University of Colorado, Department of Physics
PHYS3320, Spring 2016, HW 5
due Fri, Feb 12 by 5:00pm, in the mailbox at the entrance to the physics helproom

1. [Total: 8 pts, each part: 4 pts]
   Consider a standard coax cable as wire of infinite length and radius a surrounded by a thin conducting cylinder, coaxial with the wire, with inner radius b and outer radius c. Assume $a \ll b$ and $(c-b) \ll b$ (i.e. thin shell and wire). A current $I(t)$ flows along the wire and a corresponding current $I(t)$ in the opposite direction on the outer cylinder. Assume that we have the (quasi-static) situation in which the currents are identical in magnitude at each moment in time, and the changes in current are sufficiently slow.

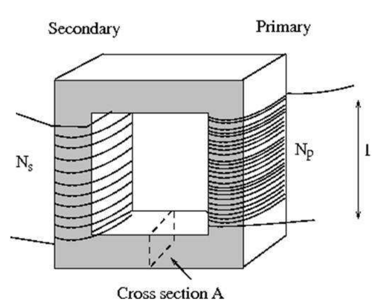
   a) Find the self-inductance per length of the cable by considering the change in flux as I is changed. Ignore the small amount of flux within the conductors themselves.

   b) Now find the self-inductance per length of the cable by considering the magnetic energy W stored per length of the cable and relating that to the energy W sored in the inductor L by current I. Does your answer agree with that from part a)?

2. [Total: 9 pts, each part: 3 pts]
   The simplest way to make a compact transformer is to use iron to "trap" the magnetic flux so that it can be "channeled" from one circuit to another in a mechanically stable and simple way. One common design uses a square frame of iron as shown in the Figure below. On the input side (the primary) a wire is wrapped around one side of the frame N_p times; on the other side (the secondary) a wire is wrapped around the frame N_s times. The coil wraps around the side in such a way that positive flux in the primary side corresponds to a positive flux in the secondary. Assume that the only role the iron core plays is to channel the flux from one coil to the other.

   a) Assuming that you can treat the primary and secondary sides as infinite solenoids (actually a good approximation because of the iron), find the mutual inductance between the primary and the secondary coil. Take the cross sectional area of the iron sides to be A and the length of each coil to be l.

   b) Again, treating the coils like infinite solenoids, find the self-inductance per unit length of each coil. Compare your results to your answer to part a).



c) Given that the current I_p in the primary is changing with time, find the ratio of the electromotive force in the primary side $\mathcal{E}_1$ to the electromotive force induced in the secondary side $\mathcal{E}_2$.

3. [Total: 8 pts, each part: 4 pts]

In this problem, remember that very small current loops can be thought of as magnetic dipoles.

Consider two very small circular loops of wire. The center of loop 1 (loop 2) lies at position $\mathbf{r}_1$ ($\mathbf{r}_2$) and has area vector $\mathbf{a}_1$ ($\mathbf{a}_2$).

a) Find the mutual inductance of the two loops.

b) Suppose that a steady current I_1 is flowing in loop 1. Until time $t = 0$, no current is flowing in loop 2. At $t = 0$ we gradually turn on a current in loop 2, that increases linearly until time τ, and then is held constant at a value I_{2f}. So as a function of time, the current in loop 2 is

$$I_2(t) = \begin{cases} 0, & t < 0 \\ t\, I_{2f}/\tau, & 0 < t < \tau \\ I_{2f}, & t > \tau \end{cases} . \tag{1}$$

Under these circumstances, some work must be done to keep the same steady current I_1 flowing in loop 1. Explain qualitatively why this is so, then find the total work that must be done to maintain the current I_1.

4. [Total: 10 points]

Consider an infinitely long cylindrical shell of charge with radius R and surface charge density σ. Assume the cylindrical shell is massless. Starting from rest, we would like to get the shell spinning with final angular velocity ω_f.

a) [5 points] As a function of $\omega(t)$, $\dot{\omega}(t)$, and the distance s from the cylinder's axis, first find the magnetic field produced by the spinning charge in the quasi-static approximation, and then find the resulting induced electric field. How much work must you exert in the form of torque to reach the final angular velocity ω_f?

b) [5 points] Determine the total energy (per unit length of the cylinder) stored in the magnetic field once the cylinder has reached angular velocity ω_f. How does your answer compare to that of part (a)? Does this make sense? Why?