3320 ch 9-1

Let's review <u>waves</u>: A wave is a traveling <u>disturbance</u>.

- Water waves are a disturbance of surface height,
Sound waves a disturbance of <u>pressure</u>, waves in a wheat field are disturbances of positions of wheat stalks...

- A "1-D" wave refers (generally) to the movement being 1-D, so e.g. a wave on a taut string is 1-D: $f = f(x, t)$

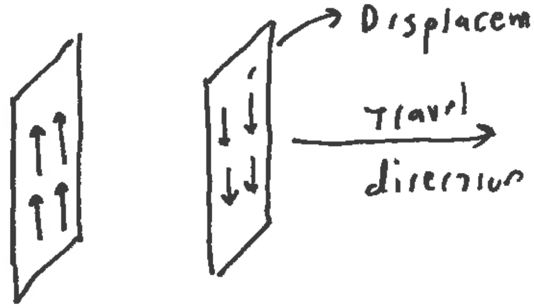
disturbance; 1 D ⇒ travels only in $\pm X$

f = some measure of disturbance

- this string might wiggle into y <u>or</u> z directions (or a mixture), but it's traveling down a string so I'll still call it 1-D.

- A 3-D wave propagates in 3-D space. It <u>might</u> still travel in a straight line, but that line is a vector in a 3-D space of possible directions. (Sound waves typically spread out in all directions)

- A "plane" wave is a 3-D wave which travels in one <u>direction</u>, in 3-D space, + the disturbance is identical for all points in the (a) plane transverse to that direction.



Displacement (e.g.) is same at <u>all</u> points in any given plane ⊥ to travel direction.

(<u>Far</u> from a source, "spherical" sound waves begin to <u>look</u> much like plane waves)

3320 9-2

The wave equation : IN 1-D, the PDE for simple waves is

$$\frac{\partial^2 f(z,t)}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 f(z,t)}{\partial t^2} \qquad \text{(1-D waves in } z\text{-direction)}$$

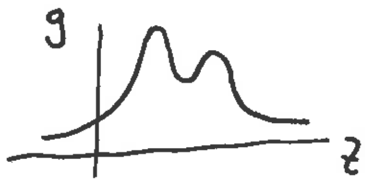
$f(z,t)$ is the disturbance. (e.g density, or longitudinal displacement = scala
or transverse displacement = vector)

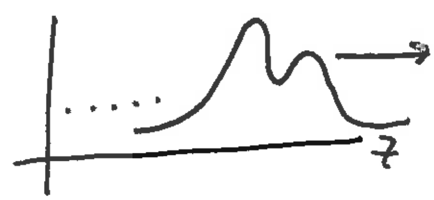
Griffiths derives this eq'n for a taut ~~sp~~ string. Many physical systems obey this same PDE!

Claim : Given any (!) 1-D function $g(z)$, then $f(z,t) \equiv g(z - vt)$ solves that wave-eq'n. (Griffiths proves it - it's just basic "partial derivative" manipulation)

This sol'n is easy to picture:

If this is $g(z)$ then this is $f(z,t)$





It's that $g(z)$ "pulse" (shape) traveling steadily right at speed V.

(Convince yourself!!)

3320 9-3

<u>Claim</u>: $f_L(z,t) = g(z+vt)$ <u>also</u> solves the wave eq'n. (Try it, check! Or, note that only v^2 appears in the wave equation!)

This is the <u>same</u> shape, traveling leftwards at speed v.

<u>Claim</u>: The <u>fully general</u> sol'n to the wave eq'n can always be written as <u>some</u> $g(z-vt) + h(z+vt)$

<u>Note</u>: you can generate sol'ns that aren't so "obvious" this way

e.g. Let $g(z) = h(z) = z$, so $f(z,t) = 2z$. This doesn't "travel"

But it certainly satisfies $\frac{\partial^2 f}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$, try it!

e.g. Let $g(z) = h(z) = \cos z$, so
$$f(z,t) = \cos(z-vt) + \cos(z+vt) = 2\cos z \cos vt$$

This wiggles but does not "travel". Still, it <u>works</u> (try it)

It's called a "standing wave".

So clearly, ∞ many sol'ns <u>solve</u> this PDE!

