

Part 2: Gauge invariance

C. Faraday's Law in differential form is

$$\vec{\nabla} \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial t}$$

Stokes' theorem says that the following integral relationship is also true:

$$\oint \vec{\mathbf{E}} \cdot d\vec{\ell} = -\frac{d}{dt} \iint \vec{\mathbf{B}} \cdot d\vec{\mathbf{a}}$$

What would be an analogous integral expression involving the vector potential $\vec{\mathbf{A}}$ and the magnetic field $\vec{\mathbf{B}}$, given that $\vec{\nabla} \times \vec{\mathbf{A}} = \vec{\mathbf{B}}$?

Combine this integral relationship between $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ from above with Faraday's law in integral form to find an integral relationship between $\vec{\mathbf{E}}$ and $\vec{\mathbf{A}}$.

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D. If $\oint \vec{E} \cdot d\vec{\ell} = 0$ says that $\vec{E} = -\vec{\nabla}V$, use the integral relationship you derived on the previous page between \vec{E} & \vec{A} to show that

$$\vec{E} + \frac{\partial \vec{A}}{\partial t} = -\vec{\nabla}V$$

Is this relationship also true in time-*independent* situations?
Why or why not?

Suppose we have two vector potentials \vec{A} & \vec{A}' , such that

$$\vec{A}' = \vec{A} + \vec{\nabla}\lambda$$

(This shift of the vector potential is part of what is called “changing the gauge”) Is it also true that $\oint \vec{A} \cdot d\vec{\ell} = \oint \vec{A}' \cdot d\vec{\ell}$? Why or why not?

Challenge Question: We showed in the previous Tutorial (part B) that changing \vec{A} in this way, $\vec{A}' = \vec{A} + \vec{\nabla}\lambda$, does not affect the \vec{B} field. Does it affect the \vec{E} field?