

FILTERS

LAB 3 INTRO: MEASURING THE FREQUENCY DEPENDANCE OF LOW PASS, HIGH PASS, AND BAND PASS FILTERS.

GOALS

In this lab, you will characterize the frequency dependence of three passive filters. You will gain more experience modeling both the response of the filters and how your measurement tools affect your measurements.

Proficiency with new equipment:

- Oscilloscope probe
- Capacitors and inductors
 - Identify polarized capacitors and determine the correct installation orientation
 - Measure capacitance and inductance with an LCR meter.

Modeling the physical system:

- Develop mathematical models of frequency dependent voltage dividers
- Determine the limitations of these models and range of applicability

Modeling measurement systems:

- Refine the model of scope measurement tool to include capacitance of the coax cable
- Refine the measurement system to reduce the effect of the capacitance of a coax cable

DEFINITIONS

Scope probe – a test probe used to increase the resistive impedance and lower the capacitive impedance compared to a simple coax cable probe.

Pass band – the range of frequencies that can pass through a filter without being attenuated.

Attenuation band - the range of frequencies that a filter attenuates the signal.

Cutoff frequency (or corner frequency or 3 dB frequency), f_c – the frequency boundary between a pass band and an attenuation band. f_c is the frequency at the half-power point or 3dB point, where the power transmitted is half the maximum power transmitted in the pass band. The output voltage amplitude at $f = f_c$ is $1 / \sqrt{2} = 70.7\%$ of the maximum amplitude.

Low pass filter – a filter that passes low-frequency signals and attenuates (reduces the amplitude of) signals with frequencies higher than the cutoff frequency

High pass filter – a filter that passes high-frequency signals and attenuates (reduces the amplitude of) signals with frequencies lower than the cutoff frequency

Band pass filter – a device that passes frequencies within a certain range and rejects (attenuates) frequencies outside that range.

Band pass filter Bandwidth – the range of frequencies between the upper (f_+) and lower (f_-) half power (3dB) points: bandwidth $\Delta f = f_+ - f_-$.

APPLICATIONS OF FILTERS

A frequent problem in physical experiments is to detect an electronic signal when it is hidden in a background of noise and unwanted signals. The signal of interest may be at a particular frequency, as in an NMR experiment, or it may be an electrical pulse, as from a nuclear particle detector. The background generally contains thermal noise from the transducer and amplifier, 60 Hz power pick up, transients from machinery, radiation from radio and TV stations, cell phone radiation, and so forth. The purpose of filtering is to enhance the signal of interest by recognizing its characteristic time dependence and to reduce the unwanted background to the lowest possible level. A radio does this when you tune to a particular station, using a resonant circuit to recognize the characteristic frequency. The signal you want may be less than 10^{-6} of the total radiation power at your antenna, yet you get a high quality signal from the selected station.

Many experiments require specific filters designed so that the signal from the phenomenon of interest lies in the pass-band of the filter, while the attenuation bands are chosen to suppress the background and noise.

This experiment introduces you to the filtering properties of some widely used but simple circuits, employing only a resistor and capacitor for high- and low-pass filters and an LCR circuit for band-pass.

FILTER BASICS

RC Low- and High-pass filters

The response of RC low-pass and high-pass filters to sine waves is discussed in FC Sections 3.9&3.10. The 3 dB frequency is

$$f_c = \frac{1}{2\pi RC},$$

where f_c is the 3 dB or half-power point.

The response of the filters to a square wave in the time domain is also interesting.

Parallel LCR Band-pass filters

See FC Section 3.12 (H&H Section 1.22). The resonant frequency and Q are given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \quad Q = \omega_0 RC = \frac{f_0}{\Delta f}$$

where $\omega_0 = 2\pi f_0$. The resonant frequency, f_0 , is the center frequency of the pass band, and the Q is equal to the ratio of the center frequency to the bandwidth Δf . (These definitions are exactly true only if $Q \gg 1$).

For a resonant LCR circuit the characteristic impedance, Z_0 , is the magnitude of the impedance of the inductor or the capacitor at the resonant frequency:

$$Z_0 = \omega_0 L = \frac{1}{\omega_0 C} = \sqrt{\frac{L}{C}}$$

USEFUL READINGS

1. FC Sections 3.4 – 3.18 and 10.1 – 10.6
2. H&H Chapter 1, especially sections 1.13-1.24. You will make frequent use of the last topic in Section 1.18, "Voltage Dividers Generalized." Appendix A on oscilloscope probes.

