DC Measurements, Voltage Dividers, and Bridges

Purpose

You will gain familiarity with the circuit board and work with a variety of DC techniques, including the Wheatstone bridge and the 4-terminal resistance measurement techniques.

Suggested reading

1. FC Sections 1.9 (Resistivity), 1.11 (Resistor circuits), 2.4 & 2.5 (Thevenin's theorem)
2. H&H Sections 1.03, 1.04, 1.05

Theory

1. The Basic Wheatstone Bridge

Bridge circuits are used to precisely compare an unknown impedance with a standard. The simplest example is the Wheatstone bridge (Fig. 2.1), a four-arm bridge with a resistor in each arm, which is usually used at DC or low frequencies. It has many applications in measurement circuits, where often the unknown resistance R_X is a resistive sensor, such as a platinum thermometer or a mechanical strain gauge (for more info see H&H 15.03). There are other types of bridges, including AC bridges (see FC 3.2) with capacitors or inductors in one or more arms, radio-frequency bridges, and bridges that use precision transformers to generate voltage ratios.

In our the basic bridge R_X, R_S, R_1, & R_2 are each >>0.1Ω so that all contact resistances (typically of order .1 Ω) can be ignored. The bridge is made from two voltage dividers, each connected to the same source voltage ε. When the division of the two dividers is adjusted to the same value the null meter reads zero voltage (ΔV=0). This occurs when $R_X/R_S = R_1/R_2$, a

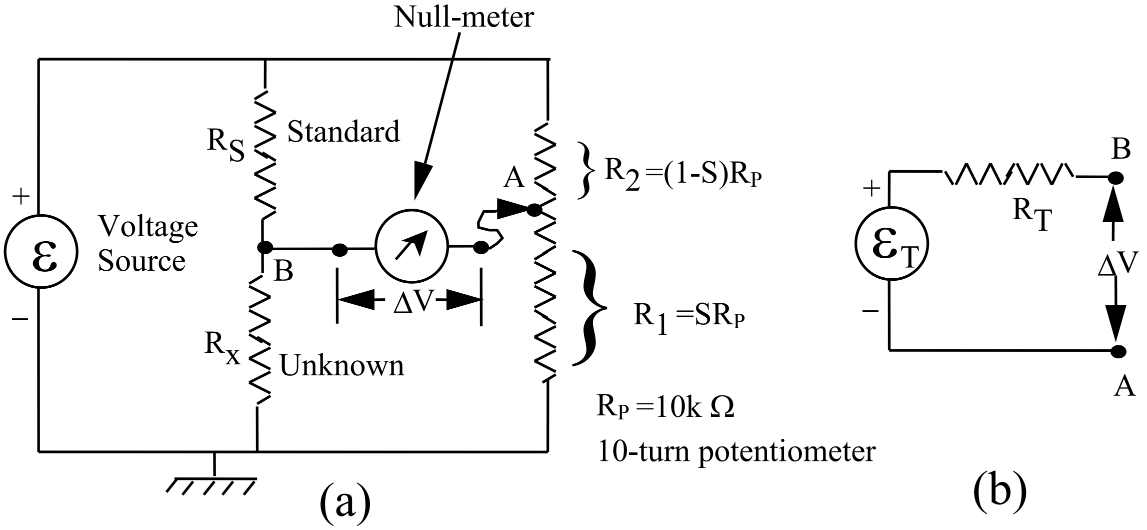

Fig. 2.1 a) Basic Wheatstone bridge. b) Thevinin equivalent circuit.

formula which can solved for the unknown resistor R_X in terms of the standard R_S and the ratio R_1/R_2.

More generally, the operation of the bridge, both in the balanced and the unbalanced states, follows directly from its Thévenin equivalent and the equations:

$$\varepsilon_T = \varepsilon\left(\frac{R_x}{R_x+R_s}-\frac{R_1}{R_1+R_2}\right), \quad R_T = \frac{R_s R_x}{R_x+R_s}+\frac{R_1 R_2}{R_1+R_2}.$$

Stare at these equations for a while and try to see why they are correct without doing any calculations.

