University of Colorado, Department of Physics
PHYS3320, Spring 2016, HW 9
due Fri, Mar 18 by 5:00pm, in the mailbox at the entrance to the physics helproom

1. [Total: 8 pts]
Consider the following expression for the electric field of an electromagnetic plane wave:

$$\tilde{\mathbf{E}} = E_0(1+i)\exp(i(ax - by - \omega t))\hat{\mathbf{z}}$$

   a) [3 pts] What are (i) the polarization direction and (ii) the propagation direction of the wave? Write down vectors.

   b) [2 pts] Determine ω in terms of E_0, a, b and other known constants.

   c) [3 pts] Write down the real electric field and the real magnetic field of the wave.

2. [Total: 10 pts]
A particle of charge q and mass m is held at the origin in the field of a 3D electromagnetic wave with angular frequency ω and phase constant $\delta = 0$. The wave is polarized in $\hat{\mathbf{y}}$-direction and propagates in $\hat{\mathbf{z}}$-direction.

   a) [3 pts] Find the real electric field and the real magnetic field of the wave.

   b) [3 pts] At $t = 0$ the particle is released. What is the acceleration of the particle immediately after the instant of release? Give both the magnitude and the direction.

   c) [4 pts] As the particle starts to move, it has the velocity $\mathbf{u}$ in the direction of the initial acceleration. What is the acceleration of the particle now after it has some velocity after some short time Δt? Assume that the particle is essentially still at the origin, and the time Δt is much less than $T/4$, where T is the period of the electromagnetic wave.

3. [Total: 12 pts]
Consider a linear homogeneous material with no free charges and no free currents.

   a) [4 pts] Derive the wave equations (i) for the electric field and (ii) for the magnetic field in such a material.

   b) [2 pts] Argue that complex plane waves solve these wave equations. What is the speed of these waves?

   c) [4 pts] Derive (i) the relation between k and ω for a complex plane wave and (ii) the relation between the amplitudes of the complex electric and the complex magnetic fields of an electromagnetic plane wave in such a material.

   d) [2 pts] Use *Maxwell-Ampere's Law* (not Faraday's Law) to show that the propagation vector and the electric field and the magnetic field vectors of an electromagnetic plane wave in such a material are pairwise orthogonal to each, following the right-hand-rule (state the order of the vectors).

4. [4 pts] Suppose $Ae^{iax} + Be^{ibx} = Ce^{icx}$, with some nonzero constants A, B, C, a, b, c and for all x. Prove that $a = b = c$ and $A + B = C$.