Homework Set #9, Physics 3320, Fall 2014, Due Wednesday, Oct 29 (start of class)

Griffiths makes several statements and claims in Chapter 9 without really working out all the details. So please show what you feel are the main steps/big ideas in the questions below:

1. In my lecture notes (and/or Griffiths section 9.3.2) we worked out the case of normally incident light, and claimed that reflected and transmitted waves have the same *polarization* as the incident wave (on the x axis). Prove this must be true! *(Hint: See my lecture notes 9.29 where I set this up…)*

2. A) Starting with Maxwell's equation in matter (in terms of the **D** and **H** fields) show that, for a linear homogeneous dielectric (**D**= ε**E**, **B**= μ**H**) with no free charges or currents ($\rho_{free} = 0$, $\mathbf{J}_{free} = 0$), both the **E** and the **B** fields obey a wave equation with a wave speed given by $v = \sqrt{1/\mu\varepsilon}$.

B) Starting from the same equations as in part A, rewrite them in integral form, and then briefly sketch out for us the reasoning which leads to all the boundary conditions on **E** and **B** at a planar interface between two different linear materials (labeled 1 and 2), with permittivities and permeabilities ε_1, μ_1 and ε_2, μ_2, respectively. (Again, assume *no* free charge or current densities)

3. In Griffiths' section 9.3.2, (Reflection and transmission at normal incidence) he finds reflection and transmission coefficients (R and T). But he has made the assumption that $\mu_1 = \mu_2 = \mu_0$.
Drop that assumption, i.e. keep μ_1 and μ_2 general, and find the *general* formulas for R and T.
To check, explictly confirm that R+T=1, still (as it must be)
Hint: Don't redo work Griffiths has done for you. Use whatever you need from section 9.3.2, just be careful not to use results where he has assumed $\mu_1 = \mu_2 = \mu_0$. *I claim you can express your final results for R and T purely as very simple functions of* β *only!*

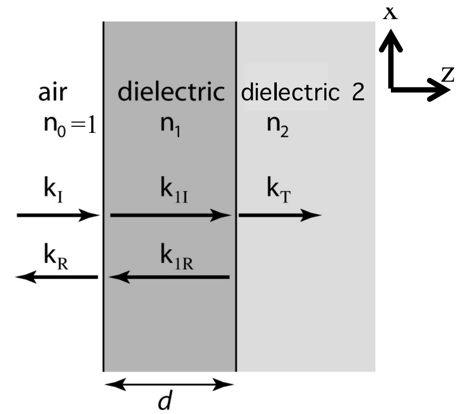
4. In my lecture notes (and/or Griffiths 9.3.3) we worked out the case of reflection and transmission at any angle. But we considered the case where the incident E-field is polarized *in* the plane of incidence. Go through that section again, but work out the different case where the E-field is polarized *perpendicular* to the plane of incidence. (You may once again assume $\mu_1=\mu_2=\mu_0$.)
Specifically, what I mean by "work out" is:

A) Make a clear sketch (modeled on Griffiths figure 9.15) of the geometry and angles for this case. Then, write out what the four boundary conditions become in this case (i.e. modify Griffiths Eq 9.101 through 9.104 appropriately for this new situation).
Finally, find the new "Fresnel Equations", i.e. a version of Eq 9.109, but for this polarization case.
- Explicitly check that your Fresnel equations reduce to the proper results at normal incidence!

B) Replicate Griffiths Figure 9.16, (but of course for this perpendicular polarization case.)
Use Mathematica (or some program) please, don't just "sketch it". Assume $n_2/n_1=2.0$
Briefly, discuss what is similar, and what is different, about this case from what Griffiths (and I) solved. Is there a "Brewster's angle" for your situation, i.e. a non-trivial angle where reflection becomes zero?

C) Replicate Griffiths Figure 9.17 (the one at the end of 9.3) but again, for this perpendicular polarization case, and again assuming $n_2/n_1=2.0$ and again using a computer to plot. Show that R+T=1 for this situation, no matter what the angle. Briefly, comment on the physics!

(continued…)



5. The figure shows 3 layers of dielectric. The index of refraction of air is $n_0=1$, the middle layer is a dielectric (n_1), and behind that is a 3rd layer, n_2. I claim that one can show that the transmission coefficient for light of frequency ω entering this setup is given by

$$T = 4n_0n_2\left[\left(n_0+n_2\right)^2+\frac{1}{n_1^2}\left(n_0^2-n_1^2\right)\left(n_2^2-n_1^2\right)\sin^2\left(\frac{n_1\omega\, d}{c}\right)\right]^{-1}$$

where d is the thickness of the middle dielectric layer. (The derivation of this expression is not required, it is an *extra credit* problem below!)

- If you are standing outside a fish tank and shine a flashlight at the fish, ($n_{glass}=1.7$, $n_{water}=1.3$), what is the maximum fraction of the light intensity that will reflect from the tank? The minimum?

- Does the fish see you any worse or better than you can see it? (Put another way, if the fish shines a flashlight at you, besides how cool that would be - how is T impacted?)

- Suppose you had a monochromatic light source and wanted to see the fish better. Further, suppose that the thickness d was "tuned" so that the $\sin^2$ term =1. Would you be better off choosing a glass whose n_1 is larger than, smaller than, or the same as n_2? (You do NOT need to compute a formula for the *optimal* n_1, that's basically extra credit option A below. I just want a qualitative answer here.)

Extra Credit: Choose *either* option A or option B. Option B is worth a bit more.

Option A: Apply the equation in Q5B. You get a summer internship with the Nikon lens company, and you are assigned the task of designing a dielectric coating that minimizes reflection of eyeglass lenses for visible light ($\lambda \approx 500$ nm) Assume air has $n_0=1$, glass has $n_2=1.5$. What value of n_1 would you choose, and how thick would you make the coating? (Explain your reasoning, briefly but clearly, in words as well as equations) What does T come out to be? Comment!
(Take your time on this one – it's not just formula plugging, think a bit!)

Or, Option B: Prove the equation in Q5B.

Step 1 - You need to know that $T=\beta\left|\frac{E_T}{E_0}\right|^2$, with $\beta=\frac{n_2}{n_0}$. Argue that this expression is correct, where does it come from? (In particular, why is that factor of β out front, and why is it the absolute value squared, and NOT just (the Real part) squared?) *(2 pts)*

Step 2 – Using the formula for T from Step 1, derive the full formula given in Question 5!

To reduce the grief, assume all μ's equal μ_0, that the incoming light is polarized in the x direction, and that $n_0=1$ (air). *Some more hints:* In region "0" (air), there is an incident wave and a reflected wave. In the rightmost region "2", (n_2), there is only a transmitted wave. In region "1" (dielectric, n_1) there are ALSO two simple plane waves, one right-moving, one left-moving. Express each of these waves in the (usual, nothing new here!) complex form, and then impose the boundary conditions associated with normal incidence (There will be TWO useful equations at each of the two boundaries. Don't forget that at the "0-1" boundary you can set z=0, but at the "1-2" boundary you are now at z=d…) *Four equations in four unknowns - the algebra is not trivial. Work it out, believe in it! Go slowly, be careful, use plenty of paper, do NOT try to skip steps. Skill with this kind of algebra will be increasingly important to you!*