HW #8, Physics 3320, Fall 2014 Due Wednesday, Oct 22 (start of class)

1. Complex EM wave notation. Consider a 3D electromagnetic plane wave in vacuum, described in complex form by $\tilde{\vec{\mathbf{E}}}(\mathbf{r},t) = \tilde{\vec{\mathbf{E}}}_{\mathbf{o}} e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$, where $\tilde{\vec{\mathbf{E}}}_{\mathbf{o}} = \tilde{E}_0\,\hat{\mathbf{y}}$, with $\tilde{E}_0 = E_0 e^{i\pi/2}$.

Assume **k** is the wave vector k $\hat{\mathbf{x}}$, ω is the angular frequency. As usual, the real field is $\vec{\mathbf{E}} = \mathrm{Re}[\tilde{\vec{\mathbf{E}}}]$

A) In which direction is this wave moving? In what direction does the E field point?
- What is the speed, wavelength, and period of the wave? What does that phase of π/2 in $\tilde{E}_0$ do?
- Sketch the *real* field **E**(*x*,*y=0*,*z*=0,*t*=0) (a 2D plot with x as the horizontal axis) and **E**(*x*=0,*y*=0,*z*=0,*t*) (a 2D plot with t as the horizontal axis).
(Clearly indicate the *direction* of the field and the *scale* of both your axes.)
- How is the field at *(0,a,0)* i.e. **E**(*x*=0,*y*=*a*,*z*=0,*t*=0), different from the case at y=0?

B) Summarize: describe in words what this mathematical expression represents physically.
- Why is this called a plane wave (where is (are) the plane(s))? Sketch or represent this in 3D.
- Describe how the direction of the electric field changes in time. If **E** always points in the same direction, the wave is said to be *linearly* polarized. Is this wave linearly polarized?

C) Find the associated magnetic field **B**(**r**,*t*) for this plane electric wave.
- Sketch the magnetic fields, **B**(*x*,*y=0*,*z*=0,*t*=0) and **B**(*x*=0,*y*=0,*z*=0,t) indicating field direction. (As above, be clear about your axes) A 3D sketch of B would be helpful here too, what's the simplest way to draw it? Please describe in words how **B** compares/contrasts with **E**.

D) Calculate the energy density u_{EM}, the Poynting vector **S**, and momentum density (given in Griffiths section 8.2.3) for these fields. Interpret the answers physically (Make sense of them, including units, signs, directions, etc!) Note: the correct way to do this is to take the real parts of the E and B fields FIRST, and then use the usual formulas to find u_{EM}, and **S.**

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2. Write down the (real) **E** and **B** fields for a monochromatic plane wave of amplitude E_0, frequency f (given in Hz), and phase angle *π/4* that is traveling in the direction from the origin to the point (1,1,0) with polarization in the x-y plane. (Please give the explicit Cartesian components of **k** and also of the **E** and **B** fields.) *Your final answer might involve new symbols, but they should all be clearly defined in terms of just the two givens: f and E_0 (and fundamental constants)*

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3. Superposition: Let's superpose two plane waves, $\mathbf{E_1}$ and $\mathbf{E_2}$, to find the total electric field. Suppose $\mathbf{E_1}$ and $\mathbf{E_2}$ are out of phase, *and* polarized in different directions. In particular:

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E_1}\cos(\mathbf{k}\cdot\mathbf{r}-\omega t) + \mathbf{E_2}\cos(\mathbf{k}\cdot\mathbf{r}-\omega t+\pi/2),$$

where $\mathbf{E_1} = E_0\,\hat{\mathbf{z}}$, $\mathbf{E_2} = E_0\,\hat{\mathbf{y}}$, and $\mathbf{k} = k\,\hat{\mathbf{x}}$ here. (E_0 and k are given constants)
- Simplify **E** as much as you can (though, there's not a ton of simplifying you can do) Still, convince us that **E** is a simple traveling wave. Which direction does it travel in? How fast?
- Let's think more about how this **E** varies in time. I think it is easiest to do this by considering some fixed spot (or spots) and picturing the wave there as time goes by, so let's consider all points in space where **k.r** = 0 (in this case, still with **k** = k $\hat{\mathbf{x}}$, describe this set of points in words!), and then describe in words and pictures what your E field looks like there (magnitude and direction) as time goes by. How might you describe the polarization state? If you look down the axis with the wave approaching you, is the **E** vector circling CW? CCW? Or, something else?)

