

EM Wave Equation

Using Maxwell's equations:

$$(1) \quad \vec{\nabla} \cdot \vec{\mathbf{E}} = \rho / \epsilon_0$$

$$(2) \quad \vec{\nabla} \cdot \vec{\mathbf{B}} = 0$$

$$(3) \quad \vec{\nabla} \times \vec{\mathbf{E}} = -\partial \vec{\mathbf{B}} / \partial t$$

$$(4) \quad \vec{\nabla} \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{J}} + \mu_0 \epsilon_0 \partial \vec{\mathbf{E}} / \partial t$$

and the following vector identity:

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{f}) = \vec{\nabla} (\vec{\nabla} \cdot \vec{f}) - \nabla^2 \vec{f}$$

show that in vacuum (where there are no charges or currents) each of the three spatial components of the *electric* field satisfy the three-dimensional wave equation.

$$\nabla^2 \vec{f}(\vec{r}, t) = \frac{1}{v^2} \frac{\partial^2}{\partial t^2} \vec{f}(\vec{r}, t)$$

What is the speed v of this wave?