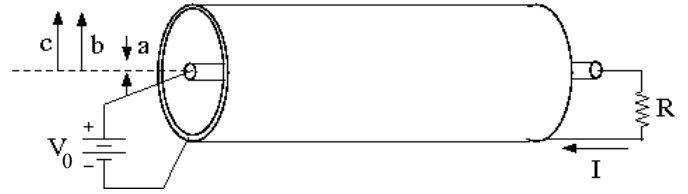


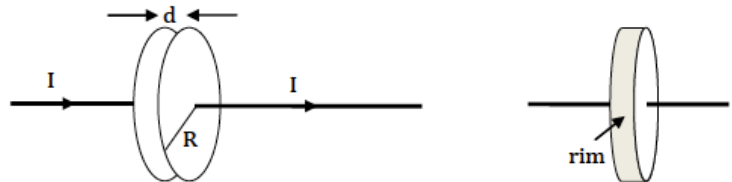
**1.) Poynting Vector:** Let's revisit our familiar coax cable, now looking at fields constant in time. Like last week, the coax has a small central wire ( $a \ll b$ ) and a *thin* outer shell ( $(c-b) \ll b$ ). Constant current  $I$  flows in the  $+z$  direction on the inner wire and total current  $I$  flows in the opposite direction in the outer shell. Assume that the coax cable (wire plus shell) is neutral, and neglect fringe fields. Note there is a constant voltage difference  $V_0$  between the wire and the shell. (see the figure)



**(a)** Find  $\mathbf{E}$  and  $\mathbf{B}$  everywhere in space, in terms of only given quantities. Neglect fields in the *metal*, considering only fields in the “coax” region  $a < s < b$ . (Outside, for  $s > c$ , convince us  $\mathbf{E} = \mathbf{B} = 0$ )  
*Hints: This is not the same as the problem you did in HW6 last week! The magnetic part here is essentially identical, but the electric part here is not solved via Faraday's law like it was last week (B is not changing with time!) We're back to Gauss' law and the basic definition of voltage. You did a similar problem in “spherical geometry” way back on HW #2.)*

**(b)** Calculate the Poynting vector  $\mathbf{S}$  everywhere. The magnitude of  $\mathbf{S}$  gives the energy flux density and represents the power per area moving through space. Does its *direction* make sense for the coax? Integrate this flux through the cross sectional area of the coax to find the power transported down the coax line. (*Be clear/careful about what your “dA” element is*) Does your answer make sense relative to the circuit maintaining the current and voltage? Briefly, comment!

**2.) More Poynting vectors, and conservation of energy:** A capacitor with circular plates of radius  $R$  separated by distance  $d \ll R$  is being charged by a steady current  $I$ . The plates are sufficiently close that fringe effects can be ignored. Assume that at  $t=0$ , the plate is uncharged.

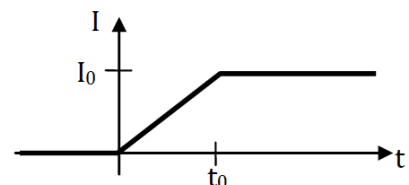


**(a)** In class we did an activity to compute the magnitude of the B-field between the plates at all distances  $r$  from the center of the plates ( $r < R$  &  $r > R$ ). Reconstruct that argument (show us your work!) and then use it to compute the Poynting vector  $\vec{S}$  (magnitude and direction) on the *rim* of the capacitor, between the plates, at  $r = R$ . (The “rim” is the ribbon of area at  $r = R$  between the plates; see the diagram.) Express your answer in terms of given quantities only.

**(b)** Compute the stored energy  $U$  in the capacitor as function of time (in terms of given quantities) Then show that the rate at which the capacitor's stored energy is increasing ( $dU/dt$ ) is equal to the rate at which field energy is entering through the rim:  $\iint_{RIM} \vec{S} \cdot d\vec{a}$ .

Comment on the *sign conventions* you used, and the basic physics – what is going on here?

**3.)** Consider a very long solenoid of length  $L$ , radius  $R$ , and turns per length  $n$ . The current  $I$  in the solenoid is linearly ramped from  $I = 0$  to  $I = I_0$  over a period  $t_0$  as shown in the graph. (Assume as usual the “quasistatic” approximation – the ramping is gentle)



- (a) Integrate the magnetic field energy density to derive a formula for the total field energy stored in the solenoid at times  $t > t_0$ .
- (b) Solve for the electric field everywhere in space at times  $0 < t < t_0$ .
- (c) Solve for the Poynting vector  $\vec{S}$  (direction and magnitude) at  $r = R$  (just inside the walls of the solenoid) as a function of time  $t$ .
- (d) Show that the total field energy/time passing through the walls of the solenoid into its interior, when integrated from  $t = 0$  to  $t = t_0$ , gives the same total energy as you computed in part (a)!
- 

4.) Consider the electric field  $\mathbf{E}(x,y,z,t) = E_0 \cos(k(x-ct)) \hat{y}$  (where  $k$  and  $c$  are known constants).

(a) What is the charge density  $\rho(x,y,z,t)$  which this  $\mathbf{E}$  field requires? What are the units of  $k$  and  $c$ ?

(b) Come up with the *simplest possible*  $\mathbf{B}$  field which satisfies Faraday's law, given this  $\mathbf{E}$  field. Then, CHECK that all the remaining free-space (*meaning, no charges and no currents*) Maxwell's equations are satisfied, *if you make the correct choice for the constant "c"*.

