Physics 2010 -- Fall 2006
Laboratory 5: Rotational Dynamics

NAME ____________________________
Section Day (circle): M Tu W Th F
Section Time: 8a 10a 12p 2p 4p
TA Name: ____________________________

This lab will cover the concepts of moment of inertia and rotational kinetic energy. These concepts play the same roles in rotational motion as mass and kinetic energy do for straight-line motion.

Instructions: Make safety a priority. This experiment uses fairly heavy rolling objects. There is a risk of toe injury as well as damage to equipment.

<u>Background Information</u>

1. Moment of Inertia

The *moment of inertia*, *I*, (also called *moment* and *rotational inertia*) of a body is a measure of how much a body resists changing its rate of rotation about some axis. Similar to mass in linear motion, the larger a body's moment, *I,* the more work it takes to increase the body's rotation rate.

The moment of inertia is always defined with reference to a particular axis of rotation ─ often a symmetry axis, but it can be any axis, even one that is outside the body. The moment of inertia of a body about a particular axis is defined as:

$$I = \sum_i m_i r_i^2 \qquad (1)$$

The moment of inertia of a point mass *m* orbiting at radius *r* about an axis, is

$$\mathrm{I} = mr^2 \qquad (2)$$

If many point masses make up the body, equation 1 can be rewritten as

$$\mathrm{I} = \mathrm{I}_1 + \mathrm{I}_2 + \ldots + \mathrm{I}_i = m_1 r_1^2 + m_2 r_2^2 + \ldots + m_i r_i^2 \qquad (3)$$

where r_i is the distance of mass m_i from the axis of rotation. If the body is a continuous object of some arbitrary shape, then performing the sum requires techniques of calculus. In this course, we tell you the answer for various shapes. For a disk of mass M and radius R (see Fig. 1), the moment of inertia through the center of symmetry is

$$\mathrm{I}_{disc} = ½\,\mathrm{MR}^2. \qquad (4)$$

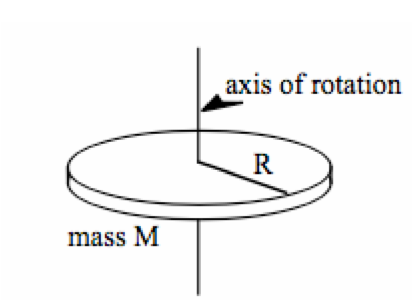

Figure 1. Solid disk with axis of symmetry, uniform mass and density.

Notice that the thickness of the disk doesn't enter into the expression for I_{disc}, so the expression for I is the same for a solid cylinder as it is for a flat disk. A cylinder is just a very thick disk.

The expression in equation $I_{disc} = ½ MR^2$ is only true for a *uniform* disk or cylinder. If the object is a hoop or a thin-walled cylinder, a different formula is used. The moment of inertia of a hoop or thin-walled cylinder is $I_{hoop} = MR^2$. Because all the mass is at the same radius from the axis, making it more resistant to changes in its rate of rotation.

In this experiment you will determine I for a wheel made of a *disk* of radius *R* and mass *M,* mounted on an *axle* of radius *r* and mass *m*, as shown in Fig. 2.

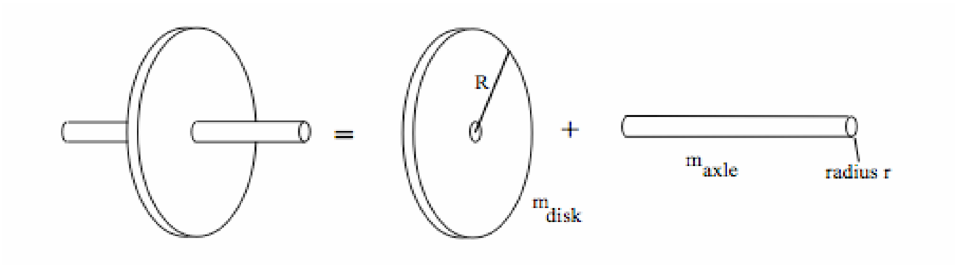

Figure 2. Axle and disk combine to form a single system, a wheel.

The disk's moment of inertia is $I_{disc} = ½ MR^2$. The axle is a uniform cylinder (remember, this is just a thick disk), so its moment of inertia is $I_{axle} = ½ mr^2$. The moment of inertia for the combined system is the *sum* of the moments of the two components:

$$I_{wheel} = I_{disc} + I_{axle} = (½ MR^2 + ½ mr^2). \qquad (5)$$

Be careful here ─ there are *two* different radii in the problem, R and r!

