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Learning to think like a physicist: A review of research-based instructional strategies

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Many studies in physics education indicate that our conventional instruction fails to achieve objectives we desire for our students. Students leave introductory courses unable to reason qualitatively about physical processes. They use primitive formula-centered problem-solving techniques. Their minds contain a small number of facts and equations that are accessible only by random searches. In recent years, research by scientists interested in cognition and pedagogy has shown that we can do much better. This paper reviews this research and the instructional strategies that are suggested by it. The following paper reports the preliminary results of using these strategies in introductory physics courses that emphasize problem solving.

I. INTRODUCTION

Students enter introductory physics courses with strongly held preconceptions that are often misconceptions. They use primitive formula-centered problem-solving strategies. Their knowledge consists of a small number of facts and equations stored randomly in the mind. Many studies indicate that students leave our courses in about the same status as they entered. They have the same preconceptions and misconceptions as when they started.¹⁻¹⁸ They still use formula-centered problem-solving methods.¹⁹⁻²² They see physics problems as springs, inclined planes, ropes and pulleys, whereas experienced physicists see the problems in terms of basic physics concepts.²³⁻²⁸

What changes in emphasis and in pedagogy can we make to address these deficiencies in student achievement following conventional instruction? In this paper, several strategies based on recent cognitive and physics-education research are described. In the following paper, we report the preliminary results of integrating these strategies into instruction in introductory physics courses that emphasize problem solving.

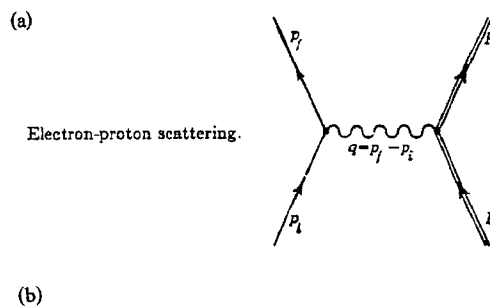
II. LEARNING TO THINK LIKE A PHYSICIST

Problem solving in physics is viewed by students as an attempt to determine the value of one or more unknown quantities. Student solutions to these problems are almost entirely formula centered—devoid of qualitative sketches and diagrams that contribute to understanding. This differs significantly from problem-solving techniques used by experienced physicists.

A physicist depends on qualitative analysis and representations to understand and help construct a mathematical representation of a physical process. For example, to determine a scattering cross section, a quantum electrodynamics (QED) theorist first constructs a Feynman diagram for the process [Fig. 1(a)].²⁹ The diagram serves several purposes. (1) It summarizes the prominent features of a process while removing noisy details that distract from understanding—in short, the diagram contributes to understanding and to physical intuition. (2) Diagrams can be strung together to reason qualitatively about more complex processes. (3) Using special rules and heuristics, the diagram can be used to construct a detailed mathematical

representation of the process [Fig. 1(b)]. Even for the relatively simple electron-proton scattering process depicted in Fig. 1(a), the math representation shown in Fig. 1(b) is complicated. For more complex processes, the diagrammatic representation becomes even more important for developing understanding and for constructing the math representation.

The problems that play an important role in physics education can be viewed in a similar manner.³¹⁻³⁶ Instead of thinking of the problem as an effort to determine some unknown quantity, we might instead encourage students to think of the problem statement as describing a physical process—a movie of a region of space during a short time interval or of an event at one instant of time. The objective



$$S_{fi} = \frac{-ie^2}{V^2} (2\pi)^4 \delta^4(P_f - P_i + p_f - p_i) \sqrt{\frac{m^2}{E_f E_i}} \sqrt{\frac{M^2}{E_f E_i}} \times [\bar{u}(p_f, s_f) \gamma_\mu u(p_i, s_i)] \frac{1}{(p_f - p_i)^2 + i\epsilon} [\bar{u}(P_f, S_f) \gamma^\mu u(P_i, S_i)]$$

Fig. 1. A quantum theorist usually starts a relativistic quantum scattering problem with a qualitative representation, such as the electron-proton scattering Feynman diagram shown in (a). To construct the mathematical representation shown in (b), the theorist uses a set of rules for changing each part of the diagram into the corresponding math representation. The diagram is an essential part in helping to understand the process and in constructing the math representation [from J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964), pp. 110-111].

