www.colorado.edu/physics/phys3320

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Homeworks due wed start of class (including this week!)

online preflights 10 AM Mon (see website for links)

- Piazza.com for Q & A

office hrs TBA (perhaps Mon 4-5, Tues 4-6?)
Location TBA

↳ or anytime electronically. Use piazza for physics/homework
email for admin/private q's

- See learn.colorado.edu for HW sol'ns, Exam sol'ns & grades

Exams: Thurs Oct 2 + Nov 13, 7:30-9:30 (see web)

To Read: For wed, Review Griffiths 1-6,
especially 2 & 5

Look @ website

For Fri, we may start Griffiths 7.1

we'll cover much of Ch. 7 to 12 this term...

What's the bigger picture? What are we studying this term, & why?

This TERM shifts us from STATICS TO DYNAMICS. Time dependence will change a lot - we can deal with realistic circuits, and EM radiation. This forms the basis for understanding light, electronics, optics, power generation & management, communication ...

Every branch of physics & astronomy works deeply with EM theory & practice, it's the core of PLASMA physics, and (combined with quantum mechanics) is central in AMO (atomic, molecular, optical) physics, & condensed matter/solid state physics. Nuclear & PARTICLE physics use E&M experimentally all the time, and although they focus on other forces (QCD / electroweak), those theories build on QED (quantum electrodynamics), which in turn rests solidly on the foundation of CLASSICAL EM theory - Maxwell's Eqns!

- Indeed, the theory side may be the core of this course - our first relativistically correct field theory (!) and the leading example of deep unification principles. There will be a lot of (useful!) MATH here too, building your tool box for applications in almost whatever direction of physics or engineering you go.

3320 - 02 - Review

Phys 3310 is mostly Electrostatics (Ch 1–3), $\vec{E}$ and V

Magnetostatics (Ch 5), $\vec{B}$ and $\vec{A}$

and Fields in presence of Matter (Ch 4, 6), $\vec{D}$ and $\vec{H}$

What do we calculate in E+M?

1) Forces and the resulting motion, of charged objects.
2) **$\vec{E}$ and $\vec{B}$ fields!!** These are vectors, defined everywhere in space, which tell you about forces on charges
3) Voltage + the vector potential $\vec{A}$.

$\vec{E} + \vec{B}$ are the big story. **Why are these fields so important?**

→ They allow us to predict motion of charges via

$$\vec{F} = q\vec{E} + q\,\vec{v} \times \vec{B}$$

[q + velocity $\vec{v}$ are properties of your object, while $\vec{E}$ & $\vec{B}$ encode information about effects of the rest of the universe]

Classical E&M is a local theory

I personally think of $\vec{E}$ and $\vec{B}$ as real, physical fields! Because, these fields carry (contain) energy, momentum, & angular momentum.

In quantum, photons are as real as particles!

(On the other hand, V and $\vec{A}$ are perhaps not so physical – e.g. you have gauge freedom, you could add any constant to V without changing physics!)

3320 - 2b

Math / Vector Calculus Quick Review:

GRADIENT $\vec{\nabla} f = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}$ "grad f"

- Tells you the directional derivative of a scalar function $f(x,y,z)$

$$\frac{df}{dn} = \vec{\nabla} f \cdot \hat{n}$$ (change as you move in the $\hat{n}$ direction)

- It inverts line integrals: If $f(\vec{r}) = \int_{\vec{r}_0}^{\vec{r}} \vec{E} \cdot d\vec{\ell}$

then $\vec{E} = \vec{\nabla} f$

- See Front Flyleaf if you're not in Cartesian coordinates!

Divergence $\vec{\nabla} \cdot \vec{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$ "Div $\vec{V}$"

- Tells you "outflow from a point", (it's the $\lim_{Vol \to 0} \frac{\text{outward flux}}{\text{unit volume}}$)

- Divergence Theorem says $\iiint_{\text{volume}} \vec{\nabla} \cdot \vec{V} \, \underbrace{dx\,dy\,dz}_{\text{or } "d\tau"} = \oint\!\!\oint_{\text{closed surface}} \vec{V} \cdot \underbrace{d\vec{A}}_{\text{outward area}}$

Curl $\vec{\nabla} \times \vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ V_x & V_y & V_z \end{vmatrix} = \hat{i}\left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right) - \hat{j} \cdots$

- Tells you about "local rotation", (it's the $\lim_{area \to 0} \frac{\text{circulation around an area}}{\text{unit area}}$)

(when you dot with that area)

- Stokes Theorem says $\iint_{\text{open surface}} (\vec{\nabla} \times \vec{V}) \cdot d\vec{A} = \oint_{\text{closed boundary line}} \vec{V} \cdot d\vec{\ell}$

Quick (condensed) review (details to follow) + overview of what's to come this term!

