University of Colorado, Department of Physics
PHYS3320, Spring 2016, HW 6
due Fri, Feb 26 by 5:00pm, in the mailbox at the entrance to the physics helproom

1. [Total: 20 pts]
   Consider our standard coax cable as a wire of infinite length and radius a surrounded by a thin conducting cylinder, coaxial with the wire, with inner radius b and outer radius c. Assume $a \ll b$ and $(c-b) \ll b$ (i.e. thin shell and wire). We have previously found the self-inductance per length of the cable. Now let's investigate the induced **E**-field and the displacement current $\mathbf{J}_d$ for a particular time-dependent current $I(t) = I_0 \cos(\omega t)$, which flows along the wire, and a correspondent current $I(t)$ flows in the opposite direction on the outer cylinder. Assume that the currents are identical in magnitude at each moment in time, and the changes in current are slow.

   a) [4 pts] Find $\mathbf{B}(s,t)$ in the 'coax region' ($a < s < b$) where s is the usual radial coordinate, and the current in the wire is I_0 in the $+z$ direction at $t = 0$. (Note: Assume we are 'quasistatic' here. Thus, assume that the changes in the electric field are small. We will later in part d) see whether this is a good assumption. Should be a familiar problem.)

   b) [6 pts] Find $\mathbf{E}(s,t)$ in the 'coax region' ($a < s < b$). Assume that the magnitude of $\mathbf{E} \to 0$ as $s \to \infty$.

   c) [6 pts] Find the displacement current density $\mathbf{J}_d$ in the 'coax region' ($a < s < b$) for this electric field **E**, and integrate it to get the total displacement current I_d.

   d) [4 pts] Using, physically reasonable numbers for a real coax (say $a = 1$ mm and $b = 1$ cm), determine the frequency ω for which I_d finally equals 1% of I_0. Briefly, comment on the following aspects: What sort of frequency is this? Do we need to worry about the displacement current for *lower* and *higher* frequencies than this?

2. [Total: 8 pts]
Consider the model for a parallel-plate capacitor, shown in Fig. 3, in which *thin* wires connect to the centers of the plates. The current I is constant, the radius of the capacitor plates is a and the separation of the plates is $w \ll a$. Assume that the current flows out over the plates in such a way that the surface charge is uniform, at any given time, and is zero at $t = 0$.

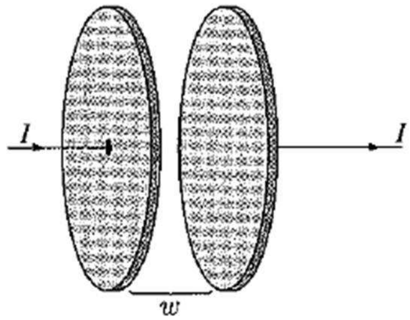

Figure 3

a) [3 pts] Find the electric field between the plates, as a function of t.

b) [5 pts] Find the displacement current through a circle of radius s in the plane midway between the plates. Using the circle as your 'Amperian loop', and the flat surface that spans it, find the magnetic field at a distance s from the axis.

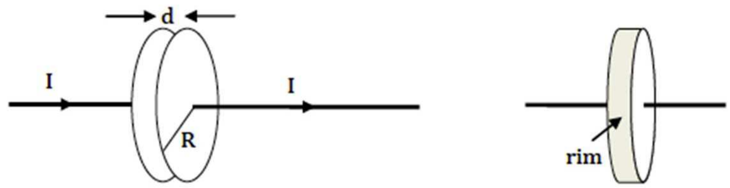

Figure 1

3. [Total: 14 pts]
A capacitor with circular plates of radius R seperated by a distance $d \ll R$ is being charged by a steady current I. The plates are sufficiently close that edge effects can be ignored.

   a) [4 pts] Compute the magnitude of the **B**-field between the plates at all distances r from the center of the plates (i.e. $r < R$ and $r > R$). Sketch the magnitude of the **B**-field vs. R.

   b) [4 pts] Compute the Poynting vector **S** (magnitude and direction) on the *rim* of the capacitor, between the plates, at $r = R$. (The 'rim' is the ribbon of area at $r = R$ between the plates; see Figure 1).

   c) [6 pts] Show that the rate at which the capacitor's stored energy is increasing is equal to the rate at which field energy is entering through the rim, $\int_{rim} \mathbf{S} \cdot d\mathbf{A}$.

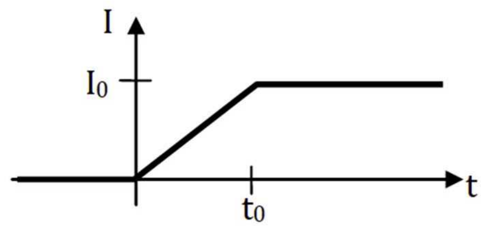

Figure 2

4. [Total: 12 pts, each part: 3 pts]
   Consider a very long solenoid of length L, radius r, and turns per length n. The current I in the solenoid is linearly ramped from $I = 0$ to $I = I_0$ over a period of t_0 as shown in the graph in Figure 2.

   a) Integrate the magnetic field energy density to derive a formula for the total field energy stored in the solenoid at times $t > t_0$.

   b) Solve for the electric field everywhere at times $0 < t < t_0$.

   c) Solve for the Poynting vector $\mathbf{S}$ (direction and magnitude) at $r = R$ (just inside the walls of the solenoid) as a function of time.

   d) Show that the total field energy/time passing from the walls of the solenoid into its interior, when integrated from $t = 0$ to $t = t_0$, gives the same total energy as you computed in part a).