LAB PREP ACTIVITIES

Answer the following questions using Mathematica. Save the complete notebook as a pdf and turn it in to D2L by midnight the day before your lab section meets. **Bring an electronic copy of your notebook to lab, preferably on your own laptop. You will use it to plot your data during the lab session.**

Question 1	<p>Low- and High-pass filters</p> <p>a. Define functions in Mathematica to calculate the cut-off or 3 dB frequency, f_c, for the low- and high-pass filters in Figure 1 (a) and (b). The input parameters to this function should be the resistance and capacitance of your circuit. Evaluate the functions using the nominal values shown in the schematic. During the lab, you can input the exact values of your components and thus quickly predict the 3 dB you expect for your circuit.</p> <p>b. Create two Bode plots (one for each filter) of the frequency response of the low- (1a) and high-pass (1b) filters in Figure 1. A Bode plot is a log-log plot of (V_{out}/V_{in}) versus frequency. See H&H Fig. 4.31 for an example. Make sure to include a large enough range in frequency to see both the pass and attenuation bands. <i>HINT: Details about making plots pretty are included in Lab Skill Activity #2.</i></p> <p>c. During the lab section, you will enter your measurements into your Mathematica notebook and plot them with your model predictions. To prepare for this, create a list of “fake data” and plot it on your Bode plots. This will allow you to compare your model and measurements in real time avoiding lost time taking lots of data when something is wrong with your circuit. The point of this part is just to have you create working code to enter a list of data and plot it along with the function. The numerical values of the fake date are unimportant. <i>HINT: There is a helpful guide on our website under the HINTS Tab titled “Plotting data and theory together in Mathematica.”</i></p>
Question 2	<p>Band-pass Filters</p> <p>a. Define functions in Mathematica to calculate the resonant frequency f_0, the characteristic impedance Z_0, and the quality factor Q for the band-pass filter in Figure 1(c). Evaluate the functions using the nominal values shown in the schematic.</p> <p>b. Create a Bode plot showing the predicted gain (V_{out}/V_{in}) versus frequency of the band-pass filter. Make sure to include a large enough range in frequency to see both the pass and attenuation bands.</p> <p>c. Create a list of “fake data” and plot it on your Bode plots. The point of this part is just to have you create working code to enter a list of data and plot it along with the function. The numerical values of the fake date are unimportant.</p>
Question 3	<p>Lab activities</p> <p>a. Read through all of the lab steps and identify the step (or sub-step) that you think will be the most challenging.</p> <p>b. List at least one question you have about the lab activity.</p>

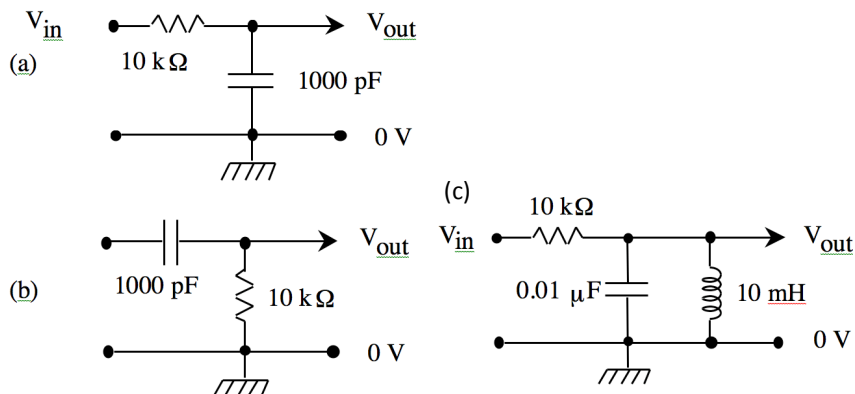


Figure 1 Filters. (a) low-pass, (b) high-pass, and (c) band-pass

SETTING UP THE CIRCUITS AND PREDICTING THE BEHAVIOR

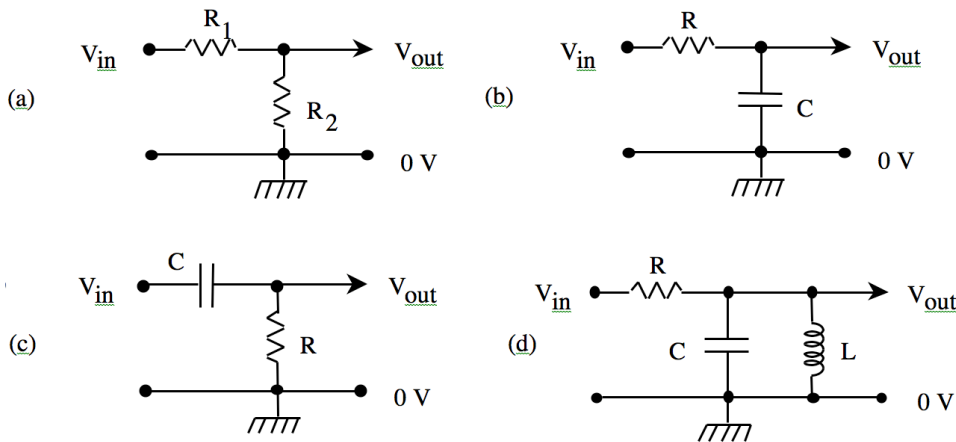


Figure 2. General Voltage Dividers. (a) resistive divider, (b) low-pass filter, (c) high-pass filter, and (d) band-pass filter.

