Filters and Waveform Shaping

Purpose

The aim of this experiment is to study the frequency filtering properties of passive (R, C, and L) circuits for sine waves. Filters are important in experiments for enhancing signals of particular interest while suppressing unwanted background.

The oscilloscope probe is introduced in this lab.

Introduction

A frequent problem in physical experiments is to detect an electronic signal when it is hidden in a background of noise and unwanted signals. The signal of interest may be at a particular frequency, as in an NMR experiment, or it may be an electrical pulse, as from a nuclear particle detector. The background generally contains thermal noise from the transducer and amplifier, 60 Hz power pick up, transients from machinery, radiation from radio and TV stations, cell phone radiation, and so forth. The purpose of filtering is to enhance the signal of interest by recognizing its characteristic time dependence and to reduce the unwanted background to the lowest possible level. A radio does this when you tune to a particular station, using a resonant circuit to recognize the characteristic frequency. The signal you want may be less than 10^{-6} of the total radiation power at your antenna, yet you get a high quality signal from the selected station.

A filter freely transmits electrical signals within a certain range of frequencies called the pass band, and suppresses signals at all other frequencies (the attenuation bands). The boundary between a pass band and an attenuation band is called the cut-off frequency f_c. We usually define f_c to be the frequency at the half-power point or 3dB point, where the power transmitted is half the maximum power transmitted in the pass band. The output voltage amplitude at $f = f_c$ is $1/\sqrt{2} = 70.7\%$ of the maximum amplitude.

There are three basic types of filter. The high-pass filter transmits signals at frequencies above the cut-off f_c and blocks them at lower frequencies. The low-pass filter transmits signals below f_c and blocks higher frequencies. The band-pass filter, often taking the form of a resonant circuit, transmits a certain band of frequencies and blocks signals outside that band. For band-pass filters the bandwidth is the range of frequencies between the upper (f_+) and lower (f_-) half power points: bandwidth $\equiv \Delta f = f_+ - f_-$.

Many experiments require specific filters designed so that the signal from the phenomenon of interest lies in the pass-band of the filter, while the attenuation bands are chosen to suppress the background and noise.

This experiment introduces you to the filtering properties of some widely used but simple circuits, employing only a resistor and capacitor for high- and low-pass filters and an LCR circuit for band-pass.

Suggested Readings

1. FC Sections 3.4 – 3.18 and 10.1 – 10.6
2. H&H Chapter 1, especially sections 1.13-1.24. You will make frequent use of the last topic in Section 1.18, "Voltage Dividers Generalized." Appendix A on oscilloscope probes.
3. D.V. Bugg, Chapter 14. The theory of series and parallel LCR circuits is presented in detail.

Theory

RC FILTERS

The response of RC low-pass and high-pass filters to sine waves is discussed in FC Sections 3.9&3.10. The most important formula for both cases is

$$f_c = \frac{1}{2\pi RC}$$

where f_c is the 3 dB or half-power point. The response of RC circuits to square waves is in FC Sections 10.1 – 10.4 (H&H Sections 1.13 and 1.14). The main result is that the exponential rise and decay times t_R and t_D are equal to RC.

LCR RESONANT CIRCUIT

See FC Section 3.12 (H&H Section 1.22). The resonant frequency and Q are given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \qquad Q = \omega_0 RC$$

where $\omega_0 = 2\pi f_0$. The resonant frequency is the center frequency of the pass band, and the Q is equal to the ratio of the center frequency to the bandwidth Δf. (These statements are exactly true only if $Q \gg 1$).

Pre-lab Problems

1. Calculate the cut-off or 3 dB frequency f_c in kHz for the low-pass filter in Figure 3.1 (a). Draw a Bode plot of the frequency response, sketching in the correct form near the half power point. (A Bode plot is a log-log plot of $|V_{out}/V_{in}|$ versus frequency. See H&H Fig. 4.31 for an example. Note that the axes are labeled in decades and the x and y step sizes are equal, so a 1/f frequency response appears as a line with a slope of −1.)

2. Calculate the cut-off frequency for the high-pass filter in Figure 3.1 (b) and draw its Bode plot.

3. Derive the expression for the resonant frequency of the resonant circuit in Figure 3.2.

4. For a resonant circuit the characteristic impedance Z_0 is the magnitude of the impedance of the inductor or the capacitor at the resonant frequency:

$$Z_0 = \omega_0 L = \frac{1}{\omega_0 C} = \sqrt{\frac{L}{C}}$$

Calculate the resonant frequency f_0, the characteristic impedance Z_0, and the quality factor Q for the band-pass filter in Figure 3.2. Make a Bode plot showing the expected attenuation versus frequency and mark on it the two half power points f_+ and f_-. This plot is a bit harder to make since it is not just straight lines. You can plot some points near the resonance by hand or use a computer plotting program.