But, we will focus our attention on one elegant, useful sol'n, the "sinusoidal traveling wave" $f(z,t) = A\cos(k(z-vt)+\delta)$

which you can rewrite (with $\omega \equiv kv$) as $A\cos(kz - \omega t + \delta)$

[It's certainly a sol'n, since it's of the form $g(z-vt)$]

(It is a <u>rightward</u> moving sol'n if $v > 0$)

3320 9-4

Sinusoidal waves: $f(z,t) = A\cos(kz - \omega t + \delta)$

Fourier tells us we could build up more complicated waves by summing many such sol'ns with different k's (and A's). By linearity, these too solve the wave eq'n. (So, sinusoidal waves keep things simple, but you could "build up" arbitrary sol'ns if you want to!)

Our sinusoidal traveling wave has a definite periodicity in z, called wavelength, $\lambda = 2\pi/k$ (check !)

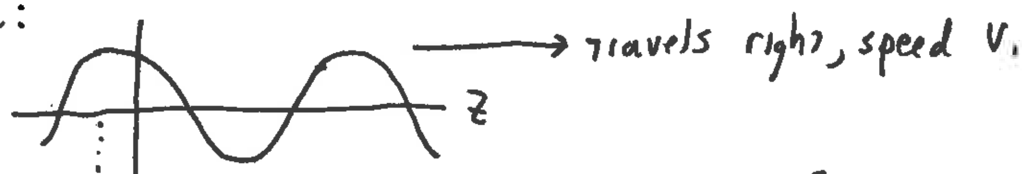
Also a definite period $T = 2\pi/\omega = 2\pi/kv$ (check!)

And thus a definite frequency $f = 1/T$. Note $f\lambda = v$!

Note: $\omega = 2\pi f = kv$, called "angular frequency"

Easy to visualize:

At $t=0$



↳ peak is shifted left by $\Delta z = -\delta/k$ at $t=0$

So it's "phase delayed" by δ.

Bad news: It's ∞, has no beginning or end or edge. (So if you're picturing a single "pulse", you may not be correctly thinking about this wave)

Note: we could've used sin instead of cos, it's just a convention (+ really, is just related to choosing δ)

3320 9-5

More conventions: For left-moving waves, we choose to write them as:

$$f_{\text{left-moving}}(z,t) = A\cos(k(z+vt)-\delta)$$ Note, I switched the sign!

$$= A\cos(kz+\omega t-\delta)$$

Why the sign switch? So this wave is also delayed at $t=0$ by δ/k!

Note: If $f_{\text{right-moving}} \equiv A\cos(kz-\omega t+\delta)$

and $f_{\text{left-moving}} \equiv A\cos(kz+\omega t-\delta) = A\cos(-kz-\omega t+\delta)$

↳ since $\cos(-x)=\cos x$

We see that $\boxed{f_L \text{ is obtained from } f_R \text{ by simply switching } k\to -k.}$

Whenever I see $f(z,t) = A\cos(kz-\omega t+\delta)$, then

I think "ω is always positive" (by convention)

and "if $\delta>0$, this wave is delayed)

and "if $k>0$, it moves right, if $k<0$, it moves left"

So, my convention is $\omega>0$, but k can flip signs.

Note: $|k|$ is "the wavenumber", $|A|$ is "the amplitude",

δ is "the phase shift", "T" is "the period" $= 2\pi/\omega$

$|v|$ is "the wave speed" $= f\lambda$.

v is also $\underline{\underline{v=\omega/k}}$, (so sign of v = sign of k.

Again, $+k \Rightarrow +v \Rightarrow$ right moving)

3320 9-6.

Complex representation : Just like with phasors!

Derivatives of e^x are always simpler than of $\cos x$, but since

$e^{i\theta} = \cos\theta + i\sin\theta$, our wave solution of previous pages is

the real part of a complex wave solution!

So henceforth, our "sinusoidal wave" will be a complex sol'n (!)

$$\boxed{\tilde{f}(z,t) = \tilde{A}\, e^{i(kz-\omega t)}}$$

(the ~ reminds me it's complex!)

Here $\tilde{A} \equiv A e^{i\delta}$ has a magnitude + the phase shift.

So $$\boxed{f_{physical}(z,t) = \mathrm{Re}\left[\tilde{f}(z,t)\right] = A\cos(kz-\omega t+\delta).}$$

Note: If we ever need to sum up waves using Fourier, we easily can:

$$\tilde{f}(z,t) = \int_{-\infty}^{\infty} \tilde{A}(k)\, e^{i(kz-\omega t)}\, dk$$ produces any wave you want!

this "sums" all waves, with complex amplitudes, it's fully general.

