

Complex Exponentials

1. Euler's Equation : $\exp(i\theta) = \cos(\theta) + i \sin(\theta)$

Recall that a complex number z can always be written in two ways:

$$z = a + ib, \quad \text{or} \quad z = A \exp(i\theta)$$

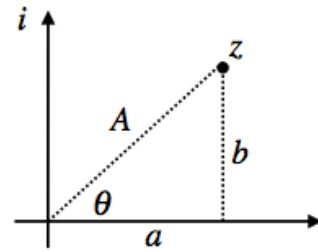
where A , a , b , and θ are real numbers such that:

$$|z| = A = \sqrt{a^2 + b^2} \quad a = A \cos(\theta) \quad b = A \sin(\theta)$$

When multiplying two complex numbers, the phase angles add:

$$z_1 = A_1 \exp(i\theta_1) \quad z_2 = A_2 \exp(i\theta_2)$$

$$z_1 \cdot z_2 = A_1 A_2 \exp[i(\theta_1 + \theta_2)]$$



Rewrite the following complex numbers in the form $A \exp(i\theta)$

$$-i = \frac{5}{i} =$$

$$1 + i = \frac{1}{1 - i} =$$

Use these last two answers and the rules for multiplying complex exponentials to find:

$$\frac{1 + i}{1 - i} =$$

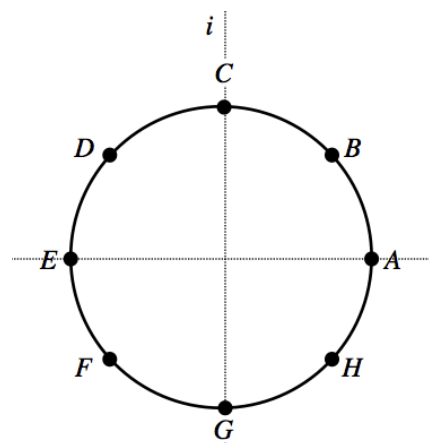
2. Identify the following complex numbers with their location in the complex plane:

$$\exp(i\pi/4)$$

$$-1$$

$$\cos(3\pi/4) - i \sin(\pi/4)$$

$$\exp(-i \cdot 3\pi/4)$$



(When you are done, answer the clicker question on the screen)

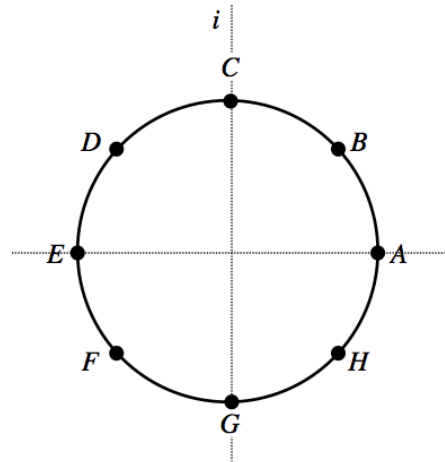
Complex Exponentials

3. Now, do the same task for the function $\exp(-i\omega t)$ at the various times given below:

$$\omega t_1 = \frac{\pi}{4}$$

$$\omega t_2 = \frac{\pi}{2}$$

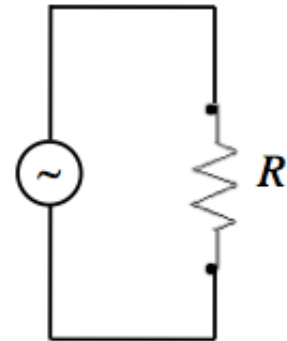
$$\omega t_3 = \frac{3\pi}{4}$$



Would an arrow representing $\exp(-i\omega t)$ in the complex plane rotate *clockwise* or *counter-clockwise* as time advances?

4. The circuit at right contains a resistor R and an AC voltage source, which we often represent by the following expression:

$$V(t) = V_0 \exp(i\omega t)$$



What is the magnitude of the physical current through the resistor when $\omega t = \pi/3$? Explain in words how you arrived at your answer.