The wheel will roll down an inclined set of rails after starting from the top, as shown in Fig. 3.

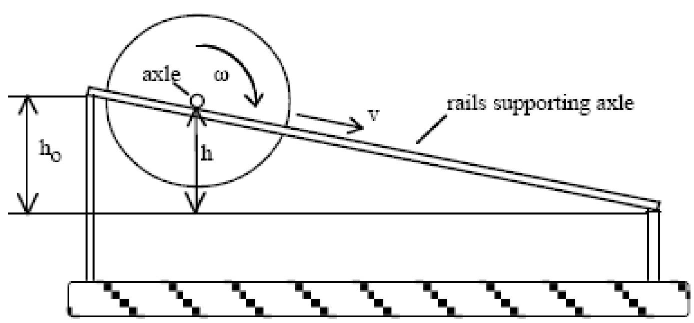

Figure 3: Axle and disc rolling down an idealized track.

The rest of the preparation will be worked out in the prelab.

Prelab Questions

1. A rolling wheel of radius r has translational velocity as well as angular velocity. Here you will relate translational velocity to angular velocity. Since the circumference of a circle of radius r is 2πr,

every time the center of mass moves a distance $2\pi r$, the object rotates a full circle (2π radians). If this happens in a time T then

$$v_{CM} = \Delta x/\Delta t = 2\pi r/T \qquad (6)$$

and

$$\omega = \Delta\theta/\Delta t = 2\pi/T \qquad (7)$$

Combine these equations to find an expression for ω in terms of v_{CM}.

2. When applying equation 6 to our system, which radius (r or R) should you use? Why?

3. Justify the general statement for any object moving under constant acceleration that if $v_{initial} = 0$, then $v_{avg} = v_{final}/2$.

4. Two disks have the same radius R and the same mass M, but one of the disks is twice as thick as the other. Disk A has thickness t, and disk B has thickness 2t. How does the moment-of-inertia of disk A compare to that of disk B? Specifically, what is the ratio I_A / I_B ?

2. Pre-setup

1. Check off and confirm with your TA the answers to pre-lab questions 1-4. Make sure these are right before continuing.
2. Check that you understand how to use the calipers by using them to measure something that you already know the size of, for example, something you can measure precisely with a ruler. If you don't know how to use the calipers, ask your instructor.

3. Make measurements in preparation for calculating moment-of-inertia I

1. Gently slide the axle out of the disk and weigh both separately to find their masses. Measure their diameters to find the radii, *r* for the axle and *R* for the disk. Write your measurements and uncertainties in the table below.

	Mass	Radius
Disk		
Axle		
Disk + Axle		

2. Calculate separately the moment of inertia of the disk, I_{disc}, and the axle, I_{axle}. Add these together to calculate the moment of inertia of the wheel, I_{wheel} as shown in equation 5. Note that using MKS units, your moment of inertia will be a small number. Be sure to keep track of an appropriate number of significant figures; this is best done in scientific notation.

	Show work used to calculate I	Moment of Inertia
I_{disc}		
I_{axel}		
I_{total}		

4. Does the axle contribute significantly to the total mass? To the total I?

4. Conservation of energy for a rolling wheel

1. One end of the rails can be raised and lowered to one of three positions (see Fig. 4). Place the rails in the lowest position, at which they are approximately level, and then use the adjustable screws in the base to make the rails exactly level. Use the bubble level to get a rough level and then place the wheel on the rails to get a precise level.