The integral has + and − k's, as it must

Also, $\tilde{f}(z,t=0) = \int_{-\infty}^{\infty} \tilde{A}(k)\, e^{ikz}\, dk$ is in the standard "Fourier form"

so Fourier's trick ⇒ $\tilde{A}(k) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{f}(z,0)\, e^{-ikz}\, dz$

=

3320 9-7

3D waves : Wave eq'n in 3D is $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$

I claim (you can check!) that any $f(k_x x + k_y y + k_z z - \omega t + \delta)$ always solves this eq'n, as long as $k_x^2 + k_y^2 + k_z^2 = \omega^2/v^2$.

I also claim (like in 1-D) this is the most general form for a wave traveling in the $\vec{k} \equiv (k_x, k_y, k_z)$ direction!

So in 3-D, we write our general sinusoidal solution as

$$\boxed{\tilde{f}(\vec{r}, t) = \tilde{A}\, e^{i(\vec{k}\cdot\vec{r} - \omega t)}}$$

Here $\vec{k} = k_x \hat{x} + k_y \hat{y} + k_z \hat{z}$ tells you the direction of travel

$|k| = 2\pi/\lambda$ tells you wavenumber

$|v| = \omega/|k|$ is the speed (called "phase velocity" since it's the rate of change of the phase in our exponential)

And as always, $f_{physical}(\vec{r}, t) = \mathrm{Re}[\tilde{f}(\vec{r}, t)]$

$$= A\cos(\vec{k}\cdot\vec{r} - \omega t + \delta)$$

(Again, this sinusoidal sol'n is "ideal", ∞ in extent, has a single definite ω, and λ, and direction of travel.)

(we can build up more realistic/complex waves by summing these up.)

If the disturbance is itself a <u>vector</u> (e.g $\vec{E}$ or $\vec{B}$!) then "f" here is in fact a vector quantity. That means

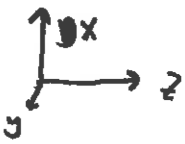
$$\vec{f}(\vec{r},t) = f_x(\vec{r},t)\,\hat{x} + f_y(\vec{r},t)\,\hat{y} + f_z(\vec{r},t)\,\hat{z}$$

And <u>each</u> of these is <u>itself</u> a wave!

Then we have the (nasty?) notation $\vec{f}_{\text{Physical}}(\vec{r},t) = \mathrm{Re}\left[\vec{\tilde{f}}(\vec{r},t)\right]$

<u>Note</u>: If $\vec{f} \perp \vec{k}$, we say the wave is "transversely polarized" or simply "transverse"

If $\vec{f} \parallel \vec{k}$, it is "longitudinally polarized", or "longitudinal".

E.g. if a wave travels in the $+z$ direction, 

, then a "vertically" or "x" polarized transverse wave would be

$$\vec{\tilde{f}}(z,t) = \tilde{A}\, e^{i(kz-\omega t)}\,\hat{x}$$

↳ this is just a <u>constant</u> complex #, $A_0 e^{i\delta}$

If $\vec{f}$ is transverse <u>and</u> $\vec{f}(\vec{r},t)$ always points in the <u>same</u> direction independent of $\vec{r}$ and t, we say it is <u>linearly</u> <u>polarized</u>.

(the wave example above is linearly polarized in the x-direction)

3320 9-9

Waves + Boundaries

In 1-D, you might imagine situations like this:

A wave coming in from the left, an "incident wave" might be, in ①:

$$\hat{f}_I(z,t) = \hat{A}_I\, e^{i(k_1 z - \omega t)}$$

with $\begin{cases} z<0 \text{ only} \\ k_1>0, \text{ it's moving right} \\ v_1 = \omega/k_1 \end{cases}$

But superposed on this we'll have a left moving reflected wave

$$\hat{f}_R(z,t) = \hat{A}_R\, e^{i(-k_1 z - \omega t)}$$

↑ flip k_1 to flip direction!

with $\begin{cases} z<0, \text{ still in region ①} \\ k_1>0 \text{ still} \\ |v_1| = \omega/k_1, \text{ but moves left} \end{cases}$

On right, in region ②, we'll have a "transmitted wave" of the form

$$\hat{f}_T(z,t) = \hat{A}_T\, e^{i(k_2 z - \omega t)}$$

with $\begin{cases} z>0 \\ k_2>0 \\ v_2 = \omega/k_2, \text{ right moving only} \end{cases}$

Claim: ω is the same in all these eq'ns! ω counts wiggles/sec, the "boundary point" has one wiggle frequency, that's ω

Boundary CONDITIONS require $f(z,t)$ and $\frac{\partial f}{\partial z}(z,t)$ be continuous.