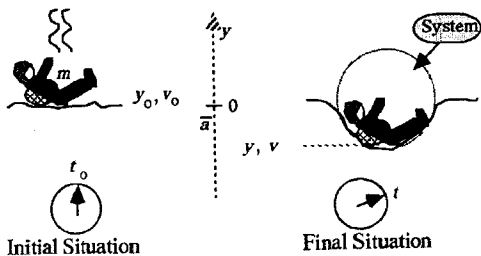
is to have students represent that process or event in ways that lead to qualitative and quantitative understanding. Consider the example shown in Fig. 2, a relatively simple end-of-chapter problem in dynamics.

The problem statement describes a physical process in words—a parachutist being stopped while sinking into a snow bank after his or her parachute did not open [Fig. 2(a)]. We ask the student to change the word description into a *pictorial representation*—a sketch that depicts the situation at the start and at the end of the process [Fig. 2(b)]. The variables used to describe the process are placed in the appropriate parts of the sketch— y_0 and v_0 at the initial location of the diver at the start of the process and y and v at the final location at the end of the process. The average acceleration during the time interval between t_0 and t is placed between the initial and final positions. The student associates the meaning of the variables with something happening in the sketch. More complex processes often need to be broken into more than one part. Such sketches are described in greater detail in other papers.^{30,31}

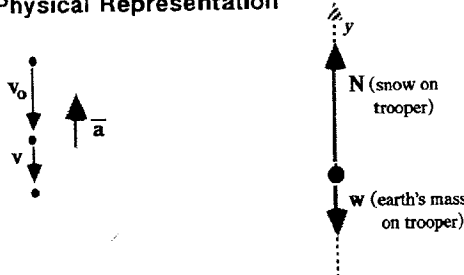
(a) Words

A parachutist whose parachute did not open landed in a snow bank and stopped after sinking 1.0 m into the snow. Just before hitting the snow, the person was falling at a speed of 54 m/s. Determine the average force of the snow on the 80-kg person while sinking into the snow.

(b) Pictorial Representation



(c) Physical Representation



(d) Math Representation

$$2\bar{a}(y - y_0) = v^2 - v_0^2$$

$$\sum F_y = N - w = m\bar{a}$$

Fig. 2. The physical processes described in problems used in introductory physics courses can be represented in qualitative sketches and diagrams that contribute to understanding. The sketches and diagrams can then be used to help construct with understanding the math representation.

Next, the student is asked to construct a *physical representation*—diagrams and graphs that are more physical depictions of the process. For example, kinematic quantities are represented by arrows in motion diagrams [Fig. 2(c)].^{30,32} The forces acting on an object are represented by arrows in a free-body diagram or in a force diagram.³⁰ The pictorial representation helps in the construction of these diagrams. For example, to construct a free-body diagram, the student circles an object (or objects) in the sketch. This object, the system, is the focus of attention when solving the problem. The student then looks for short-range forces due to objects outside the system that touch the object inside and for a long-range action-at-a-distance forces.³⁰

After constructing the diagrams, we ask the student to reason qualitatively about the process, just as a QED theorist reasons qualitatively about scattering processes using Feynman diagrams. For example, the student might be asked how the magnitude of the average upward normal force of the snow on the parachutist compares to the magnitude of her downward weight as she sinks into the snow. Many students think that the weight must be greater because the person is moving down. They are impetus thinkers. They believe that the net force acting on an object is proportional to its velocity. If questions such as this go unasked, the student remains an impetus thinker—as do about 60% or 70% of students who successfully complete the first semester of the conventionally taught engineering physics source. These same students who lack qualitative understanding routinely use the mathematical form of Newton's second law to solve problems.

The student can now use the physical representation to help construct a *mathematical representation* [Fig. 2(d)], just as the theorist uses a Feynman diagram to help construct the math representation of a scattering process. Following these representations, the equations are now used to determine the unknown quantity. The solution to the problem is the whole series of representations with the value of the unknown quantity being only a small part of the solution. Evidence provided in the following paper indicates that students who "solve" problems in this way significantly outperform conventionally taught students in qualitative understanding and in problem-solving expertise.

In summary, one objective of our instruction is to help students learn to (1) construct qualitative representations of physical processes and problems, (2) reason about the processes using these qualitative representations, (3) construct mathematical representations with the help of the qualitative representations, and (4) solve the problem quantitatively. Students are learning to think like physicists.

III. WHY DO STUDENTS RESIST USING QUALITATIVE REPRESENTATIONS IN THEIR PROBLEM SOLVING?

Every physics professor and physics teacher in the country uses diagrams, such as free-body diagrams, while working examples for students. Every textbook has diagrams that accompany the examples in the text. Yet only about 10% of students in conventionally taught precalculus introductory physics courses and 20% in engineering physics courses use diagrams to help solve problems on final exams. Why do they resist this aid that seems essential

to the professor and the teacher? There are three reasons that may account for the lack of qualitative reasoning in student problem solving.