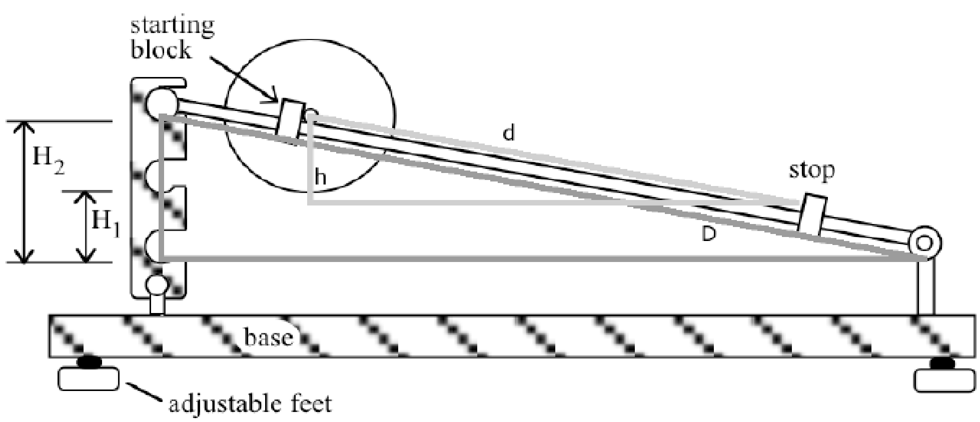

Figure 4: The rolling track you will use in the lab, including similar triangles you can use to find the height drop of the axle.

2. Raise the movable end of the rails to one of the two upper positions H_1 or H_2. Keep the rail at this height throughout the rest of the lab. Fix the two starting blocks, one on each rail, at some convenient position near the top of the track. Make sure the starting blocks are level with each other so the axle starts at rest against both blocks and rolls straight down the track when

released. Leave the starting blocks fixed from now on, so the wheel rolls the same distance for all timings.

3. Determine the height *h* by measuring *d,* D, and H_1 or H_2, and using similar triangles (pale grey and dark gray in the picture).
   a. *d* is the distance along the track through which the wheel rolls and can be measured from the starting and stopping positions of the sharp tip of the axle.
   b. D is the total length of the track measured from the center of the pivot at the bottom to the center of the support at the top.
   c. H_1 or H_2 can be measured quite precisely by measuring the separations of the notches that hold up the end of the rail. Compute h and record all values in the table below.
      Show your formula for h here:

D	*d*	H_1 or H_2	*h*

4. Measure the time t for the wheel to roll down the rail. This is best done with one person operating the stopwatch and releasing the wheel. Make a few trial runs to find the best procedure. Have each member of your team measure t a few times and record all values here. Determine an average value for t and record it.

$t_{average}$ = ________________________

5. You now have enough information to calculate the wheel's final speed v_f at the bottom of its travel. Be careful! The quantity *d/t* is the *average* speed of the wheel. Use the result from Prelab question 3 to find the final velocity v_f.

a. Calculate the initial gravitational potential energy of the wheel ($M + m$) starting from rest. Show your calculation here and enter your answer in the table on the next page.

b. Calculate the final translational kinetic energy of the wheel using $KE_{trans} = \frac{1}{2} (M + m) v_f^2$. Show your calculation here and enter your answer in the table below.

6. It seems as if some of the wheel's energy has been "lost" as it rolled down the ramp, violating the law of conservation of energy. How much energy is "missing"?

7. Observe the wheel a few times as it rolls down the ramp. The *center of mass* is moving down the ramp, but the particles that make up the wheel are also moving *around* the center of mass. No energy is "missing" — it is there as kinetic energy of these particles. A rolling object, like a wheel, can be considered to have two types of kinetic energy — translational *and* rotational kinetic energy. Calculate the rotational KE by subtracting translational KE from the total energy. $KE_{rot} = E_{tot} - KE_{trans}$

5. Make sense of the results.

Translational kinetic energy, $KE_{trans} = \frac{1}{2} mv^2$ and can be increased by increasing an object's inertia, *m*, or speed, *v*. The formula for *rotational* kinetic energy is similar to that of translational KE.

$$KE_{rot} = \frac{1}{2} I\omega^2 = \frac{1}{2} I \left(\frac{v}{r}\right)^2$$

The relationship between v and ω was developed in Prelab question 1. Use this relationship and data from above to complete the chart below. Compute KE_{rot} using this formula here and then enter the result in the table below.

	$E_{tot} = mgh$	$KE_{trans} = \frac{1}{2} mv^2$	$KE_{rot} = E_{tot} - E_{trans}$	$KE_{rot} = \frac{1}{2} I (v_f/r)^2$
Wheel (disk + axle)				

Compare the results of the last two columns. If there is a large discrepancy among the values, comment on possible sources of experimental error.

6. Are the results reproducible? Now change the height of the rail to the other notch. What things do you need to re-measure? List the items and the results of your re-measurements below. With your new results, solve for the moment-of-inertia I, and compare with the value of I that you computed on page 4.