Maxwell-Ampere Part 1

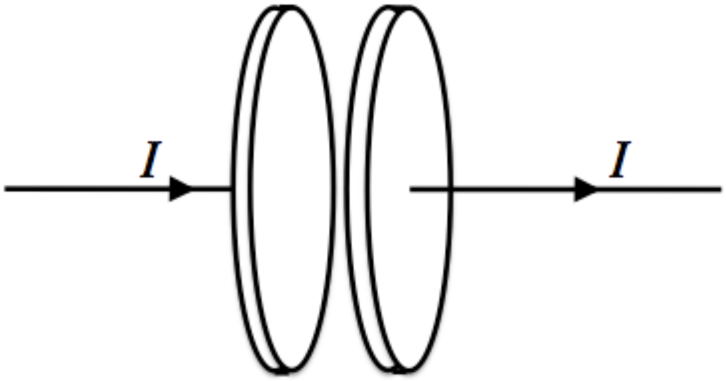
A. The full Maxwell-Ampere Law in differential form is:

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

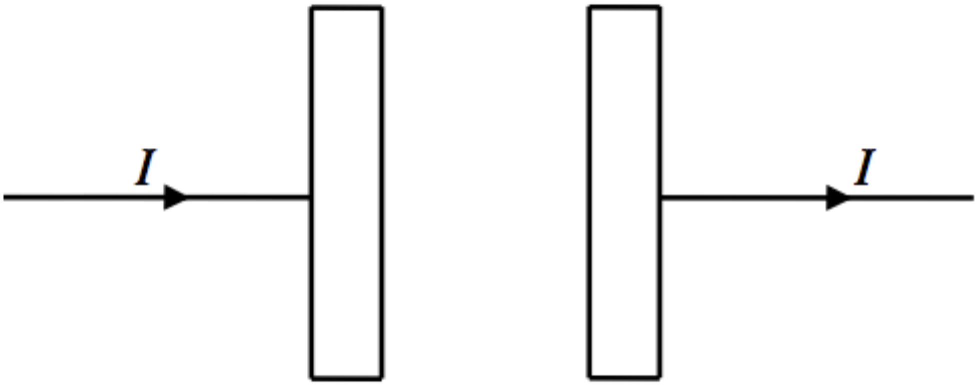
Rewrite this equation in integral form using Stokes' theorem. Be sure to show each of your steps.

▽ You may continue, but be sure to check your answer with an instructor.

B. Consider a capacitor in the process of charging up. The circular plates have radius R, area $A = \pi R^2$, and are so close together that fringe effects can be ignored. A current I is flowing in the long, straight wires.



Sketch the E-field between the capacitor plates in the diagram below, which shows the plates edge-on. Is this E-field changing with time?



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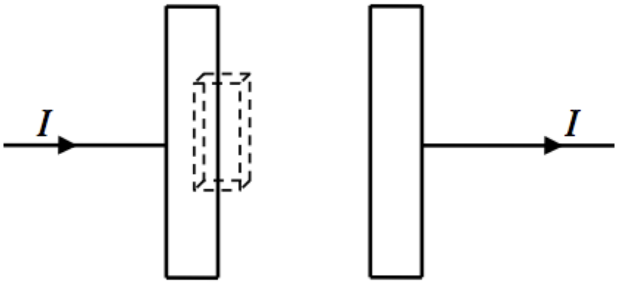
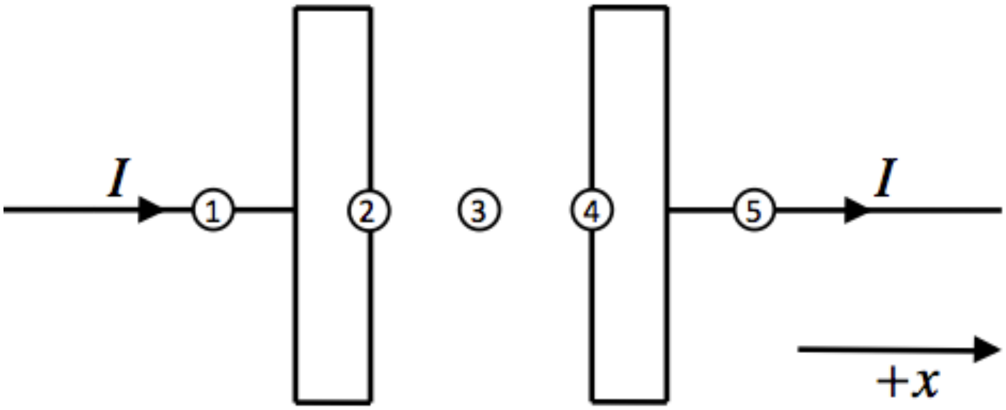
Consider the surface of an imaginary volume (dashed lines, at right) that partly encloses the left capacitor plate. For this closed surface, is the *total* flux of the current density **J** *positive*, *negative* or *zero*? Briefly explain

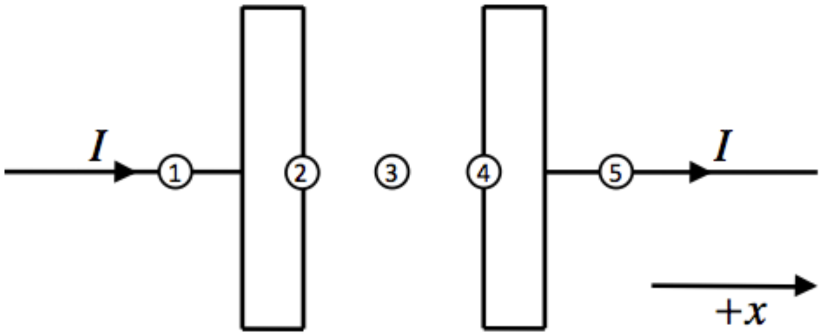




C. For each of the five points in the diagram above (labeled 1-5), fill out the table below to indicate whether the quantity in each row is *positive*, *negative* or *zero* at that point. Be sure your answers are consistent with charge being conserved.

	1	2	3	4	5
$\partial\rho/\partial t$					
$\nabla\cdot\mathbf{J}$					

Now, explain in words how your answers in each column are consistent with the conservation of charge.



D. Suppose the original Ampere's law $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$ were correct without any correction from Maxwell (it's not, but suppose for a moment that it is). What would this imply about $\nabla \cdot \mathbf{J}$ at points 2 and 4 in the diagram? [Hint: What is the divergence of the curl of a vector field equal to?] Are your answers consistent with your entries in the table on the previous page? (Conclusions?)

Still using the uncorrected Ampere's law $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$, fill out the table below to indicate whether $(\nabla \times \mathbf{B})_x$ is *positive*, *negative* or *zero* at points 1, 3 & 5.

	1	2	3	4	5
$(\nabla \times \mathbf{B})_x$		■		■	

Now, fill out the table below for points 1, 3 & 5 using the FULL Maxwell-Ampere Law (given on the first page) to indicate whether the quantities are *positive*, *negative* or *zero*.

	1	2	3	4	5
J_x		■		■	
$\partial E_x / \partial t$		■		■	
$(\nabla \times \mathbf{B})_x$		■		■	

Compare your answers for $\nabla \times \mathbf{B}$ in the two tables above (they *should* be inconsistent). Which set of answers is consistent with the equation $\nabla \cdot (\nabla \times \mathbf{B}) = 0$?