A. The following identities (I & II) are true for **any** vector field $\vec{\mathbf{F}}(\vec{\mathbf{r}},t)$ or scalar function $f(\vec{\mathbf{r}},t)$:

I. $\vec{\nabla}\cdot\left(\vec{\nabla}\times\vec{\mathbf{F}}\right)=0$

II. $\vec{\nabla}\times\left(\vec{\nabla}f\right)=0$

Maxwell's *time-independent* equations are:

$$\vec{\nabla}\cdot\vec{\mathbf{E}}=\rho/\varepsilon_0 \qquad \vec{\nabla}\times\vec{\mathbf{E}}=0 \qquad \vec{\nabla}\cdot\vec{\mathbf{B}}=0 \qquad \vec{\nabla}\times\vec{\mathbf{B}}=\mu_0\vec{\mathbf{J}}$$

Given the information above about scalar and vector fields, which of these Maxwell equations implies that $\vec{\mathbf{E}}=-\vec{\nabla}V$? Explain your reasoning.

Which of these Maxwell equations implies that $\vec{\mathbf{B}}=\vec{\nabla}\times\vec{\mathbf{A}}$? Explain your reasoning.

B. Recall the second math identity from the first page:

II. $\vec{\nabla} \times \left(\vec{\nabla} f \right) = 0$

$\vec{\nabla} \cdot \vec{\mathbf{B}} = 0$ says the magnetic field can be written as the curl of some vector potential $\vec{\mathbf{A}}$:

$$\vec{\mathbf{B}} = \vec{\nabla} \times \vec{\mathbf{A}}$$

Use Eq. II above to show that if we create a *new* vector potential $\vec{\mathbf{A}}'$, gotten by adding $\vec{\nabla}\lambda$ (where $\lambda\left(\vec{r}, t\right)$ is some scalar function of space and time) to the first vector potential $\vec{\mathbf{A}}$:

$$\vec{\mathbf{A}}' = \vec{\mathbf{A}} + \vec{\nabla}\lambda$$

then this new vector potential will also correspond to the same B-field:

$$\vec{\mathbf{B}} = \vec{\nabla} \times \vec{\mathbf{A}}'$$