(See Griffiths if this isn't simply obvious to you for a string!)

This will let us find A_T and A_R, given A_I.

(We'll return to this soon + work it all out!)

3320 ch 9-10

Back to Maxwell's Eq'ns! Let's consider ME's in vacuum

that means $\rho = \vec{J} = 0$. You might imagine that means $\vec{E} = \vec{B} = 0$,

(+ that is a sol'n!) but there are non-zero sol'ns too!

Gauss: $\vec{\nabla} \cdot \vec{E} = 0$, Faraday: $\vec{\nabla} \times \vec{E} = -\partial \vec{B} / \partial t$

$\vec{\nabla} \cdot \vec{B} = 0$, Modified Ampere: $\vec{\nabla} \times \vec{B} = \mu_0 \varepsilon_0 \, \partial \vec{E} / \partial t$

A lovely trick: Just as we chose to investigate $\vec{\nabla} \cdot$ (curl eq'ns) before, let's now investigate $\vec{\nabla} \times$ (curl eq'ns).

Try it on Faraday's Law:

you can move $\vec{\nabla} \times$ past $\partial / \partial t$

using Modified Ampere.

$$\underbrace{\nabla \times (\vec{\nabla} \times \vec{E})}_{} = -\frac{\partial}{\partial t} (\vec{\nabla} \times \vec{B}) = -\frac{\partial}{\partial t} \left(\mu_0 \varepsilon_0 \, \partial \vec{E} / \partial t \right)$$

This is a pure math identity, see (11) on front flyleaf.

$$\vec{\nabla} (\underbrace{\vec{\nabla} \cdot \vec{E}}_{=0 \text{ by Gauss.}}) - \vec{\nabla}^2 \vec{E}$$

So, $$\boxed{\vec{\nabla}^2 \vec{E} = + \mu_0 \varepsilon_0 \, \partial^2 \vec{E} / \partial t^2}$$

Claim: If you work out $\vec{\nabla} \times (\vec{\nabla} \times \vec{B})$, you similarly get

$$\boxed{\nabla^2 \vec{B} = + \mu_0 \varepsilon_0 \, \partial^2 \vec{B} / \partial t^2}$$

In Cartesian, $\nabla^2 \vec{E}$ means $\dfrac{\partial^2 \vec{E}}{\partial x^2} + \dfrac{\partial^2 \vec{E}}{\partial y^2} + \dfrac{\partial^2 \vec{E}}{\partial z^2}$

(See Griffiths p. 235)

3320 9-11

So what we really have is 3 separate eq'ns, one each for E_x, E_y, E_z:

$$\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_x}{\partial t^2} \equiv \frac{1}{v^2} \frac{\partial^2 E_x}{\partial t^2}$$

and similary for E_y, E_z, $B_{x,y,\text{ and } z}$. So each component of $\vec{E}$ & $\vec{B}$ satisfies the 3-D wave eq'n, with speed $v = 1/\sqrt{\mu_0 \varepsilon_0} = 2.998 \cdot 10^8$ m/s

whoa! This must have blown Maxwell away! The equations, in empty space have a non-zero sol'n: a traveling wave with speed c! Could it be that light is simply a traveling EM wave? It's not proven here, but it's sure tempting to postulate!)

The sol'ns to these wave eq'ns are very general. As with 1-D waves, we will consider only the simplest, ideal sol'n, the "plane waves". (Then, any general EM wave could be build up by superposing these using Fourier's method)

So, we'll be investigating monochromatic, plane wave sol'ns:
(fixed ω)

$$\tilde{\vec{E}}(\vec{r}, t) = \tilde{\vec{E}}_0 \, e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

$\longrightarrow$ the frequency, $\omega = c|\vec{k}|$.

$\longrightarrow$ The wave-vector, wave travels in $+\hat{k}$ direction

$\tilde{\vec{E}}$: a complex vector

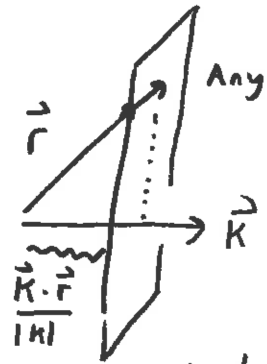
Take $\text{Re}[\tilde{\vec{E}}]$ to get true $\vec{E}$-field

$\tilde{\vec{E}}_0$: The amplitude vector.