E+M is about Maxwell's Equations! (Boxed below)

FALSE in General (Phys 3310!) But true in STATICS	TRUE Always! (* Mostly new this term!)
$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$ (Coulomb's law)	$\vec{F} = q\vec{E} + q\vec{v}\times\vec{B}$ Lorentz $\boxed{\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0}$ Gauss' Law By Divergence theorem or $\oint\vec{E}\cdot d\vec{A} = Q_{enc}/\varepsilon_0$
$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\iiint \frac{\rho(r')\hat{R}}{R^2}\,d\tau'$ $\vec{E}(\vec{r}) = -\vec{\nabla}V(\vec{r})$ (Potential) $\vec{\nabla}\times\vec{E} = 0$ ($\vec{E}$ is conservative)	$\boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}}$ Faraday By Stoke's Theorem $\oint\vec{E}\cdot d\vec{\ell} = -\frac{d\Phi_{MAG}}{dt}$
	$\boxed{\vec{\nabla}\cdot\vec{B} = 0}$ No magnetic monopoles or, $\oint\vec{B}\cdot d\vec{A} = 0$
$\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$ Ampere's Law or $\oint\vec{B}\cdot d\vec{\ell} = \mu_0 I_{through}$	$\boxed{\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\,\partial\vec{E}/\partial t}$
In conductors, $\vec{E} = 0$	Current density $\vec{J} = \rho\vec{v}$ (velocity) In conductor, $\vec{E}$ makes currents

How do we calculate $\vec{E}$ & $\vec{B}$ in statics?

If you know where charges are:

① $\vec{E}$ from Coulomb's law

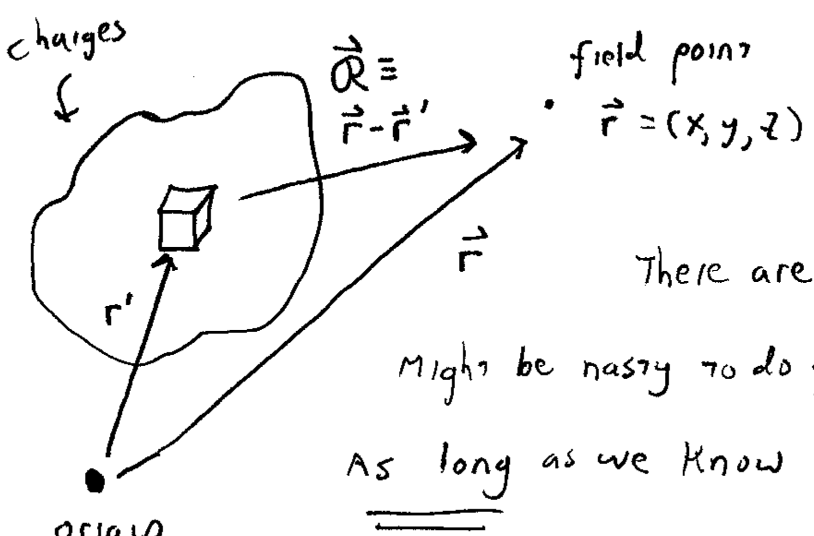
$$\vec{E}(\vec{r}) = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r} \quad \leftarrow \vec{E} \text{ from one charge } Q \text{ at origin}$$

If many charges, use superposition, $\vec{E}(\vec{r}) = \sum_{i=1}^{N} \vec{E}_i(\vec{r})$ due to single q_i at point $\vec{r}_i$

If continuous distribution of charges, this becomes

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\iiint \rho(\vec{r}\,')\frac{\vec{r}-\vec{r}\,'}{|\vec{r}-\vec{r}\,'|^3}\,dx'\,dy'\,dz'$$

(this is also $\frac{\hat{\mathcal{R}}}{\mathcal{R}^2}$, see fig!)



There are three 3-D integrals to compute.

Might be nasty to do formally, but it's "solved" in principle, As long as we know where all charges are, $\rho(r)$

If you have a conductor or matter around, ρ adjusts in the presence of q, and the problem is harder...

Computing E - continued

(2) If you know ρ, and you have some nice symmetry, you can use Gauss' Law : $\vec{\nabla} \cdot \vec{E} = \rho / \varepsilon_0$

so $\iiint \vec{\nabla} \cdot \vec{E} \, d\tau = \iiint \frac{\rho}{\varepsilon_0} d\tau = \frac{Q_{enc}}{\varepsilon_0}$

By Divergence theorem, $\oiint \vec{E} \cdot d\vec{A} = Q_{enc}/\varepsilon_0$

This is always true, but not usually useful to compute $\vec{E}$

Unless symmetry tells you

- Direction of $\vec{E}$ is simple, so dot product $\vec{E} \cdot d\vec{A}$ can be simplified to 0 or $E \cdot dA$

and that

- E is the same over that whole area, so you can pull E out of the integral giving eg. $E \cdot A = Q_{enc}/\varepsilon_0$ (or some variant)

Lots of "ifs", but when you can do this, it's simple + elegant!