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4. Radiation pressure

A. On earth, the time-averaged flux of electromagnetic energy ($\langle S \rangle$) from the sun is 0.14 watt/cm^2. Consider steady sunlight hitting 1 m^2 of earth: picture an imaginary box (containing streaming sunshine) striking this area, with a "box height" of 1 light-second. There is a certain amount of momentum stored in that box, and in one second, ALL that momentum will strike the 1 m^2 area. Assuming the EM wave is absorbed (not reflected), what force does that work out to? How does the radiation pressure from this light compare to atmospheric air pressure, Comment!

- If the earth *reflected* the sunlight, how would that affect the radiation pressure (qualitatively)?

B. We (a combination of private industry and NASA) can launch tiny satellites (CubeSats are 1 liter in volume, with a mass of about 1.3 kg) on the (relative) cheap into near-earth orbit. There is currently a Kickstarter project to attempt to boost one of these out to *lunar* orbit by using a 32 m^2 reflective "solar sail" to utilize light pressure to propel the satellite (!) The orbital mechanics of this are really complicated, but let's ignore all that and just figure out what the acceleration would be due ONLY to the light pressure alone, and thus what the corresponding (simple, Phys 1110-style) time-to-moon estimate would be? What are some advantages and disadvantages over conventional spacecraft?
(https://www.kickstarter.com/projects/aresinstituteinc/lunarsail-the-worlds-first-crowdsourced-solar-sail)

C. Fine particles of dust in interplanetary space are pushed out of the solar system by radiation pressure from the Sun too. That's why the night sky is nice and dark and transparent. Derive an expression for the size (the radius) of a particle (located 1 AU from the sun, so the solar flux is the same as in part A) that is at the *critical size* where the outward radiation pressure balances the inward pull of gravity. Insert reasonable numbers in your expression and estimate this critical size. (Would the answer you get be different if you were closer, or farther, from the sun?)

Extra Credit: Choose ONE of these two options. I suggest you look at i), even if you choose not to work it out, as the result is important!

i) Look back at Q1-part D. Note that *another* way you might consider doing part D might be to use the usual formulas, but plugging in your general (complex!!) fields, and then only AT THE VERY END taking the real part. Try this for u_{EM}, and convince yourself (and the grader) that this gives a *different* answer. (So beware! Complex notation is great, but when dealing with quadratic quantities like energy density or Poynting vector, you must work with the (real) physical fields)

OR

ii) Suppose that we add two plane waves, $\mathbf{E_1}$ and $\mathbf{E_2}$, where $\mathbf{E_1}(\mathbf{r},t) = \mathbf{E_1}\cos(\mathbf{k}\cdot\mathbf{r} - \omega t + \delta_1)$ and $\mathbf{E_2}(\mathbf{r},t) = \mathbf{E_2}\cos(\mathbf{k}\cdot\mathbf{r} - \omega t + \delta_2)$ so in this simple case both waves propagate in the same direction. Let's say the amplitudes are $\mathbf{E_1} = E_1\hat{\mathbf{z}}$ and $\mathbf{E_2} = E_2\hat{\mathbf{z}}$, where E_1 and E_2 are real. Use complex notation (taking the real part only at the very end) to find $\mathbf{E_T}(\mathbf{r},t) = \mathbf{E_1}(\mathbf{r},t)+\mathbf{E_2}(\mathbf{r},t)$ in the form $\mathbf{E_T}(\mathbf{r},t) = \mathbf{E_T}\cos(\mathbf{k}\cdot\mathbf{r} - \omega t + \delta_T)$, giving formal (if slightly ugly) expressions for the total amplitude and phase shift in terms of those from $\mathbf{E_1}(\mathbf{r},t)$ and $\mathbf{E_2}(\mathbf{r},t)$.

- Explicitly check your answer in the simpler special case $\delta_1 = \delta_2$.

Note: You will have an expression which involves the sum of two simple complex numbers, and I claim you can write that sum as a SINGLE complex number with an amplitude and a phase. That's what you're working out here, a formula for that amplitude and phase in terms of givens.