- It's a constant vector
- It has a phase, it's complex!

3320 9-12.

- If $\vec{K}\cdot\vec{r}$ = constant, I claim that describes a plane:



Any point on this plane will have the same value for $\vec{K}\cdot\vec{r}$!

So $\vec{\tilde{E}}$ is the same for all points on this plane, which is $\perp$ to $\vec{K}$

So it's a "plane wave". On that plane, e.g. the one where $\vec{K}\cdot\vec{r}=0$,

$$\vec{\tilde{E}} = \vec{\tilde{E}}_0 \, e^{-i\omega t}$$

$\hookrightarrow$ this is some fixed constant arrow, $\tilde{E}_{0x}\,\hat{x} + \tilde{E}_{0y}\,\hat{y} + \tilde{E}_{0z}\,\hat{z}$

I know we showed that ME's have wave sol'ns in free space.

But if you simply pick some wave sol'n (like our plane wave), you still have to be careful that it is consistent with all 4 ME's.

We will find there are conditions on these plane wave sol'ns:

e.g. $\vec{B}$ + $\vec{E}$ are not independent of each other, they're coupled.

And, not any old $\tilde{E}_0$ will work for a given $\vec{K}$!

Let's see how this works:

Start by assuming our plane wave $\vec{\tilde{E}}(\vec{r},t) = \vec{\tilde{E}}_0 \, e^{i(\vec{K}\cdot\vec{r}-\omega t)}$

And then we'll impose Maxwell eq'ns on this.

3320 9-13

E.g. $\vec{\nabla}\cdot\vec{\tilde{E}} = 0$ (gauss law in vacuum) must hold.

$$\vec{\nabla}\cdot\vec{\tilde{E}} = \vec{\nabla}\cdot\vec{\tilde{E}}_0\, e^{i(\vec{K}\cdot\vec{r}-\omega t)} = \frac{\partial}{\partial x}\tilde{E}_{0x}\, e^{i(K_x x + K_y y + K_z z - \omega t)}$$

$$+ \partial/\partial y\, \tilde{E}_{0y}\, e^{i(\quad " \quad)} + \frac{\partial}{\partial z}\cdots$$

$$= \tilde{E}_{0x}\, iK_x\, e^{i(\vec{K}\cdot\vec{r}-\omega t)} + iK_y\,\tilde{E}_{0y}\, e^{i(\vec{K}\cdot\vec{r}-\omega t)} + \ldots$$

$$= i\left(\vec{K}\cdot\vec{\tilde{E}}_0\right) e^{i(\vec{K}\cdot\vec{r}-\omega t)}.$$ To be true, $\vec{K}\cdot\vec{\tilde{E}}_0 = 0$!

That means $\vec{\tilde{E}}_0 \perp \vec{K}$, i.e. our plane wave in vacuum must be <u>transverse</u> (see notes p. 8). (Note: Fourier summing can't change this)

Similarly, $\vec{\nabla}\cdot\vec{B} = 0$ tells us $\vec{K}\cdot\vec{\tilde{B}}_0 = 0$, $\vec{B}$ is transverse <u>too</u>

What about Faraday? $\vec{\nabla}\times\vec{E} = -\partial\vec{B}/\partial t$ will "connect" $\vec{E}$ & $\vec{B}$.

$$\vec{\nabla}\times\vec{E} = \begin{vmatrix} \hat{x}, & \hat{y}, & \hat{z} \\ \partial_x, & \partial_y, & \partial_z \\ \tilde{E}_{0x}\, e^{i(\vec{K}\cdot\vec{r}-\omega t)}, & \tilde{E}_{0y}\, e^{i(\cdots)}, & \cdots \end{vmatrix} = \begin{array}{l} \hat{x}\,(iK_y\,\tilde{E}_{0z} - iK_z\,\tilde{E}_{0y})\, e^{i(\vec{K}\cdot\vec{r}-\omega t)} \\ -\hat{y}\,(\cdots) \\ +\hat{z}\,(\cdots) \end{array}$$

$$= -\frac{\partial}{\partial t}\vec{B} = -\frac{\partial}{\partial t}\vec{\tilde{B}}_0\, e^{i(\vec{K}\cdot\vec{r}-\omega t)} = (i\omega)\,\vec{\tilde{B}}_0\, e^{i(\vec{K}\cdot\vec{r}-\omega t)}$$

you must convince yourself the L.H.S is <u>simply</u>

$$i\left(\vec{K}\times\vec{\tilde{E}}_0\right) e^{i(\vec{K}\cdot\vec{r}-\omega t)} = i\omega\,\vec{\tilde{B}}_0\, e^{i(\vec{K}\cdot\vec{r}-\omega t)}$$