2. Four-terminal Connections

Typically, when we wish to measure the resistance of a circuit element we can simply connect the element between the two leads of a DMM and read "Ohms". With the modern digital DMM this works quite well unless the resistance of the element to be measured is small - not very much higher than the resistances of the leads or contact resistances between the sample and the leads (probably less than an ohm for your leads). If the element's resistance is not much higher than the leads, then the lead resistance will make a sizeable impact on the measurement, skewing it badly. The way around this is to use the method of 4-terminal connections, in which the current leads are separate from the voltage leads. A schematic is shown in Figure 2.2.

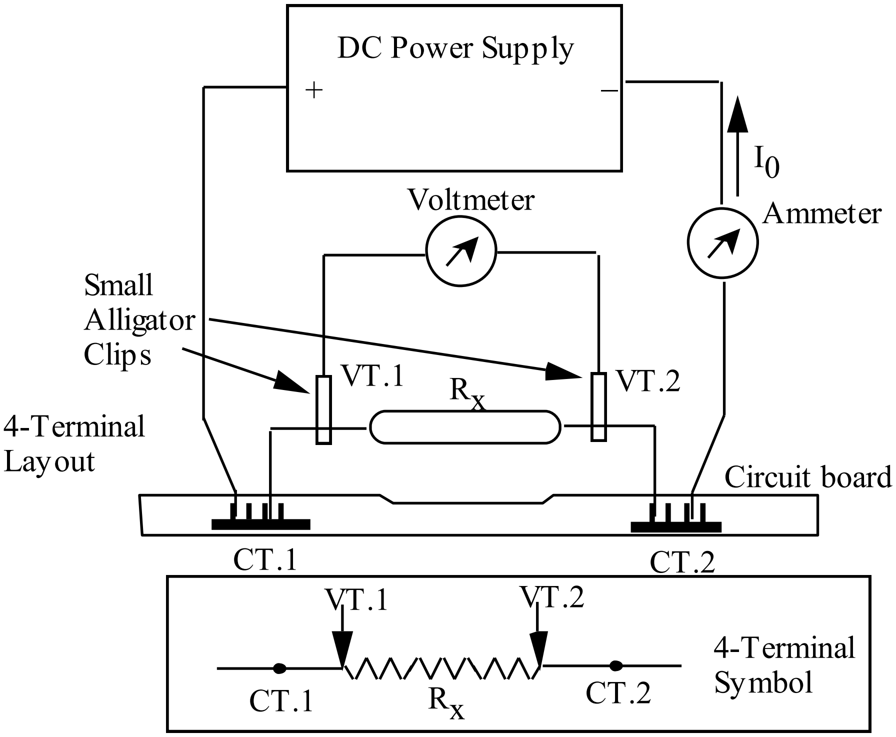

Fig 2.2 4-Terminal Connections: Physical and Schematic

A known large current passes into and out of the sample from CT.1 to CT.2. The voltage across the sample is measured at VT.1 and VT.2. Since there is essentially no current flowing through VT.1 and VT.2 there is also no voltage drop across them, even if they are of a sizeable resistance. Ohm's law is then used to determine the resistance of the sample.

The black dots CT.1 and CT.2 represent the current terminals through which current flows into and out of the sample. The arrows VT.1 and VT.2 represent voltage terminals to which the voltmeter is attached using spring loaded alligator clips. The sample length is the distance between the voltage terminals. Note that the contact resistance (about 0.1Ω) of the voltage terminals can be neglected since it appears in series with the 10 MΩ resistance of the DMM. Also, the current terminals CT.1 & CT.2 are outside the voltmeter circuit. This is so that the voltage drops in these contacts will not be measured by the voltmeter.

Apparatus and Methods

1. Prototyping boards

Your instructor will give your team a prototyping board to use for building your projects. Write your team member's names on it. Your team will use the same board all semester. An incomplete experiment can be left on the board and finished later. Store the board on the shelf labeled for your section. <u>Components</u> (resistors, capacitors, transistors, etc.) are available from the community stock. Take what components you need for the experiment. When it is over, stick them in a piece of foam or store them in a cardboard box for future use until the end of semester. <u>Do not take a new component</u> for an experiment unless you don't have it already. The complete circuit board contains a <u>front panel</u>, and a <u>plug-in circuit board</u>.

