

University of Colorado, Department of Physics
PHYS3320, Spring 2016, HW 1

due Fri, Jan 15, at the start of class (no late HWs will be accepted)

On all homework assignments this term, please show your work and explain your reasoning. We try to grade for clarity of explanation as much as we do for mere "correctness of final answer". Please read the syllabus and be sure you understand the course policy regarding collaboration on homework assignments.

1. [Total: 8 pts] Consider the following vector functions

$$\mathbf{v}_a = x^2 \hat{\mathbf{x}} + 3xz^2 \hat{\mathbf{y}} - 2xz \hat{\mathbf{z}} \quad (1)$$

$$\mathbf{v}_b = (r \cos \theta) \hat{\mathbf{r}} + (r \sin \theta) \hat{\boldsymbol{\theta}} + (r \sin \theta \cos \phi) \hat{\boldsymbol{\phi}} \quad (2)$$

- a) [4 pts] Calculate the curl of \mathbf{v}_a and the curl of \mathbf{v}_b .
b) [4 pts] The divergence of the curl of a vector function \mathbf{v} is always zero:

$$\nabla \cdot (\nabla \times \mathbf{v}) = 0 \quad (3)$$

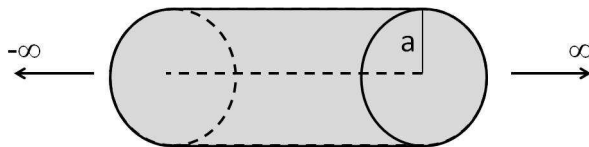
Check it for the vector functions \mathbf{v}_a and \mathbf{v}_b .

2. [Total: 8 pts] Consider an electric field given by

$$\mathbf{E}(\mathbf{r}) = cy \hat{\mathbf{y}} \quad (4)$$

in some region in space, where c is a constant.

- a) [2 pts] What are the SI units of c ?
b) [3 pts] What is the charge density $\rho(\mathbf{r})$, that generates this electric field?
c) [3 pts] What is the voltage $V(\mathbf{r})$ associated with this electric field? Is your answer unique?
3. [Total: 14 pts] An infinitely long linear charge distribution with density λ_0 is located on the axis of an infinitely long solid cylinder of uniform volume charge density ρ_0 and radius a (see Figure). The net charge per length of the combined charge distributions is zero.



- a) [4 pts] Find the relation between λ_0 and ρ_0 that must be satisfied for the net charge per length to be zero.
b) [6 pts] Find the electric field \mathbf{E} both inside and outside the solid cylinder. Express your answer in terms of λ_0 and a . Sketch the magnitude of \mathbf{E} as a function of the radial distance s from the axis of the cylinder.
c) [4 pts] Assuming, as usual, that the potential $V(s \rightarrow \infty) = 0$: What is the potential at $s = a/2$? Express your answer in terms of λ_0 or ρ_0 and a .

4. [Total: 20 pts] Suppose an electric field in a region of space is given by

$$\mathbf{E}(\mathbf{r}) = \frac{B_0}{2\tau} (y\hat{\mathbf{x}} - x\hat{\mathbf{y}}) \quad (5)$$

where τ is a constant with units of time, and B_0 is also a constant.

- a) [2 pts] What are the units of the constant B_0 ?
- b) [3 pts] Sketch the electric field in the xy -plane. Do it by hand, but if you want check yourself by using e.g. Mathematica.
- c) [4 pts] Calculate the closed line integral

$$\oint \mathbf{E} \cdot d\mathbf{l} \quad (6)$$

where the path is a circle of radius R , parallel to the xy -plane, centered at $(0, 0, z_0)$. Integrate counter-clockwise as viewed from "above".

(*Hint: It is useful to work in cylindrical coordinates here.*)

- d) [4 pts] Calculate the closed line integral, Eq. (6), again, but this time where the path is a rectangle (sides of length a and b) oriented parallel to the xy -plane. Again, integrate counterclockwise as viewed from "above".
- e) [4 pts] Calculate $\nabla \times \mathbf{E}$ and describe the resulting vector field in words. Show that the closed line integral values, which you obtained in parts c) and d), are equal to the corresponding surface integrals of $\nabla \times \mathbf{E}$. Relate this to the Stokes theorem.
- f) [3 pts] Determine the scalar potential that is related to the electric field \mathbf{E} given in Eq. (5), or explain why no such potential exists.

Does this problem strike you as a mere mathematical exercise without any physical reality? Well, we'll come back to this electric field in this course.