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In general, from here on out, you can "read off" these vector operators:

$$\vec{\nabla} \times \vec{\tilde{A}}_0 \, e^{i(\vec{k}\cdot\vec{r}-\omega t)} = i \vec{K} \times \vec{\tilde{A}}_0 \, e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$\vec{\nabla} \cdot \vec{\tilde{A}}_0 \, e^{i(\quad)} = i \vec{K} \cdot \vec{\tilde{A}}_0 \, e^{\cdots}$$

$$\frac{\partial}{\partial t} \vec{\tilde{A}}_0 \, e^{i(\cdots)} = -i\omega \vec{\tilde{A}}_0 \, e^{\cdots}$$

It's simple. This is why we moved to exponentials in the first place!

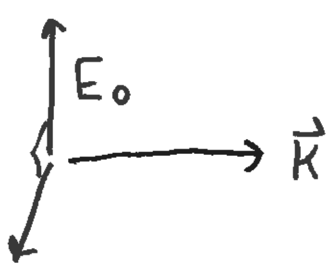
So summarizing, Faraday $\Rightarrow$ $\vec{K} \times \vec{\tilde{E}}_0 = \omega \vec{\tilde{B}}_0$

If e.g. $\hat{K} = \hat{z}$, this says $\vec{\tilde{B}}_0 = \frac{K}{\omega}(\hat{z} \times \vec{\tilde{E}}_0) = \frac{1}{c}(\hat{z} \times \vec{\tilde{E}}_0)$

If wave moves in $\hat{z}$ direction, Gauss $\Rightarrow$ $\vec{E}_0$ + $\vec{B}_0$ are both transverse,

and Faraday $\Rightarrow$ $\vec{B}_0$ and $\vec{E}_0$ are also perp to each other

and $|B_0| = \frac{1}{c}|E_0|$, and $\vec{B}_0$ is in phase with $\hat{E}_0$ (no complex funny business!)



$B_0 = \left|\frac{E_0}{c}\right|$

This is the general description of our travelling EM waves!

3320 9-15

There is one last ME, But it doesn't add anything new.
(check if you like, it's redundant)

So our full plane wave sol'n which obeys all 4 ME's in vacuum:

$$\tilde{\vec{E}}(\vec{r},t) = \tilde{E}_0 \, e^{i(\vec{k}\cdot\vec{r}-\omega t)} \, \hat{n}$$

→ this is the polarization direction
the direction of $\tilde{\vec{E}}_0$
and $\hat{n}\cdot\hat{k} = 0$, it's transverse

$$\tilde{\vec{B}}(\vec{r},t) = \frac{\tilde{E}_0}{c} \, e^{i(\vec{k}\cdot\vec{r}-\omega t)} \, (\hat{k}\times\hat{n}) = \frac{\hat{k}\times\vec{E}}{c}$$

and remember, $\vec{E}_{physical}(\vec{r},t) = \mathrm{Re}[\tilde{\vec{E}}] = E_0\cos(\vec{k}\cdot\vec{r}-\omega t+\delta)$

where δ is the phase that is hiding in the constant $\tilde{E}_0$

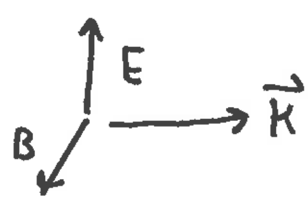
and $\vec{B}_{physical} = \dfrac{\hat{k}\times\vec{E}}{c}$

The math looks formidable, but the physics is simple,

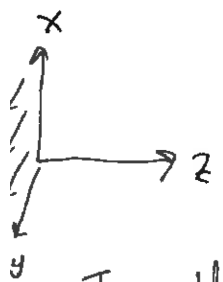
we have traveling sin waves,

moving at speed c,

with $\vec{E} \perp \vec{B} \perp \vec{k}$,



3320 9-16 (supplement: An example from Feynman)