On the front panel, you will find:
- <u>BNC cable jacks</u> that carry electric signals between your circuit on the board and the function generator and oscilloscope.
- <u>Colored banana jacks</u> to bring in dc power for transistors or chips from an external power supply (+15 V red, -15 V blue, +5 V orange, and 0 V black).
- A precision 10 kΩ ten-turn <u>potentiometer</u> (linearity ± 1/4%), and several <u>switches</u>.
- A wire or component on the board might be broken, or might break during the semester. Don't worry – you will be able to repair the board as you go.

The circuit board contains arrays of holes, interconnected by buried conductors, into which components are plugged to build your circuit. The description we give below is generally accurate, but there are several varieties of boards in the lab; use the multimeter to verify the connections inside your board. In general, you can never be sure that any two contacts are really connected, or any wire is really continuous, unless you test it yourself, so get into the habit of testing things.
- The long lines of connected holes are used for power lines (+15 V red and -15 V blue) and ground lines (0 V black). They are never used for signals.
- The 5 holes in each short group at right angles to the long lines are interconnected, but separate from every other group of 5. A given short group is used to make the junction between two or more components of your circuit.
- There are four color coded binding posts on the circuit board. They are wired to the associated banana jacks on the front panel.

- Good electrical contact is essential when you plug in components or wires. Use only 22 or 24 gauge solid wire, not stranded wire. Push in each wire until you feel the contacts grip. A common fault is to plug in enamel insulated wire, used for winding inductors. First burn off the enamel with a lighter flame for 1/2" at the end and clean with emery paper. Good contacts will measure less than one third of an ohm.
- Remember, if you get confused about which holes are connected together by buried conductors, explore the board with an Ohm meter. You will have to have to connect short lengths of solid wire (preferably red wire for + and black for –) to the meter leads in order to make measurements on the prototyping board.

Reliable ground connections (0 V), readily accessible from any point on the board, are essential to the good functioning of most circuits. The front panel is the ground for your circuit board since the coax cable shields connect the front panel of your circuit board to the ground of other instruments in your experiment.

2. DC power supplies

For various experiments you will need dc power. The Tektronix regulated dc power supplies convert 115 V, 60 Hz, ac power from the wall outlets into the dc required for your circuits.

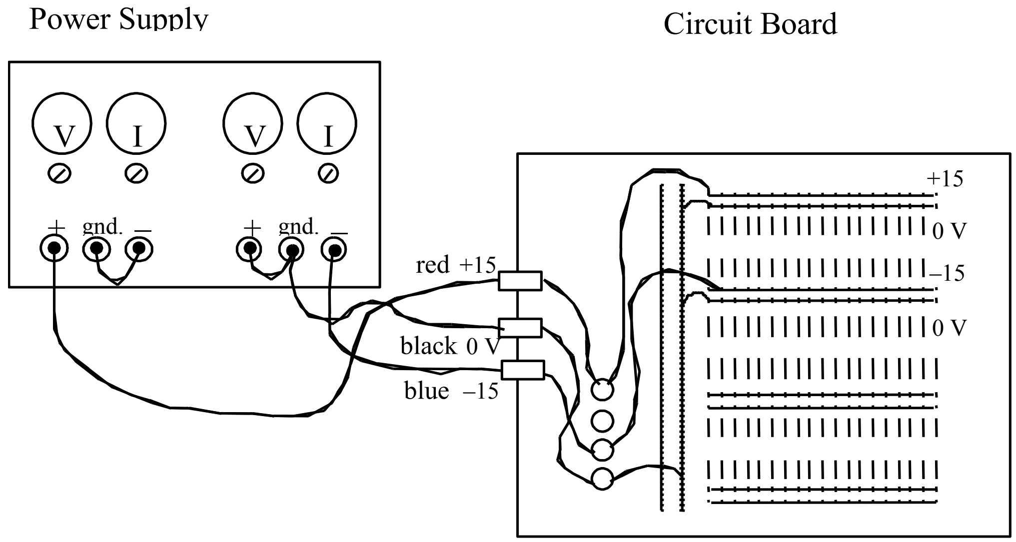

Figure 2.3 DC Power Connection for the Power Supplies

Figure 2.3 shows an example of a power supply with two outputs set to supply +15 V and –15 V. The Tektronix supply has a digital display; you change between current and voltage readout with a button.

