University of Colorado, Department of Physics
PHYS3320, Spring 2016, HW 3
due Fri, Jan 29 by 5:00pm, in the mailbox at the entrance to the physics helproom

1. [4 pts] Show that for a material with a *non-uniform* conductivity $\sigma(\mathbf{r})$ and a steady flow of current the charge distribution inside the material is given by

$$\rho(\mathbf{r}) = -\frac{\epsilon_0 \mathbf{E}(\mathbf{r}) \cdot \nabla\sigma(\mathbf{r})}{\sigma(\mathbf{r})} \tag{1}$$

Of course, this equation holds for a uniform conductivity $\sigma(\mathbf{r}) = \sigma_0$ as well. What is the result in this case?

2. [Total: 22 pts]
The region between two concentric metal spherical spheres (with radius a and b, respectively, and $a < b$) is filled with a weakly conducting material of conductivity σ. Assume that the outer shell is electrically grounded, and a battery maintains a potential difference of $|V| = V_0$ between the two shells.
(*In this problem, don't confuse the conductivity σ with the surface charge density. Also, for this problem, ignore any dielectric properties of this weakly conducting material.)*

   a) [4 pts] What total current I flows between the shells?

   b) [2 pts] What is the total resistance R of the weakly conducting material between the shells? Then adapt your equation for the resistance to the situation where a conducting sphere of radius a is embedded in a large uniform volume with conductivity σ, and held at a potential of V_0 with respect to some boundary very far away. What would be the resistance for this arrangement?

   c) Suppose the battery would be suddenly disconnected at $t = 0$. Thus, at $t = 0$ the voltage difference between the shells is V_0, but there is no battery to *maintain this* any more.

      (i) [2 pts] Describe qualitatively what you expect happens over time.

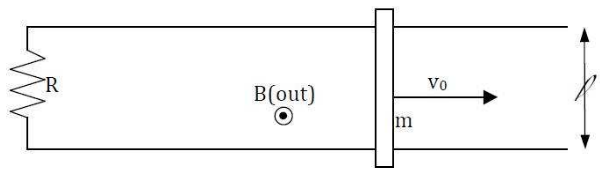
      (ii) [2 pts] Determine the net charge on the shells as a function of t in terms of the resistance R and capacitance C.

      (iii) [2 pts] Then, calculate the voltage, and the current that flows between the two shells, i.e. find $V(t)$ and $I(t)$. Does your result agree with your qualitative prediction? Discuss whether/how your answer depends on the specific (spherical) geometry of this situation.

   d) [4 pts] Now assume that the region between the two concentric metal spherical spheres (with radius a and b, respectively) is filled with a weakly conducting material of *non-uniform* conductivity $\sigma(\mathbf{r}) = cr$ where c is a constant. As before, assume that the outer shell is electrically grounded, and a battery maintains a potential difference of $|V| = V_0$ between the two shells. Calculate the electric field $\mathbf{E}(\mathbf{r})$ and the voltage $V(\mathbf{r})$ in the region between the shells as well as the total resistance R of the weakly conducting material.

   e) [6 pts] Check your results, which you obtained in part d), by calculating the charge distribution $\rho(\mathbf{r})$ in the region between the shells in three different ways: (i) using Eq. (1) (see problem 1), (ii) using Gauss' Law and (iii) using Poisson's Equation.

3. [Total: 12 pts] A metal bar with mass m is sliding without friction on two parallel conducting rails a distance l apart, as shown. The circuit of rails plus bar is completed through a resistor R. The bar, rails, and resistor are in a region of space with uniform magnetic field $\mathbf{B}$ pointing out of the page. At a given time $t = 0$, the bar is moving to the right with speed v_0.



a) [2 pts] Find the emf in the circuit using the Lorentz force law, showing the contribution of each piece of the circuit.

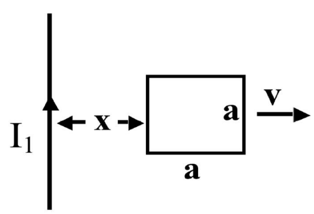
b) [2 pts] Calculate the emf in the circuit using the flux rule. Does it agree with your answer to part a)? If not, why not?

c) [2 pts] Find the magnitude and the direction of the current through the resistor using the emf from part b). You can ignore any effects of self-inductance.

d) [3 pts] Determine the motion of the bar after $t = 0$, that is, find an expression for $v(t)$.

e) [3 pts] Can you test your result obtained in part d) using energy conservation? If yes, do so. If not, explain why not.

4. [Total: 12 points] A long wire carries a steady current I. Nearby to the long wire is a square loop of wire (side length a) with resistance R. You apply a force on the loop away from or toward the wire so that the loop maintains a constant velocity $\mathbf{v}$.
(*Ignore any possible self-inductance effects in this problem, i.e. assume that the magnetic field produced by current in the loop is small compared to the field produced by the wire.*)



a) [Each part: 2 pts] Assume that the nearby edge of the loop is at a distance x away from the wire and find:
(i) the magnetic flux through the loop,
(ii) the emf around the loop,
(iii) the magnitude of the current circulating around the loop,
(iv) the power dissipated in the loop.

b) [4 pts] Determine the magnetic force on the loop, and the power you need to supply to keep the loop moving at a constant velocity, both as a function of position x. Show that the power is always positive, independent of whether the applied force is toward or away from the wire. Compare your result to that from part (a)-(iv). Does the result make sense?