University of Colorado, Department of Physics
PHYS3320, Spring 2016, HW 11
due Fri, Apr 15 by 5:00pm, in the mailbox at the entrance to the physics helproom

1. [Total: 22 pts]
   Let the interface between two linear media be the x, y-plane ($z = 0$), and the incident electric wave be
   $$\tilde{\mathbf{E}}_{in}(\mathbf{r}, t) = \tilde{E}_{0,in} \exp[i(\mathbf{k} \cdot \mathbf{r} - \omega t)]\hat{\mathbf{y}}$$
   The propagation vector is $\mathbf{k}_{in} = k_{in} \sin(\theta_{in})\hat{\mathbf{x}} + k_{in} \cos(\theta_{in})\hat{\mathbf{z}}$. Assume medium 1 has permittivity, permeability and index of refraction ϵ_1, μ_1 and n_1; those of medium 2 are ϵ_2, μ_2 and n_2. This is the case of polarization perpendicular to the plane of incidence; in lecture, we covered the different case of polarization lying in the plane of incidence.

   a) [8 pts] Find the Fresnel equations for $\tilde{E}_{0,r}$ and $\tilde{E}_{0,t}$.
   b) [4 pts] Sketch $\tilde{E}_{0,r}/\tilde{E}_{0,in}$ and $\tilde{E}_{0,t}/\tilde{E}_{0,in}$ as functions of θ_{in} for the case $\mu_1 n_2/\mu_2 n_1 = 1.5$.
   c) [4 pts] Compute the Fresnel equations for the case of normal incidence. Compare your results with those given in Griffiths, section 9.3.2.
   d) [4 pts] Compute the reflection and transmission coefficients R and T, and check that they add up to 1.
   e) [2 pts] For this polarization case and with distinguishable media (ϵ, μ and n are different for the two media), is there a Brewster's angle where the amplitude of the reflected wave is zero? If so, find an expression for the angle in terms of the media properties (e.g., n_1, n_2).

2. [9 pts total; 3 pts each part] Griffiths 9.20

3. [Total: 10 pts]
Consider a point charge q at rest at the origin. The scalar and vector potentials of this charge distribution are given by

$$V(\mathbf{r}, t) = \frac{q}{4\pi\epsilon_0 r} \tag{1}$$

$$\mathbf{A}(\mathbf{r}, t) = 0 \tag{2}$$

a) [4 pts] Check to see if we are in the Coulomb gauge, the Lorentz gauge, or perhaps, both, or maybe, neither.

b) [6 pts] Now introduce a gauge transformation $\mathbf{A}' = \mathbf{A} + \nabla f$ and $V' = V - \partial f/\partial t$ with

$$f(\mathbf{r}, t) = \frac{qt}{4\pi\epsilon_0 r}\,. \tag{3}$$

Calculate $\mathbf{A}'$ and V'. Briefly, discuss your results by considering

* Are these new potentials time-independent (static) or time-dependent?
* Do these potentials represent the same physical situation?
* Are we in the Coulomb gauge, or in the Lorentz gauge now?
* What is the **B**-field based on the transformed potentials V' and $\mathbf{A}'$?

4. [Total: 10 pts]
Say the potentials throughout space and time are

$$V(\mathbf{r}, t) = 0$$

$$\mathbf{A}(\mathbf{r}, t) = A_0 \cos\left[k(z - ct)\right]\hat{\mathbf{x}}$$

where A_0 and k are given constants, and c is the speed of light.

a) [4 pts] Find the **E** and **B** fields everywhere in space and time, and comment on the *physics* here - what have we got going on?

b) [4 pts] Are we in the Coulomb gauge, the Lorentz gauge, both, or neither?

c) [2 pts] Is it possible, in principle, to find a different gauge for this problem (following the usual procedure for gauge transformations) in which $\mathbf{A}(\mathbf{r}, t) = 0$? If your answer is *yes*, you do not necessarily need to find the particular gauge transformation. I am just asking if it is possible *in principle*, and how do you know.