 Computing $\vec{E}$, continued. Voltage

③ If you can compute the potential $V(\vec{r})$, then $\vec{E} = -\vec{\nabla} V$, simple enough!

Since $\vec{\nabla}\times\vec{E} = 0$ in statics, I know $V(\vec{r})$ exists (*see below*)

There are many tricks to find V, often simpler than finding $\vec{E}$ directly

e.g. if (again) you know where all charges are, $\rho(r)$, then

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\iiint \frac{\rho(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}\,d\tau'$$

Just one 3-D integral, often easier.

(Taking $-\vec{\nabla}V$ is usually way easier than the extra integrals to get $\vec{E}$)

If $\rho = 0$ ("vacuum" or "free space")

then $\vec{\nabla}\cdot\vec{E} = 0$ + so $\nabla^2 V = 0$ Laplace's Eq'n

Ch. 3 is filled with tricks to solve ↑ this 2nd order PDE

you need Boundary Conditions, here you need V on the "edges" of your region.

(Note: you need V on boundary, not ρ, so it's a different approach, and is especially (useful when you have conductors or materials ...))

($\nabla^2 V = 0$, with boundary conditions, can always be solved numerically! If need be)

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* If $\vec{\nabla}\times\vec{E} = 0$, Stoke's says $\oint \vec{E}\cdot d\vec{l} = 0$, which says $\int_A^B \vec{E}\cdot d\vec{l}$ is independent of path, so $\int_{\vec{r}_0}^{r} \vec{E}\cdot d\vec{l}$ is unique, well defined function of $\vec{r}$!

Griffith's TRIANGLE summarizes the last few pages.

$$\rho$$

$$V = \frac{1}{4\pi\varepsilon_0}\iiint \frac{\rho(r')\,d\tau'}{|\mathcal{R}|} \qquad \vec{E} = \frac{1}{4\pi\varepsilon_0}\iiint \frac{\rho(r')}{\mathcal{R}^2}\,\hat{\mathcal{R}}\,d\tau'$$

$$\nabla^2 V = -\rho/\varepsilon_0 \qquad \vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0 \quad \left(\text{and } \vec{\nabla}\times\vec{E} = 0\right)$$

$$V \longleftrightarrow \vec{E}$$

$$\vec{E} = -\vec{\nabla} V$$

$$V(\vec{r}) = -\int_{\vec{r}_0}^{\vec{r}} \vec{E}\cdot d\vec{\ell}$$

Given any of the 3 (ρ, $\vec{E}$ or $V(r)$) you can find the other two.

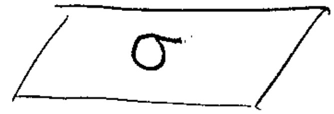
If solving $\nabla^2 V = -\rho/\varepsilon_0$, you want V (or $\frac{\partial V}{\partial n}$) on the boundary.

If solving $\begin{cases} \vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0 \\ \vec{\nabla}\times\vec{E} = 0 \end{cases}$ Directly, you want to know about behaviour of $\vec{E}$ at Boundary.

"Boundary" here is generally a surface, so we're often interested in boundary conditions in the presence of a wall or sheet, with surface charge density σ. See next page!

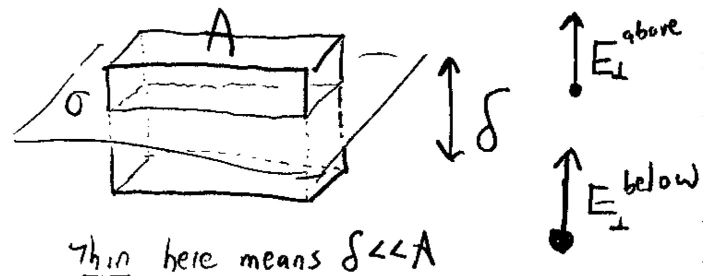
Boundary Conditions

Consider a sheet with charge density



σ (Coul/m²)

Imagine a very thin, small box as shown + use Gauss' law on this imaginary box:



$$\oiint \vec{E}\cdot d\vec{A} = Q_{enc}/\varepsilon_0$$

Thin here means $\delta \ll A$

But small means A is small too!

$$E_\perp^{above} A - E_\perp^{Below}\cdot A = \frac{\sigma\cdot A}{\varepsilon_0}$$

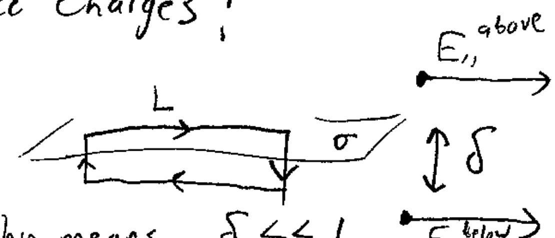
No contribution from other 4 sides since $\delta \ll A$, so those areas are tiny

Note my "$E_\perp$" refers to the component of $\vec{E}$ in direction perp to area. It is a signed quantity, it's not an absolute value. Convince yourself of the minus sign in my equation above!

so $$E_\perp^{above} - E_\perp^{below} = \sigma/\varepsilon_0$$ ← A Boundary condition!