Consider turning on an ∞ sheet of current in x-y plane @ $t=0$

(Surface current is $K_s \hat{y}$ on that plane)

I would expect to see a $\vec{B}$ field (Ampere's law!) Indeed, I remember this from 3310, use a little "Amperian Loop" (+ symmetry!), I expect

$$\vec{B} = \left.\begin{matrix} B_0 \hat{x} & \text{for } z>0 \\ -B_0 \hat{x} & \text{for } z<0 \end{matrix}\right] \text{ And " " tells me } B_0 = \mu K_s/2$$

Well, it can't be that instantly, that'd fill space with energy in an instant!

No, I anticipate this $\vec{B}$ to "sweep outwards" in a wave, from the $z=0$ plane

Of course, if $\vec{B}$ "sweeps" then it's changing, + I expect to see $\vec{E}$ (from Faraday!)

But that will "sweep" out too, so this might affect $\vec{B}$ (?)

It's hard to deduce the sol'n (we'll find a better method later), but let's guess a sol'n + verify that it "works" (i.e, is consistent with Maxwell's eq'ns)

Near the sheet, I expect $\vec{B}$ (given above) is correct. Any additional "flux of E" that might add in would be tiny if we consider a "loop" region tiny + near the origin.

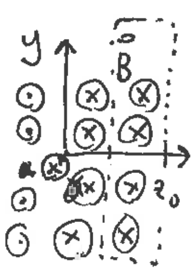
So let's propose $\vec{B} = \pm B_0 \hat{x}$ out to some z_0, + then $B=0$ past that.

Just right / Just left of $z=0$

(Symmetry demands B is independent of $y+x$!)

So, move out to the "leading edge", + draw a Faraday loop:

consider this (tilted) view



Around this loop, we must have $\oint \vec{E}\cdot d\vec{\ell} = -\dfrac{d\Phi_B}{dt}$

And, since $\vec{\nabla}\times\vec{E} = -\partial\vec{B}/\partial t$ points in $-\hat{x}$ direction,

I'm expecting $\vec{E} = -E_0 \hat{y}$ here. (for $z<z_0$)

and 0 past the "front".

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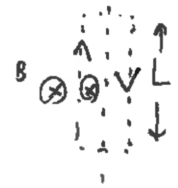
So in that little Faraday loop

going CW



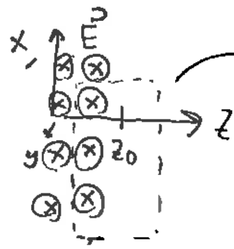
$$+E_y \cdot L = -\frac{d\Phi_B}{dt} = -B_x L V$$

$$\text{or} \quad E_y = -B_x V \qquad (1)$$

$\nabla \times B = \mu_0 \varepsilon_0 \, \partial E/\partial t$

So Now, with this $\vec{E}$ appearing out of thin air, let's examine

Consider this view



New, Amperian loop! Around this loop

$$\underbrace{\oint \vec{B} \cdot d\vec{\ell}} = \mu_0 \varepsilon_0 \frac{d}{dt} \Phi_E$$

$$B_x \cdot L = -\mu_0 \varepsilon_0 \cdot E_y L V$$

So $B_x = -\mu_0 \varepsilon_0 E_y V$ which is $\frac{1}{c^2} E_y V$ (2)

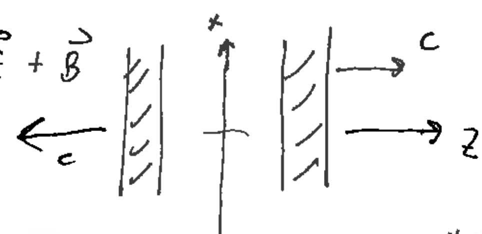
So (1) says $E_y / B_x = -V$

(2) " $E_y / B_x = -\frac{c^2}{V}$

together, $\frac{c^2}{V} = V$ or $\underline{\underline{V = c}}$

So Maxwell's eq'ns here support this "traveling wave front" sol'n, but only if $V = c$. And, we have $E \perp B \perp z$, and $\frac{|E|}{|B|} = c$, all as we claimed!

Note that if you turned the current sheet back off, you'd have a thick travelling "pulse" of $\vec{E} + \vec{B}$ (two, actually)



heading off at speed c. An EM "plane wave pulse"!