Step 1	<p>Building the Circuits</p> <ol style="list-style-type: none"> Gather all the components to be able to build the four circuits shown in Fig. 2. If you cannot find components in stock with the specified values, take the nearest in value that you can find, within 30% if possible. <ul style="list-style-type: none"> ○ Resistive divider: $R_1 = 10 \text{ k}\Omega$, $R_2 = 6.8 \text{ k}\Omega$ ○ Low-pass filter: $R = 10 \text{ k}\Omega$, $C = 1000 \text{ pF}$ ○ High-pass filter: $R = 10 \text{ k}\Omega$, $C = 1000 \text{ pF}$ ○ Band-pass filter: $R = 10 \text{ k}\Omega$, $C = .01 \text{ }\mu\text{F}$, $L = 10 \text{ mH}$ Measure all components before placing them into the circuit. Record the values in your lab book. Draw diagrams of all the circuits. Make sure to use the same labels on the diagrams and for the values of the components. Build all four circuits on your proto-board (make sure they are all separate)
Step 2	<p>Use the Mathematica models to predict the behavior of the filters.</p> <ol style="list-style-type: none"> Calculate the expected attenuation of the divider. Calculate the expected values of the cut-off frequencies for the high- and low-pass filters using the actual component values. Calculate the expected resonant frequency f_0 and quality factor Q for the band-pass filter using the actual component values. <p>HINT: You should have already done these calculations in your lab prep notebook. Just enter the exact values of your components.</p>
Step 3	<p>Use the Mathematica models to plot the expected the behavior of the filters.</p> <ol style="list-style-type: none"> Plot your mathematical models of all three filter circuits (three independent plots) using your actual component values. The frequency range should cover at least $f = 10^{-3} f_c$ (or f_0) to $f = 10^3 f_c$ (or f_0) to show the full behavior. After the lab is completed and you have your measurements on these plots as well, you will print off the plots and tape them into your lab book. Make sure to leave room in your lab book for the plots. <p>HINT: You should have already made these plots in your lab prep notebook. Just enter the exact values of your components.</p>

SETTING UP TEST AND MEASUREMENT EQUIPMENT

Step 4	<p>Prepare to test the circuits</p> <ol style="list-style-type: none"> Connect the circuit board to the function generator and the oscilloscope as shown in Fig. 3. It is always helpful to display both the input voltage as well as the output voltage on the scope at the same time. Test your setup by creating a 1 kHz sine wave at 1 volt p-p using the function generator and confirm the waveform frequency and amplitude by measuring the signal on the scope. Trigger the scope on the Sync. output of the function generator.
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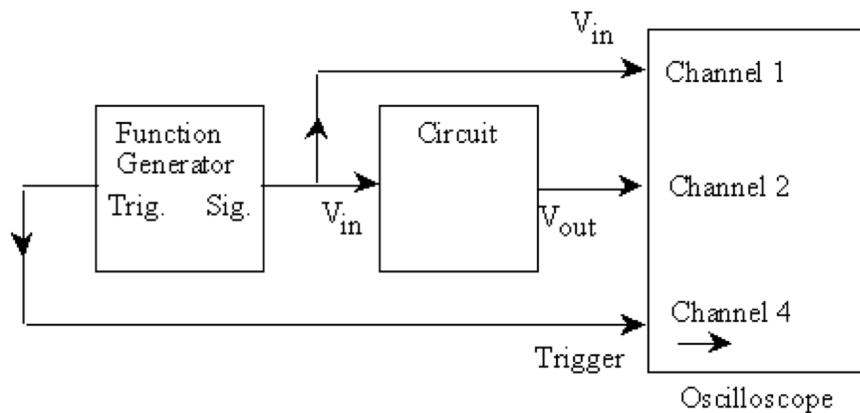


Figure 3. Test and Measurement Set-up.