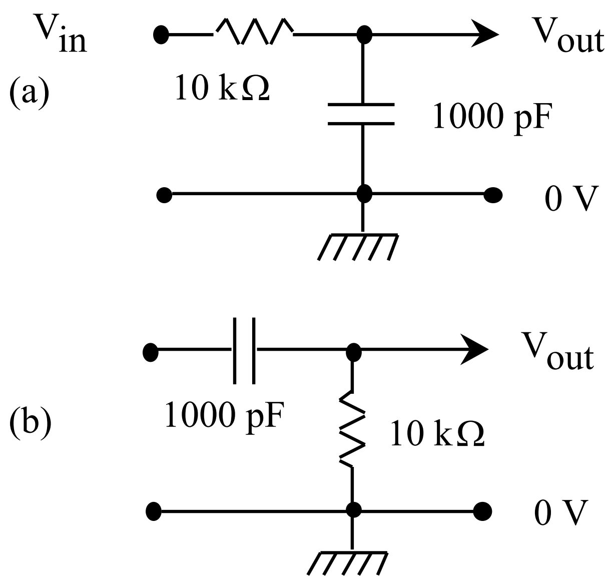

Figure 3.1

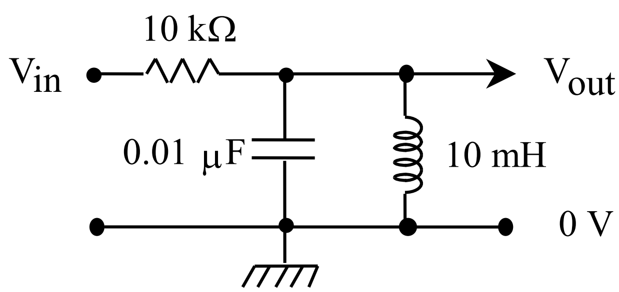

Figure 3.2

The Experiment

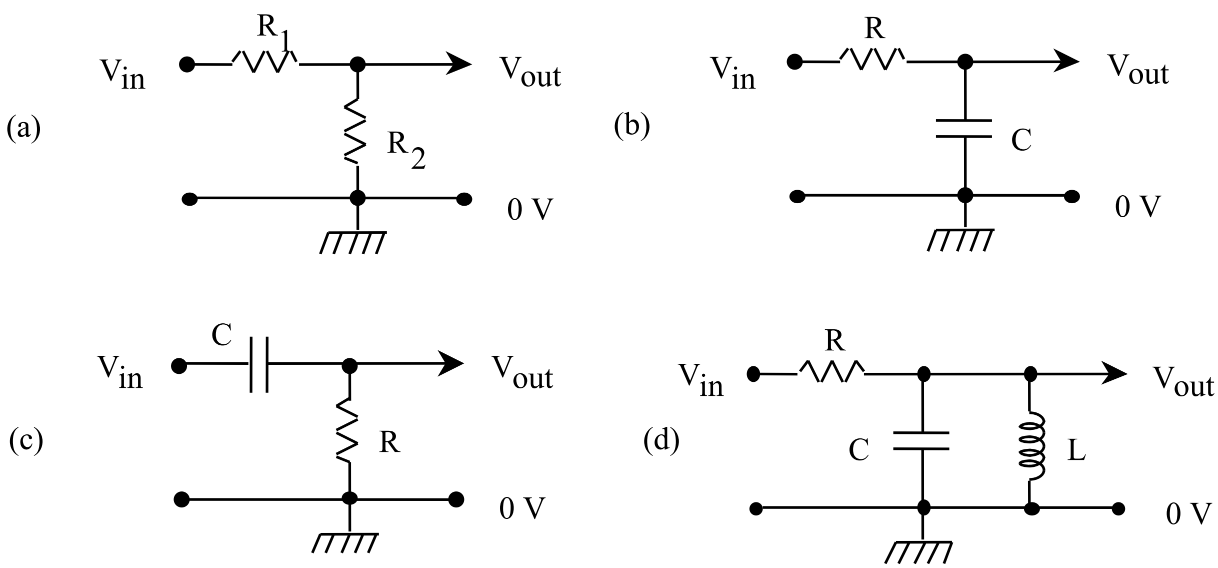

Figure 3.4

<u>THE CIRCUITS</u>

Make up the following circuits on your board (Figure 3.4):

a) <u>Resistive divider</u>.

R = 10 kΩ

R = 6.8 kΩ

b) <u>Low-pass filter</u>

R = 10 kΩ

C = 1000 pF

c) <u>High-pass filter</u>

R = 10 kΩ

C = 1000 pF

d) <u>Bandpass filter</u>

R = 10 kΩ

$C = .01\ \mu F$

$L = 10\ mH$

If you cannot find components in stock with the specified values, take the nearest in value that you can find, within 30% if possible.

1A. Measure the value of each resistor and capacitor before you insert them. Why before?

1B. Calculate the expected attenuation of the divider. Calculate the expected values of the cut-off frequencies for the high- and low-pass filters using the actual component values. Calculate the expected resonant frequency f_0 and quality factor Q for the band-pass filter using the actual component values.

1C. Sketch approximate Bode diagrams for each circuit. Make the axes large and clear so you can plot data on these same graphs as you observe the actual behavior of the circuits below. The frequency range should cover at least $f = 10^{-3} f_c$ to $f = 10^3 f_c$.

TEST SET-UP

Connect the circuit board to the function generator and the oscilloscope as shown in Figure 3.5.

