University of Colorado, Department of Physics
PHYS3320, Spring 2016, HW 4
due Fri, Feb 5 by 5:00pm, in the mailbox at the entrance to the physics helproom

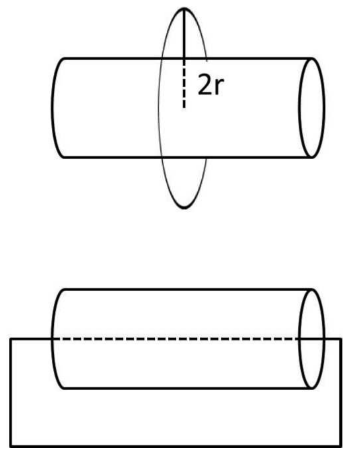

Figure 1: Set-up for problem 1b) (upper panel) and 1c) (lower panel).

1. [Total: 15 pts]
A very long insulating cylinder of length L and radius r ($L \gg r$) has a charge Q uniformly distributed over its outside surface. The cylinder is made to rotate with angular velocity ω_0 about its axis. (Ignore edge effects).

   a) [7 pts] Find the magnetic field (magnitude and direction) inside the cylinder.

   b) [5 pts] A single-turn loop of wire with radius $2r$ and resistance R is wrapped around the cyclinder, as shown in Fig. 1 (upper panel on the right). The rotation of the cylinder is slowed down linearly as a function of time, i.e. $\omega(t) = \omega_0(1 - t/t_0)$. What is the magnitude of the induced current in the coil? In what direction does the current flow?

   c) [3 pts] Instead of the coil in part b), a one-turn loop of wire is placed through the cyclinder as shown in Fig. 1 (lower panel on the right), and the cylinder is slowed down as before. How much current will now flow?

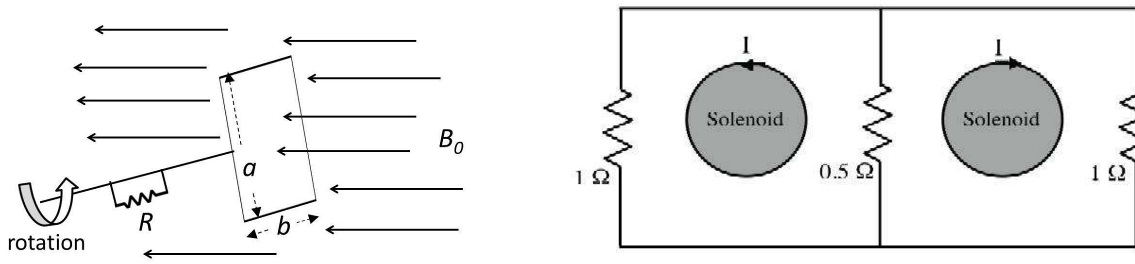

Figure 2: Left: Electric generator for problem 3. Only one conducting loop is shown. It is not obvious from the figure, but when the loop is "vertical" (a-direction vertical and b-direction horizontal), the magnetic field is normal to the plane of the loop. Another way of saying this is that the maximum magnitude of magnetic flux through the loop is $|B_0 ab|$. Right: Set-up for problem 4.

2. [Total: 12 pts]
   A conducting disk with radius a, height $h \ll a$, and conductivity σ is immersed in a time varying but spatially uniform magnetic field parallel to its axis, $\mathbf{B}(t) = B_0 \sin(\omega t)\hat{z}$.

   a) [6 pts] Ignoring the effects of any induced magnetic fields, find the induced electric field $\mathbf{E}(\mathbf{r}, t)$ and the current density $\mathbf{J}(\mathbf{r}, t)$ in the disk. Sketch the current distribution.

   b) [3 pts] If the power dissipated in a resistor is $P = IV$, show that the power dissipated per unit volume is $\mathbf{J} \cdot \mathbf{E}$.

   c) [3 pts] Use your results from parts a) and b) to calculate the total power dissipated in the disk at time t, and the average power dissipated per cycle of the field.

3. [Total: 10 pts]
   An electric generator consists of a coil (N conducting loops) rotating in a constant external magnetic field at the frequency ω (e.g. propelled by a steam or a water turbine). Each loop has an area ab, as shown in Fig. 2 on the left.

   a) [4 pts] Calculate the electromotive force and the current in the coil as a function of known parameters.

   b) [2 pts] Calculate the heat dissipated in the resistor averaged over one period.

   c) [4 pts] The heat output needs to be compensated by the turbine. Calculate the torque exerted on the current-carrying coil and use the result to calculate the mechanical power of the turbine driving the generator.

4. [8 pts] Consider the electrical circuit shown in Fig. 2 (on the right). There are two infinitely long solenoids (shaded, shown end-on in the Figure). The solenoids are identical, both with the same cross sectional area of 0.1 m^2, and the *magnitude* of the magnetic field inside both is equal and increasing at a constant rate of 1 T/s. However, note the *direction* of the current in the two solenoids is such that the **B**-fields are increasing in opposite directions. The resistors have the values as shown. Determine the current passing through each resistor (magnitude and direction).