Note that the power supply produces only a voltage difference between the "+" output and the "-" output. The actual terminal voltage is determined only when one of the outputs is connected to a definite reference potential. The minus terminal can be grounded to give a positive dc voltage at the "+" output, or the positive terminal can be grounded to give a negative dc voltage at the "–" output. It is important to connect the power supply correctly.

3. DC power connections

First, connect the power supply to your circuit board as shown in Fig. 2.3. Connect wires from the +V, 0 V, and –V color coded binding posts to the long lines on the plug board. You won't be using –15 V for this week's lab, but connect it anyway. Then, connect the power supply to the circuit board front panel. To reduce confusion it's a good idea to use the standard colors: red wire for + V, black for 0 V (ground) and blue for –V.

Turn on the power supply. Set voltages for + V and –V using the panel meters. Set the current limit (knob) initially to about 100 mA by temporarily short circuiting the output and reading the current meter. If your circuit draws more current than 100 mA, raise the limit; this is mainly for safety when we start connecting integrated circuits. If the current is limited, then a mistake is less likely to generate smoke.

Test the dc level on each connected long line of holes on the circuit board with your DMM to see that it is correct. Also check that there is 0 volts between the ground lines and the front panel. The front panel is the reference ground since the coax cable shields connect it to the ground of every other instrument. Later, when you start to use sensitive components like opamps and transistors, be sure to turn off the power supply while you are building the circuit and until you check that the connections are correct.

Pre-lab Problems - Turn these in to your instructor at the agreed upon time for your lab period (check website).

1. Voltage dividers. An ideal voltage source drives current around the loop of resistors shown in Figure 2.4(a).

(A) Find a formula for the current I and the voltage V_{out}. What is V_{out} if V = 10 V, R_1 = 2 MΩ, and R_2 = 1 MΩ ?

(B) For these component values, what is the Thévenin equivalent circuit?

(C) Calculate the voltage V_{out} for the modified circuit shown in Figure 2.4(b) with R_3 = 10 MΩ and the other components unchanged.

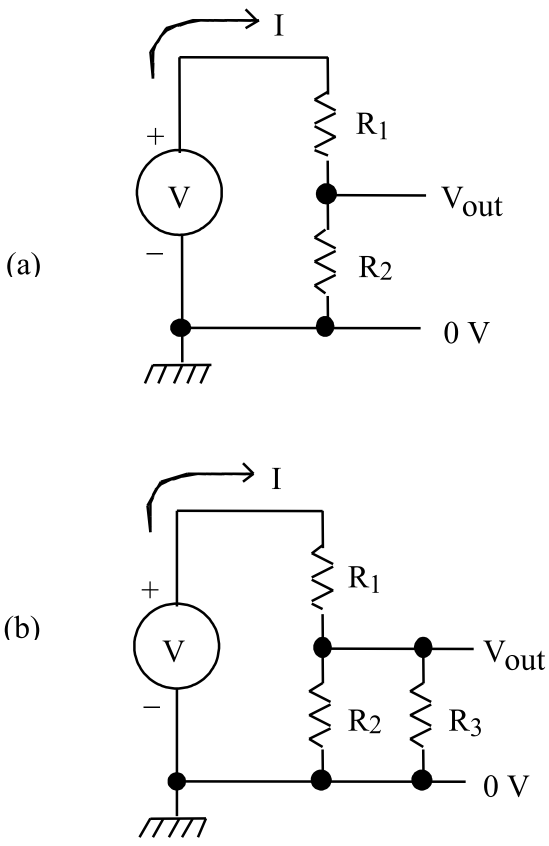

Figure 2.4 Voltage dividers

2. (A) What is the diameter of a typical copper wire 6-ft long if its resistance (at room temperature) is 0.1 Ω? Assume that the resistivity of copper at room temperature is 1.68 μΩ-cm.