This is a Boundary Condition on E, it says $E_\perp$ "jumps" in value, it is discontinuous when you cross surface charges!

Or, imagine a small, thin loop as shown:



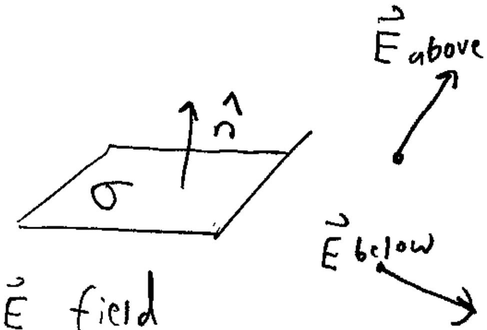
$$\oint \vec{E}\cdot d\vec{l} = 0$$

thin means $\delta \ll L$

(Small means L is also small)

$$E_{//}^{above} L - E_{//}^{below} L = 0$$ (No contrib from 2 small edges, 'cause $\delta \ll L$.)

This says $E_{//}$ is always continuous, another Boundary condition!



you can summarize the previous page as

$$\boxed{\vec{E}_{above} - \vec{E}_{below} = \frac{\sigma}{\varepsilon_0}\hat{n}}$$ B.C. on $\vec{E}$ field

Since $V = \int_{r_0}^{\vec{r}} \vec{E} \cdot d\vec{l}$, if you look at $V_{above} - V_{below}$

this is just $E \cdot \Delta L$, vanishes as $\Delta L \to 0$. So, For finite E (<u>even</u> if it's discontinuous)

$$\boxed{V_{above} = V_{below}}$$ B.C. on Voltage !

These B.C.'s will constrain V when solving $\nabla^2 V = -\rho/\varepsilon_0$

or $\vec{E}$ when solving Maxwell's eq'ns directly.

When <u>Conductors</u> are present, charges ad<u>just</u> to make $\vec{E} = 0$ in the bulk, so conductors are <u>equipotentials</u>, and thus the B.C. is <u>simple</u>, you often know V(conductor), experimentally

(And in this case, solving V gives you $\vec{E}$, and then the B.C. on E <u>tells</u> you what σ is, if you're interested!)

Energy

Since Work $= \int \vec{F} \cdot d\vec{l}$, we can interpret

Voltage V = Work on a test charge / unit charge

or $\underset{\text{from } \infty \text{, brought to } \vec{r}}{W_{\text{on } Q}} = Q\,V(\vec{r}) - \underbrace{Q\,V(\infty)}_{\text{often \underline{set} to } 0}$.

Building up a continuous $\rho(\vec{r})$ requires building up ρ bit by bit, and

$$W_{\substack{\text{to build}\\ \rho(\vec{r})}} = \underbrace{\frac{1}{2}}\iiint \rho(\vec{r})\,V(\vec{r})\,d\tau$$

Needed to avoid double counting work done to bring together pairs of chunks!

This implies energy is stored by/in charges, in presence of other charges.

But Griffiths show this formula is equal to

$$\text{Energy} = \frac{1}{2}\varepsilon_0 \iiint_{\text{all space}} E^2(\vec{r})\,d\tau$$

This implies, instead, that energy is stored in $\vec{E}$ fields!

$$\boxed{u = \frac{\text{energy}}{\text{unit volume}} = \frac{\varepsilon_0}{2}E^2.}$$

Simple to compute, only need to know fields.

Fields have energy!

 (OPTIONAL - TO RETURN TO SOON)

In matter, charges rearrange, modifying $\vec{E}$.

e.g $\vec{E}_{ext}$ can create dipoles $\overset{+q}{\underset{-q}{\uparrow}}$ $\vec{p} = q\vec{d}$, where $\vec{d}$ points from $-q$ to $+q$

These in turn create their own $\vec{E}$ which superposes with $\vec{E}_{ext}$.

We'll come back to this soon. For now, realize Maxwell's eq'ns are always true, e.g $\vec{\nabla}\cdot\vec{E}_{total} = \rho/\varepsilon_0$, (but it might be convenient to think about $\rho = \rho_{free} + \rho_{bound}$)

ρ_{bound}: The "response" charge distribution, which just happens, you don't get to pick / control this!