RESISTIVE VOLTAGE DIVIDER

Step 5	<p>Measure the frequency dependence of the voltage divider</p> <ol style="list-style-type: none"> Connect the signal from the function generator to the input of the voltage divider. Measure the transfer function (attenuation) ($=V_{out}/V_{in}$) over a large range in frequency (1 kHz to 15 MHz in approximately decade (X10) steps). Record your measurements in your lab book. At low frequencies (1 kHz), compare your measured value of the attenuation to what your model predicted using your actual component values. Does your measurement agree with your prediction? Explicitly record what criteria you used to determine whether or not the model and measurements agree.
Step 6	<p>Refining the measurement system of the voltage divider</p> <ol style="list-style-type: none"> If there is a high frequency cut-off (3 dB frequency), measure its value (where the voltage is reduced to 0.7 of the low frequency value). Record the cut-off frequency. A voltage divider containing only resistors should not have any frequency dependence. However, a coax cable has a capacitance of ~ 25 pF/foot. You could refine your model to include this capacitance. However, in this case, refine your physical system instead by using a scope probe (see definitions) in place of the coax cable to reduce the capacitance of the measurement probe. Repeat the measurements (and record them in your lab book) of Question 5 part (a) using the 10x probe to measure the output of the circuit. Note that the oscilloscope knows when the 10X probe is attached and automatically adjusts the scales to give the correct value. Does your original model of just two resistors now predict the behavior of the circuit when you use a 10X probe?

LOW AND HIGH PASS FILTERS

You have shown with data that the 10X probe perturbs your measurements less than the coax cable. Use the probe for the rest of your measurements.

Step 7	<p>Measure the frequency dependence of the filters.</p> <ol style="list-style-type: none"> Determine and record the cut-off frequency for the low- and high-pass filter. Compare your measured half power point ($V_{out}/V_{in} = 0.707$) with the cut-off frequency computed from the actual component values used. Include your comparison in your lab book. Measure and record the attenuation vs. frequency at decade intervals from $f = 10^{-3} f_c$ to $f = 10^3 f_c$ if possible. Test the predicted frequency response by plotting your data points directly on your two Bode plots. Does the model agree with your data? Explicitly record what criteria you used to determine whether or not the model and measurements agree.
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BAND PASS FILTER

Step 8	<p>Measure the frequency dependence of the band-pass filter.</p> <ol style="list-style-type: none"> On resonance, V_{out} will be a maximum and the phase shift between the input and output waveforms will be zero. Find the resonant frequency f_0 both ways. Adjust the frequency so that (1) the output has maximum amplitude ($V_{out}/V_{in}=\max$), (2) there is zero phase difference between V_{out} and V_{in}. Record both measurements. Which method is more precise? The LCR meter measured the inductance of your inductor at a particular frequency. Your inductor's inductance changes slightly at different frequencies. Use your measurements of f_0 to get a more accurate measure of L on resonance by doing the following. Compare the measured f_0 with the expected value $1 / (2\pi\sqrt{LC})$. Refine the model of the inductor by calculating a corrected value of L from the measured values of f_0 and C, and use this refined value below. Compare this value of L to the value you measure using the LCR meter in the lab. Determine the quality factor Q by measuring the frequencies at the two half-power points f_+ and f_- above and below the resonance at f_0. Record your measurements. Recall that $Q = \frac{\text{Resonant frequency } f_0}{\text{Bandwidth } \Delta f} \quad \text{where } \Delta f = f_+ - f_- .$ <p>HINT: The half-power points are where $V_{out} = V_{out}(\max)/\sqrt{2}$ not $V_{in}/\sqrt{2}$.</p> Compare the measured value of Q with that predicted from measurements of component values. Do they agree? It is common in all electrical circuits to find Q values that are somewhat lower than values you predict using measured component values. This is due to additional losses in the circuit, in this case losses are in the inductor. Measure the inductor's "equivalent series resistance" (ESR) using a DMM. You can refine your model by including this resistance in your circuit. Draw a schematic that includes this resistor. What is the predicted Q when you include this resistance in your model? HINT: See hints section below. Does this have better agreement with your measured Q? Measure the transfer function (attenuation) ($=V_{out}/V_{in}$) as function of frequency. Use your model prediction to decide what values of frequency to take data. Plot your measurements on the same graph as your model. Note, your transfer function did not include the refined value of Q.
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HINTS: REFINED LCR BAND-PASS FILTER MODEL

Inductors often have considerable resistance as they are just wires wrapped around a ferrite core. One can include this resistance as a resistor in series with the inductor. The refined model of the Q of this system is

$$Q_{refined} = \frac{R/R_L}{R\sqrt{\frac{C}{L}} + \frac{1}{R_L}\sqrt{\frac{L}{C}}}$$

where R_L is the equivalent series resistance of the inductor. This is non-trivial to derive.