(B) What is the maximum error in measuring the diameter that you can tolerate if the resistance is to have a 1% uncertainty?

3. The American standard wire gauge gives a logarithmic measure of the diameter of a wire,

$$\text{AWG gauge} = 30 - 20 \log_{10}(d/0.01'').$$

Wires usually are available only in even gauges. What gauge should you use for the 6-ft, 0.1Ω wire?

4. Consider the basic Wheatstone bridge of Fig 2.1. The divider ratio S of the adjustable arm is defined by $S=R_1/R_P$ where $R_P=R_1+R_2$. Suppose that a resistance R_{xo} installed as the unknown, and the bridge is balanced giving a potentiometer divider ratio S_o. Now leave the potentiometer alone, but let the unknown resistance change by a small amount ΔR_x. (A) Show that, as a consequence, the null-meter will measure an (approximate) unbalanced potential difference

$$\Delta V \approx \varepsilon\left(\Delta R_x / R_{x0}\right)\left(1 - S_0\right)S_0 .$$

Hint: the math is simplified if you use $1/(1+\varepsilon) \approx 1-\varepsilon$.

(B) What value of S_o gives you maximum sensitivity, $\Delta V/\Delta R_x$?

(C) What does your answer imply about choosing a standard resistance if you know the approximate value of the unknown resistance?

Lab Experiment:

No formal error analysis is required, but you are required to estimate your errors and use them to evaluate whether the theory being tested is correct.

1. Build three copies of the simple voltage divider circuit of prelab Figure 2.4(a), with both resistors approximately equal to each other. In the first circuit use relatively low resistances of about 1 kΩ. In the second use resistances near 1 MΩ, and in the third use resistors of 10 MΩ or more.
   (A) Measure each resistor with your DVM before inserting them into your circuit and record the values.
   (B) Apply a DC voltage to the input and measure the output voltage of your dividers first using first your DVM and second your oscilloscope. Record your findings. (Do not have the DVM and the oscilloscope connected at the same time because each will perturb the measurement differently.)
   (C) Compare the voltages you expected to the voltages you measured. What does this tell you about the input impedances of your instruments?

2. Build the basic Wheatstone bridge on your circuit board. Validate the null procedure by using the balanced bridge to compare a cheap (10%) 1 kΩ resistor as R_X with a 1 kΩ, 1% resistor as the standard R_S. You can find S after the bridge is balanced by taking the pot out of the circuit and measuring R_1 and R_2 with the DMM, or you can use the markings on the dial. Compare your result for R_X with a measurement using a DMM.

3. Now, without changing R_S or S, test the theory for the voltage output by slightly unbalanced bridge by using a value of R_X that is about 5% different from the value that balances your bridge.

4. Cut a 6-ft length of copper wire of a diameter such that the resistance will be about 0.1 Ω. Measure the resistance of the sample with a digital multimeter (DMM). Be sure that the enamel is removed from the ends so that a good connection is made.

5. Determine the resistivity of copper metal in μΩ-cm by measuring the resistance of the sample and its dimensions. Use 4-terminal connections (Fig 2.2) and measure the voltage drop along the sample with a known current passing through it to find the resistance. When measuring the diameter of the wire, remember that the thickness of the enamel coating may not be negligible. The power supply will give you the current on its display, or you can measure it with a DMM on the current range. Hint: The correct result appears in prelab question 2.

6. Is the resistance measurement in 4 above from measuring ohms directly more or less accurate that the resistance measurement in 5 where V and I are measured? Why? See page 11 of the DMM manual for the accuracy of the meter on each scale. An accuracy of ±(0.5%+2) means ±0.5% and an additional error of ± 2 in the least significant digit.