The lingo here is $\vec{P}$ = polarization = $\frac{\text{electric dipole moment}}{\text{Unit Volume}}$

In a polarized medium, charges separate, creating charge distribution

$\rho_{bound} = -\vec{\nabla}\cdot\vec{P}$ (Griffith's Ch. 4 explains / proves this formula)

so $\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0} = \frac{\rho_{free} + \rho_{bound}}{\varepsilon_0} = \frac{\rho_{free} - \vec{\nabla}\cdot\vec{P}}{\varepsilon_0}$

so $\vec{\nabla}\cdot(\underbrace{\varepsilon_0\vec{E} + \vec{P}}) = \underbrace{\rho_{free}}$

this is called $\vec{D}$ — This is externally under your control!

In "Linear Dielectrics", $\vec{P} = \chi_\epsilon \vec{E}$ and $\vec{D} = \varepsilon\vec{E} = \varepsilon_0(1+\chi_\epsilon)\vec{E}$

A general comment:

you learned many tricks to find E and/or V in E+M 1

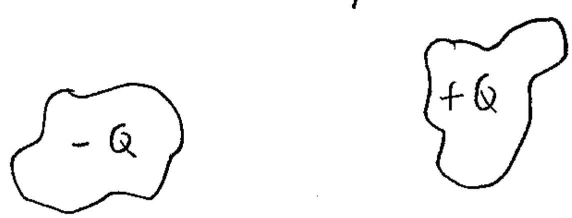
e.g. $\iint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$ simplifies to $E A = Q/\varepsilon_0$

in situations with considerable symmetry (like $\rho(r)$, spherically symmetric)

A nasty integral problem is replaced with simple algebra. Sweet!

Such tricks are <u>great</u>: review them, find out + practice where / how to rediscover / reinvent / locate / look up such things!

Here's another:



A general, <u>nasty</u> pair of oppositely charged conductors

Maxwell's equations tell us $\Delta V \propto Q$

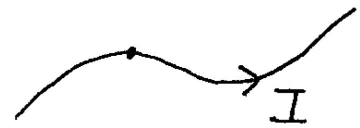
so $Q = C\,\Delta V$

↳ Nasty to calculate in general, but you could <u>measure</u> it! Let nature / experiment do it for you. Then, Maxwell's eq'ns tell you that this C is a <u>constant</u> if the geometry is fixed, and you know ΔV for <u>ANY</u> Q at all on those conductors

Again, poisson's eq'n with <u>unknown</u> charge distribution simplifies to one step algebra, sweet!

What about $\vec{B}$ fields & Magnetostatics?

Current $I = \dfrac{\text{charges passing}}{\text{second}}$

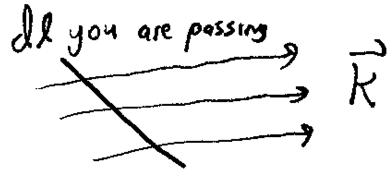


If linear charge density λ moves with velocity $\vec{v}$,

$$\vec{I} = \lambda \vec{v}$$ (units: $\lambda \frac{\text{charges}}{\text{meter}} \times \vec{v} \frac{\text{meters}}{\text{sec}} = \frac{\text{charges}}{\text{sec}}$ ✓)

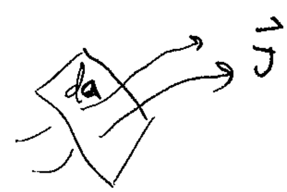
If surface charge density σ moves,

$\vec{K} = \sigma \vec{v}$, where $\vec{K}$ = surface current density $= \dfrac{d\vec{I}}{d\ell_\perp}$



If volume charge density ρ moves,

$\vec{J} = \rho \vec{v}$, where $\vec{J}$ = volume current density $= \dfrac{d\vec{I}}{da_\perp}$



Magnetic Force $\vec{F}_{mag} = q\, \vec{v} \times \vec{B}$ (Lorentz force)

For a wire, $\vec{F} = \int \vec{I} \times \vec{B}\, d\ell$ (which you might also write as $\int I\, d\vec{\ell} \times \vec{B}$)

For a sheet, $\vec{F} = \iint \vec{K} \times \vec{B}\, dA$

For a volume current, $\vec{F} = \iiint \vec{J} \times \vec{B}\, d\tau$

How do you compute $\vec{B}$ (A lot like finding $\vec{E}$!)