- Set the oscilloscope to display CH 1 and CH 2. Connect the sync or trigger output of the generator to CH4 and trigger the scope on CH 4. Use dc coupling.
- Test the system with 1 kHz sine waves at 1 volt p-p. Display CH 1 and CH 2 waveform on the screen. Depending on settings, the Agilent 33120A Function Generator may display an output voltage that is ½ the actual output voltage when it is connected to a high impedance load. This because the generator has a 50 Ω output impedance. When it is connected to a matched 50 Ω load, the displayed output voltage will be correct.

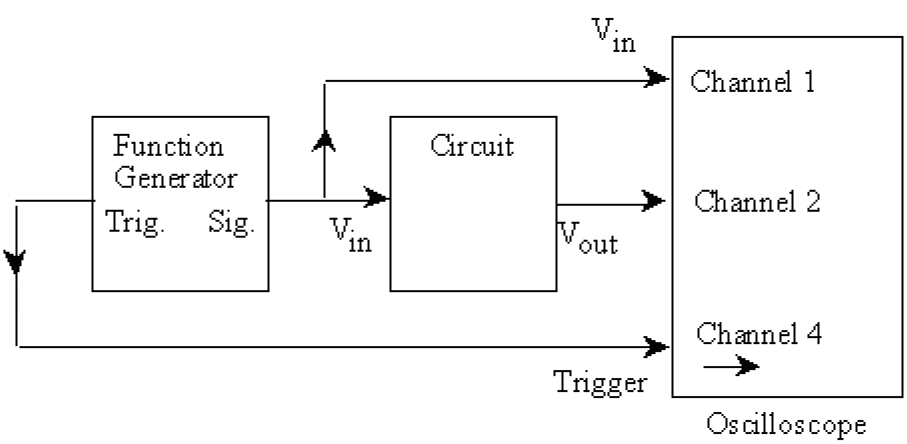

Figure 3.5 Test Set-up

MEASUREMENTS WITH SINE WAVES

First make connections to the voltage divider. Vary the frequency over a wide range with the function generator. Verify that the attenuation ($=V_{out}/V_{in}$) is independent of frequency up to a very high frequency. Compare the measured value of attenuation with that predicted from component values. In the questions below, you do not need to do an error analysis. We do expect that you will notice when your measurement is very different from what is expected and will correct your mistakes.

2A. What is the phase difference (in degrees) between V_{in} and V_{out}? Is the value what you expect? (See Fig. 1.60 of H&H for an example of a phase plot.)

2B. If there is a high frequency cut-off, measure its value (where the voltage is reduced to 0.7 of the input value) and explain the cut-off quantitatively. Consider the capacitance of the cable (about 10 pF per foot) and the scope input.

2C. Repeat the measurement using the 10x probe to observe the output of the circuit. Explain your observation. Note that the oscilloscope knows when the 10X probe is attached and automatically adjusts the scales to give the correct value.

> Use the 10X probe for the remaining measurements because it is less of a perturbation to your circuits.

Next study the high-pass and low-pass filters.

3A. Determine the frequency at the half power point f_c for each filter. Compare your measured half power point ($V_{out}/V_{in} = 0.707$) with the cut-off frequency computed from the actual component values used, rather than the nominal values in the Pre-lab.

3B. Measure the attenuation vs. frequency at decade intervals from $f = 10^{-3} f_c$ to $f = 10^3 f_c$ if possible. Test the predicted frequency response by marking your data points directly on your two Bode plots.

3C. Measure the phase shift between input and output sine waves at $f = 0.1 f_c$, f_c , and $10 f_c$. Plot these data on a phase diagram. (See H&H Fig. 1.60 for an example of a phase diagram.)

Finally look at the band-pass filter. Find the resonant frequency f_o two ways.

4A. Adjust the frequency so that:

a) The output has maximum amplitude (V_{out}/V_{in}=max).

b) There is zero phase difference between V_{out} and V_{in}.

4B. Which method is more precise?

4C. Determine the quality factor Q by measuring the frequencies at the two half-power points f_+ and f_- above and below the resonance at f_o. Recall that

$$Q = \frac{\text{Resonant frequency } f_0}{\text{Bandwidth } \Delta f} \qquad \text{where } \Delta f = f_+ - f_- .$$

4D. Map out the shape of the resonance curve by measuring the attenuation at f_o, $f_o \pm \Delta f/4$, $\pm \Delta f/2$, $\pm \Delta f$, $\pm 2\Delta f$, $\pm 5\Delta f$ and plot the result.

4E. Compare the measured f_o with the expected value $1/\left(2\pi\sqrt{LC}\right)$. Calculate a corrected value of L from the measured values of f_o and C, and use this value below. Compare this value of L to the value you measure using the impedance bridge in the lab.

4F. Compare the measured value of Q with that predicted from measurements of component values. It is common in all electrical circuits to find Q values that are somewhat lower than values you predict using measured component values. This is due to additional losses in the circuit, in this case losses in the inductor. If you want to understand this in detail, you will have to find a way to measure the inductor's "equivalent series resistance" (ESR) at the frequency of interest, and then come up with an equation that shows how the ESR reduces the Q.