First, many studies indicate that students do not understand the meaning of basic quantities and concepts that are represented in the diagrams. (Refs. 1 and 2 summarize these results.) For example, if given qualitative questions concerning kinematics, students think that objects at the same position have the same velocity. The confuse velocity and acceleration—the faster object has a greater acceleration. Students cannot correctly identify the forces acting on objects in simple situations. They believe in impetus forces, ma forces, the force of inertia, and the force of momentum. Eighty percent of the beginning engineering students at New Mexico State University, and probably at other colleges, believe that the net force acting on an object is proportional to its velocity. This number is based on the results of special test constructed to uncover students' qualitative understanding. Unfortunately, examination of questions answered by conventionally taught engineers following their first semester of introductory physics indicates that about 60% of the students that successfully complete this course still believe that the net force is proportional to the velocity (Fig. 3). If students cannot understand the quantities and concepts represented in the diagrams, how can we expect them to use the diagrams to help solve problems? The understanding must come before students start using math in problem solving. The equations become crutches that short-circuit attempts at understanding.

A second reason why students may avoid qualitative representations to help in their problem solving is that they have little opportunity to develop special techniques needed to construct these representations.^{34,36-38} In many other activities, such as learning to play a musical instrument, to paint, or to play golf or tennis, the student is carefully taught individual skills. The student then practices these skills immediately after they are introduced and in isolation from other activities. A violin student with violin in band is taught special techniques, such as playing vibrato. The student practices vibrato with the instructor and in isolation from other techniques. Basketball players while on the court are taught strategies for three-on-two fast breaks. They practice these breaks regularly in isolation from other parts of the game. In physics education, students are passive observers as their instructor demonstrates the entire game. The student never gets on the playing floor until a day or two after lecture when they try their first problem. There is very little explicit instruction and practice with individual skills such as constructing pictorial representations, free-body diagrams, motion diagrams, and changing a free-body diagram to Newton's second law in component form. How can we expect students to integrate these techniques routinely into their problem solving if they have only cursory understanding and practice with the techniques?

Third, the lecture method of instruction as practiced in most physics courses assumes that the student can accept clearly presented knowledge as given. Yet, the student mind holds many preconceptions that have been stored numerous times during 20 or more years of living.¹⁻¹⁸ These preconceptions are often misconceptions that conflict with the concepts being taught. A computer would become very confused if given two conflicting sets of operating instructions, one stored many times over a period of years and the

1. A ball rolls down an incline and off the horizontal ramp. Ignoring air resistance, what force(s) act on the ball as it moves through the air after leaving the horizontal surface?



Standard instruction (percent)	OCS instruction (percent)
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- | | | |
|---|----|----|
| (a) <u>The weight of the ball, vertically downward.</u> | 38 | 94 |
| (b) A horizontal force that maintains the motion. | 1 | 0 |
| (c) A force whose direction changes as the direction of motion changes. | 2 | 0 |
| (d) The weight of the ball and a horizontal force. | 29 | 3 |
| (e) The weight of the ball and a force in the direction of motion. | 30 | 3 |

2. A hollow, circular tube rests on a frictionless, horizontal table. A ball enters one end at high speed and comes out the other end at the position shown. While moving on the table after leaving the tube, what force(s) act on the ball? Ignore air resistance.



- | | | |
|--|----|----|
| (a) The weight of the ball, vertically downward. | 4 | 3 |
| (b) A force from the table vertically upward. | 4 | 0 |
| (c) A horizontal force in the direction of motion. | 4 | 0 |
| (d) <u>The first two forces above.</u> | 32 | 92 |
| (e) All three forces above. | 56 | 5 |

3. A ball swings at the end of string in a circular path in a vertical plane. Which free-body diagram shown below best represents the forces acting on the ball when at the bottom of the circular path and moving toward the right. The ball moves at constant speed.