If you know where currents are, use

① $\vec{B}$ from Biot-Savart Law

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} I \int \frac{d\vec{l}' \times \hat{R}}{R^2}$$

or for sheets, $\frac{\mu_0}{4\pi}\iint \frac{\vec{K}(r')\times\hat{R}}{R^2}\,da'$

or for volume flow, $\frac{\mu_0}{4\pi}\iiint \frac{\vec{J}(r')\times\hat{R}}{R^2}\,d\tau'$

This reminds me of coulomb,

with $\frac{1}{4\pi\varepsilon_0} \leftrightarrow \frac{\mu_0}{4\pi}$

$$\rho\,\hat{R} \leftrightarrow \vec{J}\times\hat{R}$$

It involves three different (triple) integrals for the 3 components,

+ requires you know what the currents are.

If you have nice symmetry, you can use

② $\vec{B}$ from Ampere's Law

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$$

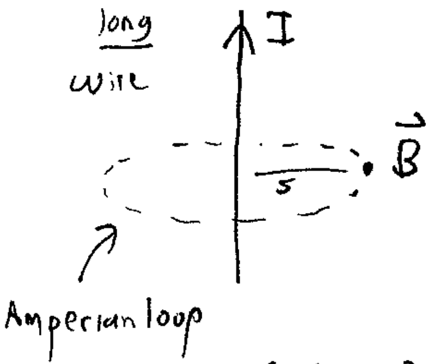
so $\iint(\vec{\nabla}\times\vec{B})\cdot d\vec{A} = \mu_0\iint\vec{J}\cdot d\vec{A} = \mu_0 I_{thru}$

use Stokes

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{thru}$$

Like Gauss: Always true, but $\vec{B}$ is inside a line integral, so this only helps you compute $\vec{B}$ if symmetry tells you direction of $\vec{B}$ (to eliminate dot product) + that $|\vec{B}|$ is constant (to pull it out of the integral)

Example: with Ampere's Law



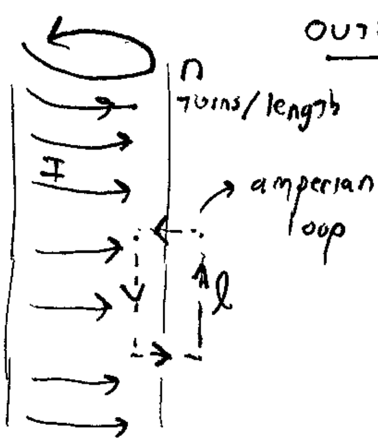
$\vec{B}$ can't be radial (Violates $\nabla \cdot B = 0$)

- can't point along I, because it would be uniform out to $s \to \infty$!
- Can't depend on θ either, so

$$\underbrace{\oint \vec{B} \cdot d\vec{l}}_{B(s) \cdot 2\pi s} = \mu_0 I$$

$$B(s) \cdot 2\pi s = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi s} \hat{\varphi}$$

Example Solenoid



Inside, $\vec{B} = B_z \hat{z}$, convince yourself there's no B_φ and no B_r

It's not totally trivial, think it through!

outside, $\vec{B} = 0$ (convince yourself, it's not totally trivial!)

So Ampere on dashed loop says

$$B_z \cdot l + 0 + 0 + 0 = \mu_0 I n l$$

or $B = \mu_0 n I \hat{z}$ inside

$ 0$ outside

3320 - 16. Computing B: continued

Vector Potential Just as $\vec{\nabla}\times\vec{E} = 0$ guarantees $V(r)$ exists, with $\vec{E} = -\vec{\nabla}V$

so $\vec{\nabla}\cdot\vec{B} = 0$ tells us $\vec{A}$ exists, with $\vec{B} = \vec{\nabla}\times\vec{A}$ } Not totally trivial, but true!

