University of Colorado, Department of Physics
PHYS3320, Spring 2016, HW 8
due Fri, Mar 11 by 5:00pm, in the mailbox at the entrance to the physics helproom

1. [Total: 4 pts; each part: 2 pts]

   a) If you add two sinusoidal traveling waves, with the same angular frequency and wavelength, but traveling in opposite directions, you get a standing wave. Use complex notation for waves to show that the sum $A\cos(kx-\omega t)+A\cos(kx+\omega t)$ is a standing wave. Make a sketch of this wave, showing its shape at two different times and indicate on the sketch where the nodes and anti-nodes are.

   b) Consider the sum of two traveling waves,
   $$f_3(x,t)=A_1\cos(kx-\omega t+\delta_1)+A_2\cos(kx-\omega t+\delta_2). \quad (1)$$
   In class we used complex notation to show that $f_3(x,t)=A_3\cos(kx-\omega t+\delta_3)$. Find A_3 and δ_3 in terms of A_1, A_2, δ_1 and δ_2.

2. [Total: 10 pts]
   Consider the electric field given by $\mathbf{E}(\mathbf{r},t)=E_0\cos(k(x-ct))\,\hat{\mathbf{y}}$, where k and c are known constants.

   a) [2 pts] What is the charge density $\rho(\mathbf{r},t)$ associated with this **E**-field?

   b) [2 pts] What are the units of k and c?

   c) [2 pts] Come up with the *simplest possible* **B**-field that satisfies Faraday's law, given this **E**-field.

   d) [4 pts] Then, check for your result from part c) that the remaining free-space Maxwell equations are satisfied, if you make the right choice for the constant c. What is the current density $\mathbf{J}(\mathbf{r},t)$ associated with these **E**- and **B**-fields?

3. [Total: 30 pts]
   Consider a 3D electromagnetic wave in vacuum, described in the complex form by $\tilde{\mathbf{E}}=\tilde{\mathbf{E}}_0\exp(i(\mathbf{k}\cdot\mathbf{r}-\omega t))$, in which $\tilde{\mathbf{E}}_0$ is a constant vector equal to $\tilde{E}_0\hat{\mathbf{x}}$ with $\tilde{E}_0=E_0\exp(i\pi/2)$. Assume $\mathbf{k}$ is the wave vector $k\hat{\mathbf{y}}$ and ω is the angular frequency. As usual, the real field is $\mathbf{E}=\mathrm{Re}[\tilde{\mathbf{E}}]$.

   a) [3 pts] Describe in words what this mathematical expression represents physically. You may use sketches, but if you do, they should be well described. Then answer the following questions:
   - In which direction is the wave moving?
   - What is the speed, wavelength, and period of the wave?
   - What is the polarization of the wave? Does it change in time?

   b) [3 pts]
   (i) Sketch the real field $\mathbf{E}(x=0,y,z=0,t=0)$ (a 2D plot with y as horizontal axis).
   (ii) $\mathbf{E}(x=0,y=0,z=0,t)$ (a 2D plot with t as horizontal axis).
   (iii) How is the field at $x=a$, i.e. $\mathbf{E}(x=a,y,z=0,t=0)$, different from the field at $x=0$?

c) [3 pts] Why is this called a plane wave? Where is (are) the plane(s)? Sketch or represent this in 3D.

d) [4 pts]
(i) Find the associated magnetic field $\mathbf{B}(\mathbf{r}, t)$ for this plane electric wave.
(ii) Describe in words how $\mathbf{B}$ compares/contrasts with $\mathbf{E}$.
(iii) Sketch the magnetic field, $\mathbf{B}(x = 0, y, z = 0, t = 0)$ and $\mathbf{B}(x = 0, y = 0, z = 0, t)$ indicating the field direction.

e) [3 pts] Calculate the energy density for these fields. Interpret the answer physically, i.e. make sense of them, including units, signs, directions, etc.

f) [3 pts] Calculate the Poynting vector for these fields. Interpret the answers physically, i.e. make sense of them, including units, signs, directions, etc.

g) [3 pts] Calculate the momentum density for these fields. Interpret the answers physically, i.e. make sense of them, including units, signs, directions, etc.

h) [3 pts] Calculate the 3×3 Maxwell stress tensor for these fields. Interpret the answers physically, i.e. make sense of them, including units, signs, directions, etc.

i) [5 pts] Suppose now we add two plane waves, $\mathbf{E}_1(\mathbf{r}, t) = \mathbf{E}_1 \cos(\mathbf{k} \cdot \mathbf{r} - \omega t + \delta_1)$ and $\mathbf{E}_2(\mathbf{r}, t) = \mathbf{E}_2 \cos(\mathbf{k} \cdot \mathbf{r} - \omega t + \delta_2)$ and the amplitudes are $\mathbf{E}_1 = E_1 \hat{\mathbf{z}}$ and $\mathbf{E}_2 = E_2 \hat{\mathbf{z}}$. Use complex notations (taking the real part only at the every end) to find $\mathbf{E}_{total}(\mathbf{r}, t) = \mathbf{E}_1(\mathbf{r}, t) + \mathbf{E}_2(\mathbf{r}, t)$ in the form $\mathbf{E}_{total}(\mathbf{r}, t) = \mathbf{E}_{total} \cos(\mathbf{k} \cdot \mathbf{r} - \omega t + \delta_{total})$, giving the expressions for the total amplitude and phase shift in terms of those from $\mathbf{E}_1(\mathbf{r}, t)$ and $\mathbf{E}_2(\mathbf{r}, t)$. Explicitly check your answer in the special case $\delta_1 = \delta_2$.

4. [6 pts] Consider a localized wave packet that satisfies the one-dimensional wave equation from a sum of sinusiodal waves using Fourier's integral method:

$$f(x, t) = \int_{-\infty}^{\infty} A(k) \exp(ik(x - ct))dk$$

(Note that $A(k)$ is a complex-valued function of k.) Show that $f(x, t)$ satisfies the wave equation with velocity c.