University of Colorado, Department of Physics
PHYS3320, Spring 2016, HW 10
due Fri, Apr 1 by 5:00pm, in the mailbox at the entrance to the physics helproom

1. [4 pts] Griffiths 9.14

2. [12 pts] Griffiths 9.15

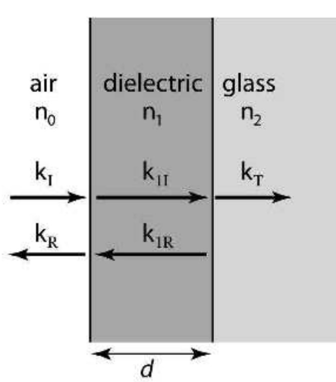
3. [Total: 12 pts]
A thin coating of dielectric material is deposited onto the surface of a glass lens, so that the light entering the lens first goes through an air-dielectric interface before going through a dielectric-glass interface and then into the glass (see Figure). The index of refraction of air is n_0 (which you can set to 1, as a good approximation), of the dielectric is n_1, and of the glass is n_2. You can assume that all μ's are equal to μ_0, and that the incoming light propagates in the $\hat{\mathbf{z}}$-direction and is polarized in the $\hat{\mathbf{x}}$-direction.

   a) [4 pts] Show that, for normal incidence, the transmission coefficient is given by:

   $$T = \beta \left| \frac{\tilde{E}_{0,t}}{\tilde{E}_{0,in}} \right|^2$$

   where $\tilde{E}_{0,in}$ and $\tilde{E}_{0,t}$ are the complex electric field amplitudes of the incident and the transmitted wave, respectively, and $\beta = n_2/n_0$.
   (*Note: remember that the transmission coefficient is defined to be the ratio of transmitted to incident intensity.*)

   b) [4 pts] Write down all (relevant) boundary conditions for the electric and magnetic fields.



c) [4 pts] Using the boundary conditions one can show that the transmission coefficient for light of frequency ω entering the lens material is given by:

$$T = \frac{4n_2}{(1+n_2)^2 + \frac{1}{n_1^2}\left(1-n_1^2\right)\left(n_2^2-n_1^2\right)\sin^2\left(\frac{n_1\omega d}{c}\right)}$$

where d is the thickness of the dielectric layer. (You don't have to derive this formula!) Using this result, assume you work for the Nikon lens company, and you are assigned the task of designing a dielectric coating that optimizes the transmission of glass lenses for light at $\lambda = 500$ nm. Assume air has $n_0 = 1$ and glass has $n_2 = 1.5$. What value of n_1 would you choose, and how thick would you make the coating? Explain your reasoning briefly, but clearly, in words as well as equations. What does T come out to be?

4. [Total: 14 pts]
Later in the semester we will consider electromagnetic radiation from a "pointlike" antenna, and will get what are called spherical waves rather than plane waves. Here is a simplified mathematical formula for such a spherical wave (simplified meaning that it is accurate for large distance from the origin):

$$\mathbf{E}(r,\theta,\phi;t) = A\frac{\sin(\theta)}{r}\cos(kr-\omega t)\hat{\phi} \quad (1)$$

(Think about: Why would we call this a "spherical wave"?)

a) [2 pts] Show that this wave satisfies Gauss' law in free space.

b) [4 pts] Use Faraday's law to find the formula for the corresponding magnetic field. Since we are assuming we are far from the origin, you should neglect any terms which drop off faster than $1/r$.

c) [4 pts] Verify that the formulas for $\mathbf{E}$ and $\mathbf{B}$ satisfy the remaining two Maxwell equations in free space. (Remember, that you can neglect any terms that drop off faster than $1/r$.)

d) [4 pts] Find the magnitude and direction of the Poynting vector? Does the direction makes sense? Argue that your result for the magnitude makes sense, given simple arguments about energy conservation / flow from a point source.