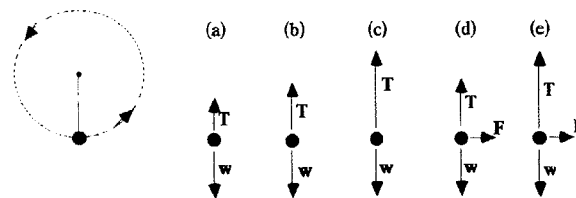


Fig. 3. Shown above are student responses for three qualitative questions concerning the forces that act on objects in simple situations (the correct answers are underlined). These responses were provided by 152 engineering students all in the same lecture room during the first class of the second semester of introductory physics. Of these students, 114 had completed the first semester of standard instruction (most had finished the course 1 month earlier) and 38 had completed the first semester 8 months earlier when taking overview, case study physics (described in the following paper). Note that about 60% of the students that had completed the standard instruction still believed that a force must push an object in the direction of motion. The students are "impetus" thinkers.

other provided a few times during a 1- or 2-week period. If we want the computer to assimilate a new operating system, we must bring up the old system and dump it before the new system can be saved. Having both operating systems present at the same time leads only to confusion. Students must confront and dump their misconceptions if they are to acquire a new set of operating rules for the universe.

IV. KNOWLEDGE ORGANIZATION AND ACCESS

Research suggests that experts in a field organize knowledge in large-scale functional units built around basic physical principles.^{22-28,34} These units are systematically broken into smaller units of increasing detail to locate ap-

appropriate concepts for solving problems. Within this knowledge structure the expert sees relationships and similarities between diverse pieces of information. (This is a simplified model for a complex and only partly understood expert knowledge structure.)

Students, at the end of their conventional study, have little structure to their knowledge. Their understanding consists of random facts and equations that have little conceptual meaning. When given a problem, they identify some structural feature described in the problem (an inclined plane, a rope, a spring, etc.). They then search randomly for and inappropriately use an equation they associate with that feature.^{23,27} An objective for our instruction is to help students construct a knowledge structure around the basic concepts of physics and to learn cues and techniques for accessing the appropriate knowledge needed to analyze physical processes.

This lack of knowledge organization and access was very apparent during a final exam in one of our conventionally taught precalculus physics courses. The test included a conservation of energy problem in which a spring launched an object up into the air. For many of the students, this was a spring problem. They searched their minds for spring equations. Over 50% of the students used the most recent "spring" equation that they had encountered—an equation for simple harmonic motion. These students had "covered" 16 chapters in 15 weeks. Each chapter seemed like a separate world built around its own set of equations. For them, the final test was an effort to find an equation to solve spring problems, inclined plane problems, cable problems, and so forth.

Our instruction must provide opportunities for students to see that a small number of concepts are the basis for many diverse applications—to see the whole knowledge structure. This very difficult task might be done in part by

providing hierarchical charts such as shown in Fig. 4.³⁴ One fundamental idea at the top of this chart is the basis for about 50% of the study in a typical first-semester physics course. The idea is applied in diverse situations below. Students should learn to access the knowledge from top down, that is, from the general to the specific. They need cues to help them decide when a particular concept is appropriate for use.

Another idea that can contribute to seeing physical processes in a unified way is called *system physics* (see Fig. 5).³⁹ All of the conceptual knowledge for the first semester of the standard introductory course and much of it for the second semester fits into this system physics scheme. The objects in a system interact in various ways (forces, impulses, torques, work, heat transfer) with objects in the environment outside the system. If the environmental interactions with the system do not add to zero, then the system change in some way. If the interactions with the system add to zero, then some quantity in the system is conserved. This one way of thinking of processes in nature produces considerable efficiency in learning and reduces the time needed to introduce new concepts. For example, work and energy seem like natural extensions of Newtonian physics. The interactions are now work and heat transfer rather than forces or torques. The changes in the system are now energy changes rather than momentum or angular momentum changes. Students see physics as a discipline with a small number of concepts that can be applied to a diverse array of phenomena. In contrast, during conventional instruction students have little chance to see any conceptual unity while rushing through 15 or more chapters in 15 weeks.

In summary, another objective of our instruction is to help students learn to (1) form a knowledge hierarchy about a small number of basic concepts at the top with