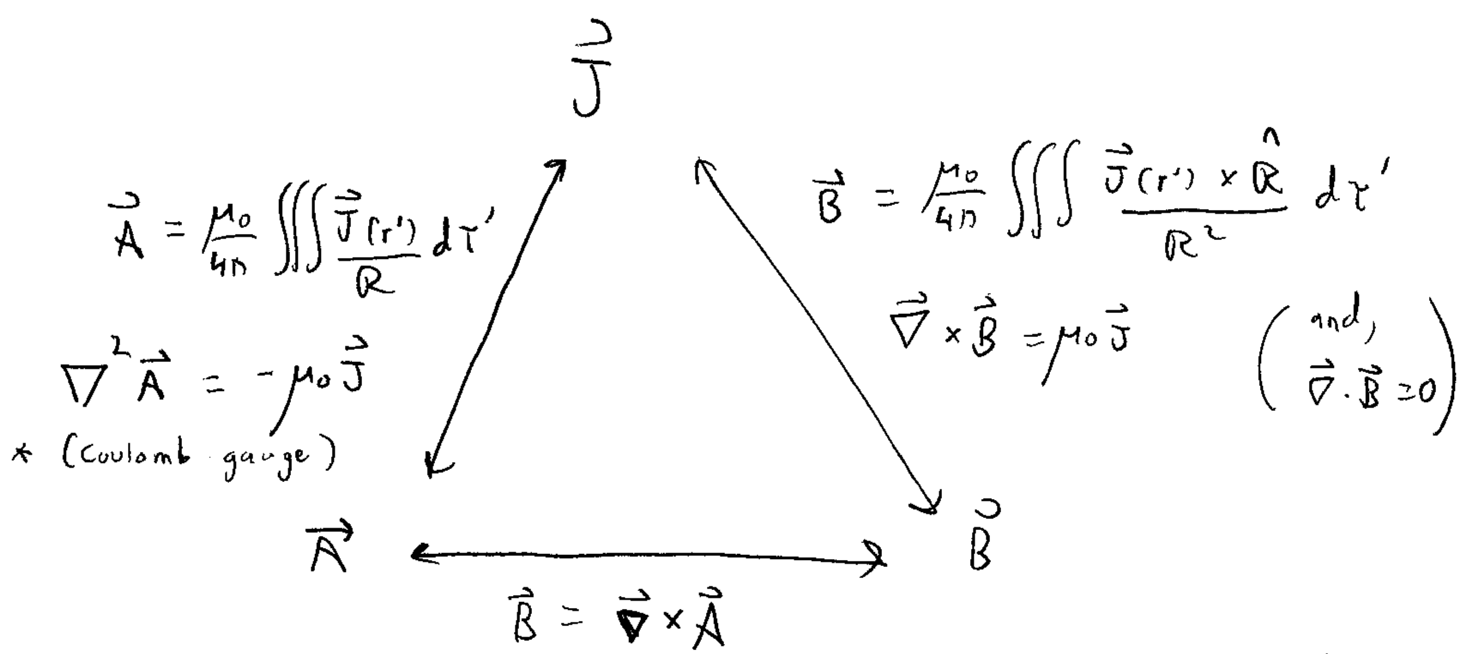
Just as we got $V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\iiint \frac{\rho(r')}{|R|}\,d\tau'$

so we get $\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\iiint \frac{\vec{J}(r')}{|R|}\,d\tau'$ ← $\vec{A}$ sort of "follows currents"

(See next page for why)

It's 3 eq'ns, not <u>one</u>, so it's not quite as <u>handy</u> as V was!

Griffith's other triangle summarizes the story:



$\vec{A} \stackrel{?}{=}$ I'm not sure... I'd "go around the other 2 legs"

(* $\vec{\nabla}\cdot\vec{A} = 0$ in Coulomb gauge)

OPTIONAL DISCUSSION (for now)

Gauges: Since $\vec{E} = -\vec{\nabla} V$, you can always add any constant to V and not change any physics. That's a choice you make, it doesn't matter!

Similarly, since $\vec{B} = \vec{\nabla} \times \vec{A}$, you can add to $\vec{A}$ + keep the physics, $\vec{B}$, unchanged. There's more freedom here than just adding a constant!

If you require $\vec{\nabla} \cdot \vec{A} = 0$, turns out you eliminate most of this freedom (can still add a constant, though!)

This choice is called "choosing the Coulomb gauge"
(You can do this + still find an $\vec{A}$ to go with your $\vec{B}$ field)

Comments:

① $$\Phi_{MAG} \equiv \iint \vec{B} \cdot d\vec{a} = \iint (\vec{\nabla} \times \vec{A}) \cdot d\vec{a} \overset{\text{Stokes}}{=} \oint \vec{A} \cdot d\vec{l}$$

so, just as current creates $\vec{B}$ via $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$

so flux creates $\vec{A}$ via $\oint \vec{A} \cdot d\vec{l} = \Phi_{MAG}$

② Since $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$, then $\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \mu_0 \vec{J}$

If you choose the gauge with $\nabla \cdot \vec{A} = 0$, a front fly leaf identity says

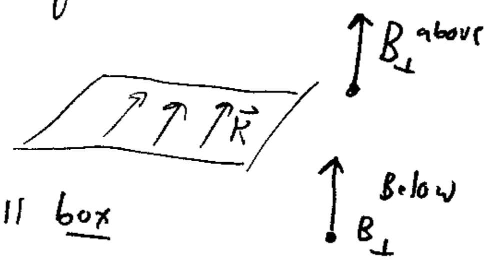
$\nabla^2 \vec{A} = \mu_0 \vec{J}$. This reminds me of $\nabla^2 V = -\rho/\varepsilon_0$

and it's where I came up with my formula for $\vec{A}$ in the triangle!

