Complex Plane Waves

A. Using complex exponential notation, construct an expression for a plane wave with the following properties:

Amplitude = A_0

Traveling in the $+\hat{z}$-direction

Linearly polarized in the x-direction

Wavelength = 2 meters

Frequency = 10 Hz

Phase angle = $+\pi/2$

▼ You may continue, but be sure to check your answers with an instructor.

B. A plane wave is given by the expression $\tilde{\mathbf{A}}\exp\left[i\left(ax - by - \omega t\right)\right]$, where $\tilde{\mathbf{A}} = A_0(1+i)\,\hat{z}\,)$ [a, b, ω & A_0 are all real numbers with appropriate units.]

What is the amplitude of this plane wave?

Re-write the argument of the complex exponential in the form $\left(\vec{k}\cdot\vec{r} - \omega t\right)$ to determine the direction of propagation for this plane wave.

What is the complex phase angle associated with this plane wave?

▼ You may continue, but be sure to check your answers with an instructor.

C. Suppose the E- and B-fields of an electromagnetic plane wave traveling in free space are given by the following expressions, where the vectors $\vec{\mathbf{E}}_0$ and $\vec{\mathbf{B}}_0$ are real constants.

(1) $$\vec{\mathbf{E}}(\vec{r},t) = \vec{\mathbf{E}}_0 \exp\left[i\left(\vec{k}\cdot\vec{r} - \omega t\right)\right]$$

(2) $$\vec{\mathbf{B}}(\vec{r},t) = \vec{\mathbf{B}}_0 \exp\left[i\left(\vec{k}\cdot\vec{r} - \omega t\right)\right]$$

Using expression (1) for the electric field, what is the partial derivative with respect to x of the x-component of $\vec{\mathbf{E}}(\vec{r},t)$?

$$\frac{\partial}{\partial x}E_X(\vec{r},t) = ?$$

Use the above result to find an expression for the divergence of $\vec{\mathbf{E}}$ for this plane wave, **written in terms of a dot-product of the vectors $\vec{k}$ and $\vec{\mathbf{E}}$.**

$$\frac{\partial E_X}{\partial x} + \frac{\partial E_Y}{\partial y} + \frac{\partial E_Z}{\partial z} = ?$$

According to Maxwell's equations $\vec{\nabla}\cdot\vec{\mathbf{E}} = \rho/\varepsilon_0$. What is the divergence of the electric field equal to for a plane wave traveling in vacuum?

$$\vec{\nabla}\cdot\vec{\mathbf{E}} = ?$$

According to Maxwell's equations, what is the divergence of $\vec{\mathbf{B}}$ for a plane wave traveling in vacuum?

$$\vec{\nabla}\cdot\vec{\mathbf{B}} = ?$$

Using the information on this page, what can you conclude about the <u>direction</u> of $\vec{\mathbf{E}}$ & $\vec{\mathbf{B}}$ relative to the vector $\vec{k}$?

(1) $\vec{\mathbf{E}}(\vec{r},t) = \vec{\mathbf{E}}_0 \exp\left[i\left(\vec{k}\cdot\vec{r} - \omega t\right)\right]$ (2) $\vec{\mathbf{B}}(\vec{r},t) = \vec{\mathbf{B}}_0 \exp\left[i\left(\vec{k}\cdot\vec{r} - \omega t\right)\right]$

D. Use Faraday's Law $\vec{\nabla} \times \vec{\mathbf{E}} = -\partial\vec{\mathbf{B}}/\partial t$ to make a convincing argument that $\vec{\mathbf{E}}$ & $\vec{\mathbf{B}}$ for an electromagnetic plane wave in vacuum are related by:

$$\vec{k} \times \vec{\mathbf{E}} = \omega\vec{\mathbf{B}}$$

[Hint: Do not spend your time trying to explicitly calculate the curl of $\vec{\mathbf{E}}$. Instead, think about what happens any time you take a partial derivative of the complex exponential.]

E. Use your conclusions from this and the previous page to determine the magnitude and direction of the Poynting vector $\vec{\mathbf{S}}$ for this electromagnetic plane wave.

$$\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}} \times \vec{\mathbf{B}} = ?$$

Summarize in words how the different spatial orientations of $\vec{\mathbf{E}}$, $\vec{\mathbf{B}}$, $\vec{k}$ & $\vec{\mathbf{S}}$ are all related to each other for an electromagnetic plane wave in free space