in Q3, I said "R is small". Small compared to *what?*