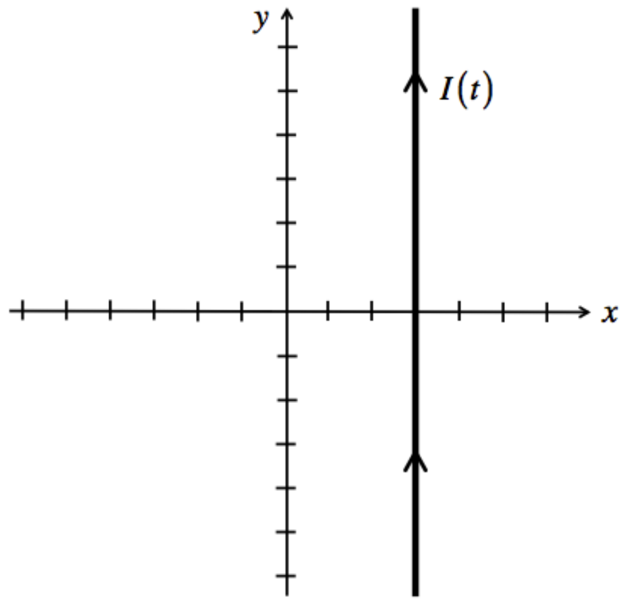
A. An infinitely long wire has a current that is turned on abruptly at $t = 0$. The function $I(t)$ describing this current is:

$$I(t) = \begin{cases} 0 & \text{for } t < 0 \\ I_0 & \text{for } t \geq 0 \end{cases} \qquad I_0 = \text{constant}$$

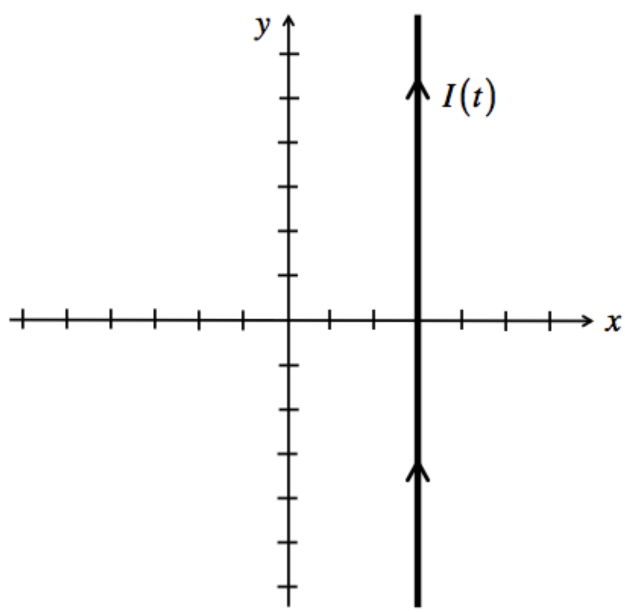
NOTE: The *x* and *y* dimensions on the graphs below are scaled so that each tick mark represents the distance light would travel in one time unit. You can use the "time ruler" on the handout to measure distances.

Indicate on the graph all of the points in space $\vec{r}$ where an observer would be aware of a non-zero current in the wire at $t = 2$.

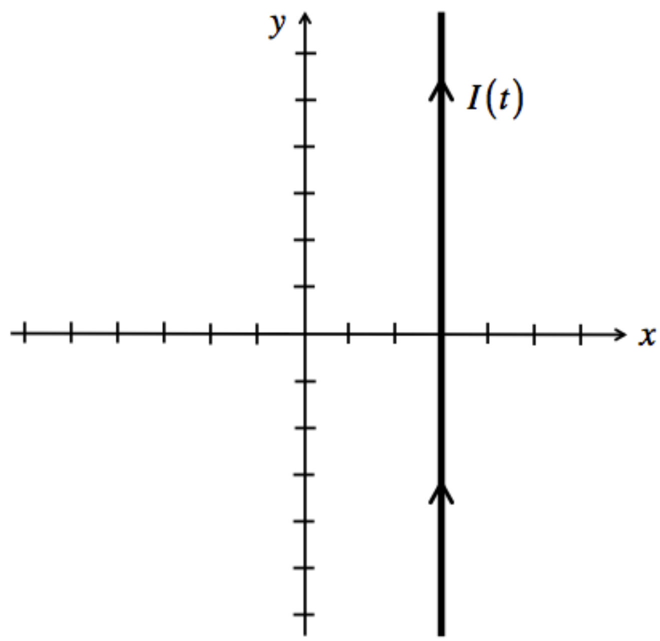


You are an observer at the origin ($\vec{r} = 0$).
- Use your "time ruler" to indicate the distance $\vec{r}'$ from the origin to several points up and down the wire.
- What is the retarded time t_R at each of these points when $t = 5$?

$$t_R = t - \frac{|\vec{r} - \vec{r}'|}{c}$$



For an observer at the origin, indicate on the graph all of the points on the wire for which the observer is aware of a non-zero current at $t = 5$. That is, where on the wire is $I(t_R) = I_0$?



B. Recall that in the Lorentz gauge, the retarded vector potential will be:

$$\vec{\mathbf{A}}(\vec{r},\, t) = \frac{\mu_0}{4\pi}\int_{-\infty}^{+\infty} \frac{I(t_R)}{|\vec{r} - \vec{r}'|}\, dy' \;\; \hat{y}$$

Re-write this equation for the vector potential **at the origin** in terms of the constant I_0. Be sure to specify the correct limits of integration, which will be a function of time.

Challenge Questions:

We want to be able to compute the electric field from $\vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{A}}}{\partial t}$.

Recall the fundamental theorem of calculus in 1-D:

$$\frac{d}{dx}\left[\int_0^{x(t)} f(x')\ dx'\right] = f(x)$$

Then, by the chain rule:

$$\frac{d}{dt}\left[\int_0^{x(t)} f(x')\ dx'\right] = \left[\frac{d}{dx}\int_0^{x(t)} f(x')\ dx'\right]\cdot\frac{dx}{dt} = f(x)\cdot\frac{dx}{dt}$$

Use this to compute the electric field $\vec{\mathbf{E}}(t)$ at the origin.

What is the electric field at the origin in the limit $t \to \infty$? Explain in words why your answer makes sense.