- What is this required "c"? What is the magnitude of the  $\mathbf{B}$  field?

(c) What is the Poynting vector,  $\mathbf{S}$ , associated with these fields?

- What is the energy density at any point  $(x,y,z, \text{time } t)$ ? How is it related to  $|\mathbf{S}|$ ?

- Describe in words what this  $\mathbf{E}$  and  $\mathbf{B}$  field look like. Can you "interpret" them physically? What does the constant "k" tell you?

For you to think about: does this set of  $\mathbf{E}$  and  $\mathbf{B}$  provide a valid, self-consistent, physically possible solution to Maxwell's equations? What's the physics here?

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**5. Warmup for Ch 9, waves:** Two points on a string are observed as a traveling sinusoidal wave passes. The points are at  $x_1 = 0$  and  $x_2 = 1$  m. The two points are known to be less than one wavelength apart. The transverse motions of the two points are observed to be

$$y_1 = 0.2 \cos(3\pi t)$$

$$y_2 = 0.2 \cos(3\pi t - \pi/8), \text{ where all numbers are in SI units.}$$

(a) What is the frequency of this wave in hertz? What is the wavelength? What is the wave speed? Can you tell if this wave is moving to the right or to the left? If so, which way is it moving?

(b) In general, for any complex plane wave  $\vec{\mathbf{E}} = \vec{\mathbf{E}}_0 \exp[i(\vec{\mathbf{k}} \cdot \vec{\mathbf{r}} - \omega t)]$ , (where  $\vec{\mathbf{E}}_0$  is a constant, independent of position and time, and  $\mathbf{k}$  is also a given constant wave vector), prove that

$$\vec{\nabla} \times \vec{\mathbf{E}} = i\vec{\mathbf{k}} \times \vec{\mathbf{E}}.$$

(Hint:  $\vec{\mathbf{r}} = (x, y, z) = x\hat{i} + y\hat{j} + z\hat{k}$ , right? And  $\vec{\mathbf{k}} = (k_x, k_y, k_z) = k_x\hat{i} + k_y\hat{j} + k_z\hat{k}$  is just a constant vector...)

(After you're done with the formal proof, stare at it and try to convince yourself that this is what you might write down "by inspection" if you had to take a guess, knowing how one takes derivatives of exponentials. We'll use this later in the term, see if you can "internalize" it so it looks obvious!?)

To test yourself – what do you think  $\vec{\nabla} \cdot \vec{\mathbf{E}}$  is here?)

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**Extra Credit: You have a choice between 2 extra credit options. (Credit only for one of them)**

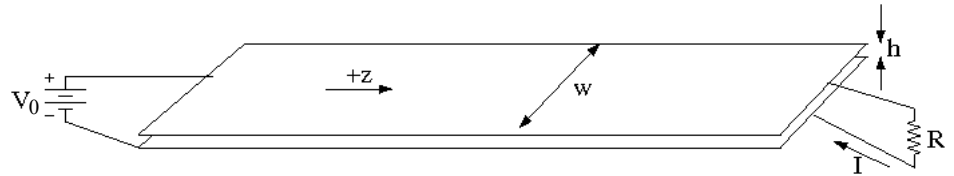
The first is *very* similar to Q1. I would strongly suggest you do it if you feel like you're not sure you fully understand the "big ideas" of that question, it will be a nice review.

The second option is just for fun – do it if you want to learn the material we skipped in the latter part of Ch 8. It is *not* computationally very hard, but you will need to read a section of the book I didn't require (Note: material on tensors is not needed for our exams!)

**EC Option #1: Revisit Q1 in a different geometry:**

Let's investigate energy flow in a different type of coax cable. Again, we will only look at fields constant in time. This coax consists of two infinitely long flat parallel "ribbons" of width  $w$  and separation  $h$ , as shown. The width is much greater than the separation,  $w \gg h$ . One can assume therefore any piece of the coax looks like infinite plane capacitors or infinite planes of surface current. Similar to Q1a above,

assume that a constant total current  $I$  flows in the  $+z$  direction on the upper ribbon and that total current  $I$  flows in the opposite direction on the lower ribbon. There is a constant voltage difference  $V_0$  between the upper and lower ribbons (as shown).



- Find  $\mathbf{E}$  and  $\mathbf{B}$  everywhere between the ribbons. Again, the two-ribbon coax is neutral, and we neglect fringe fields. (This is just like Q1a, nothing fancy here, everything is "quasistatic". Write your final  $\mathbf{E}$  and  $\mathbf{B}$  in terms of only *given* quantities)

- Then, calculate the Poynting vector  $\mathbf{S}$  inside the gap region. As in Q1b, does its direction make sense? Integrate the flux of  $\mathbf{S}$  through the cross sectional area of the coax to find power transported down the coax. Does your answer make sense relative to the circuit maintaining the current and voltage?

OR

**EC OPTION #2: Play with the stress tensor!**

Use Griffiths formulas for the stress tensor in section 8.2.2 and write out the full  $3 \times 3$  stress tensor for the traveling wave of problem 4. It's surprisingly simple! Can you interpret the result physically? (Read the portion of Griffiths where he talks about "flow of momentum" as related to the stress tensor.)