Magnetic Boundary Conditions

Just like (Notes p.8) with $\vec{E}$, Maxwell's Eq'ns tell us about B.C.'s

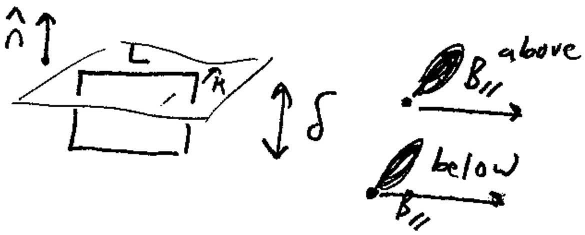
e.g, given a current sheet, $\vec{K}$



$\oint\!\!\oint \vec{B}\cdot d\vec{a} = 0$ tells us, drawing a small box

that $B_\perp^{above} = B_\perp^{below}$ ($B_\perp$ is continuous at any boundary)

But $\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{enc}$



$B_{//}^{above} \cdot L - B_{//}^{Below} L = \mu_0 (KL)$

(where // refers to both surface, and the perpendicular direction to $\vec{K}$)

or, all combined,

$$\vec{B}_{above} - \vec{B}_{below} = \mu_0 \vec{K} \times \hat{n}$$

(And as Before, since $\oint \vec{A}\cdot d\vec{\ell} = \Phi$, this will vanish if $\delta \ll L$, with voltages $\vec{A}_{above} = \vec{A}_{below}$ in Coulomb gauge)

As with $\vec{E}$ fields, these Boundary Conditions are needed to find $\vec{B}$ when solving

$$\begin{cases} \oint \vec{B}\cdot d\vec{\ell} = \mu_0 \vec{J} \\ \oint\!\!\oint \vec{B}\cdot d\vec{a} = 0 \end{cases}$$

3320 - 19 (optional)

In matter, $\vec{B}$ fields orient + modify magnetic dipoles at the level of atoms + molecules. Materials magnetize,

$$\vec{M} = \frac{\text{magnetic dipole moment}}{\text{unit volume}} = \text{Magnetization}$$

+ this Magnetization itself superposes with external fields.

So, knowing $\vec{J}_{free}$ isn't enough, the magnetization modifies current,

$$\vec{J}_{total} = \underbrace{\vec{J}_{free}}_{\text{you control}} + \underbrace{\vec{J}_{bound}}_{\text{matter responds}} = \vec{J}_{free} + \underbrace{\vec{\nabla} \times \vec{M}}_{\text{see Griffiths!}}$$

Maxwell's eqns are still TRUE, $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$

But that means $\vec{\nabla} \times \vec{B} = \mu_0 (\vec{J}_{free} + \vec{\nabla} \times \vec{M})$

or $\vec{\nabla} \times \underbrace{\left(\frac{\vec{B}}{\mu_0} - \vec{M} \right)}_{\text{call this } \vec{H}} = \underbrace{\vec{J}_{free}}_{\text{this is exp'tal under your control... + easy to measure!}}$

(For Linear materials, $\vec{B} = \mu \vec{H}$)

We'll come back to this, but not so important this term, since for most ordinary materials $\mu \approx \mu_0$, $\vec{M}$ is quite small.

Bottom Line summary, 3310 in one page! In statics

$\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0$

$\vec{\nabla}\times\vec{E} = 0$

$\vec{\nabla}\cdot\vec{B} = 0$

$\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$

Maxwell's (time independent) Equations, and, Lorentz Force law:

$\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$

Voltage (potential) $V(\vec{r}) = -\int_{\vec{r}_0}^{\vec{r}} \vec{E}\cdot d\vec{l}$

$\vec{E} = -\nabla V$

} with gauss' law, $\nabla^2 V = -\rho/\varepsilon_0$ ($\nabla^2 V = 0$ if $\rho = 0$)

~~Vec~~ Vector potential $\vec{A}$ satisfies $\vec{B} = \vec{\nabla}\times\vec{A}$

Griffith's triangles let us move back + forth $\rho \leftrightarrow \vec{E} \leftrightarrow V$, $\vec{J} \leftrightarrow \vec{B} \leftrightarrow \vec{A}$

Fields contain Energy $u = \frac{\text{energy}}{\text{volume}} = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0}B^2$

Boundary conditions help us solve Maxwell's eq'ns throughout bounded regions:

$\vec{E}_{above} - \vec{E}_{below} = \sigma_{surface}\,\hat{n}/\varepsilon_0$

$E_\perp$ can jump if σ is there

$E_\parallel$ is continuous

$\vec{B}_{above} - \vec{B}_{below} = \mu_0 \vec{K}_{surf}\times\hat{n}$

$B_\perp$ is continuous

$B_\parallel$ can jump if $\vec{K}$ is there

In "linear" Matter, $\vec{D} = \varepsilon\vec{E}$, $\vec{B} = \mu\vec{H}$

$\vec{\nabla}\cdot\vec{D} = \rho_{free}$

$\vec{\nabla}\times\vec{H} = \vec{J}_{free}$

} Use these, work out appropriate B.C.'s + solve!