University of Colorado, Department of Physics
PHYS3320, Spring 2016, HW 2
due Fri, Jan 22 by 5:00pm, in the mailbox at the entrance to the physics helproom

1. [Total: 21 pts] Suppose the vector potential in a region of space is given by:

$$\mathbf{A} = A_0 \exp\left(-\frac{x^2 + y^2}{2a^2}\right)\hat{\mathbf{z}} \tag{1}$$

a) [2 pts] What are the units of the given constants a and A_0?

b) [4 pts] Determine the magnetic field from this vector potential and then determine the current density **J** from the magnetic field.
(*Eq. (1) is given in Cartesian coordinates, but you may use another coordinate system, if that would make the calculation easier. Briefly, but clearly justify your choice.)*

c) [6 pts] Separately sketch the vector field **A**, the magnetic field **B** and the current density **J** you have found (using any representation you feel conveys the most useful information). Briefly, for each of the plots use words to describe what they look like. In particular, if your representation "hides" information, state what it is.
(*You can use e.g. Mathematica to plot these fields - preferably the one(s) you feel is (are) hard to visualize.)*

d) [4 pts] Integrate the current density to mathematically show that the total current flowing through *any* infinite plane parallel to the $x - y$ plane is zero. Then, give an argument (without doing any formal integral) why you could have known before calculating that this must be the case.

e) [3 pts] Calculate the divergence of **J**. What does your result imply?

f) [2 pts] Does this set of $\mathbf{A} - \mathbf{B} - \mathbf{J}$ fields strike you as an unphysical mathematical excercise, or can you imagine some physical system or electronic device which might at least approximately described/represented in this problem. Briefly, discuss.

2. [Total: 20 pts] Consider a parallel-plate capacitor attached to a battery of constant voltage V_0. The plates are separated by a distance d and have a square area L^2, where $L \gg d$ so that edge effects are negligible. One plate lies at $x = 0$ (in the yz plane), and the other lies at $x = d$ (parallel to the yz plane). The space between the plates is filled with a weakly conducting material that has a *non-constant* conductivity $\sigma(x) = \sigma_0 + \sigma' x$, where σ_0 and $\sigma' = \frac{d\sigma}{dx} > 0$ are constants.

   a) [4 pts] Consider the following quantities: current I, current density $\mathbf{J}$, electric field $\mathbf{E}$, and voltage V. Which of these quantities is constant, independent of position, in the space between the plates? Explain your choices.

   b) [6 pts] Solve for the electric field between the plates in terms of the battery voltage V_0 and the other known quantities. Make a qualitative sketch of the magnitude of the field vs. position x.

   c) [3 pts] Derive an expression for the charge density ρ between the plates in terms of the known quantities in the problem.

   d) [3 pts] Check your answer in part c) by considering the limit $\sigma' \to 0$, i.e. the limit of *constant* conductivity. Explain briefly.

   e) [4 pts] Derive an expression for the resistance R of the material between the plates (using $R = V_0/I$). Show that your expression makes sense by considering the limit $\sigma' \to 0$, i.e. the limit of constant conductivity.