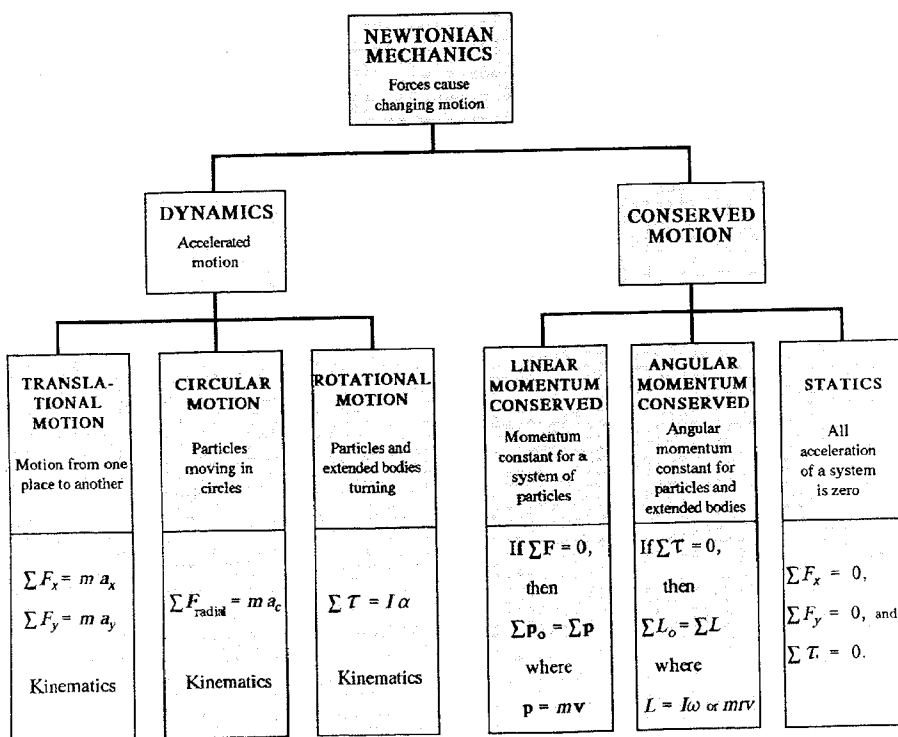


Fig. 4. The diagram above is a hierarchical chart that represents Newtonian physics. Students can see that one idea is the basis for the diverse applications in Newtonian physics.

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Interactions (→) cause change in system

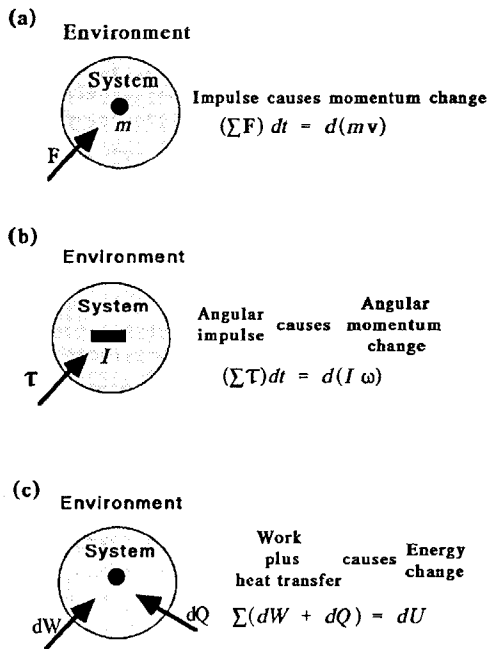


Fig. 5. A system physics approach can be used throughout the entire first semester of an introductory physics course. Interactions between a carefully identified system and its environment cause changes in the system. If the interactions add to zero, some property of the system is conserved. This unity of thought produces efficiency in student learning.

detailed applications below (to see the unity of the diverse applications), and (2) to learn cues for accessing that knowledge from top down, that is, from the general to the specific.

V. PEDAGOGICAL TECHNIQUES FOR LECTURES

We have described objectives for instruction that place more emphasis on the development of qualitative understanding, on the use of multiple-representation problem-solving techniques, and on the formation of a hierarchical knowledge structure with cues for accessing the knowledge when needed. What pedagogical techniques can help us achieve these objectives?

Recent laboratory innovation combined with the use of technology has shown considerable promise for improving student achievement. Thornton's microcomputer-based laboratories⁴⁰ and Hake's Socratic Dialog Inducing (SDI) laboratories³ are notable examples. In this paper, we will restrict our attention to paper and pencil activities that can be easily integrated into lecture and recitation instruction in large classes.

A. Active, cooperative learning in lectures

"Lectures in physics can be incredibly passive experiences for students, particularly dangerous for those who believe that if they can follow the professor, they've mastered the material."⁴¹

The graduate student in creative writing who made this observation was one of several advanced students who recorded their impressions of the introductory calculus-based physics course for engineers while taking that course.⁴¹ The student, according to her professor, "...could easily have been a physics major, and a good one." Her comment summarizes a concern about the passive nature of university instruction in science and math.

Historically, we have relied on expository lectures—telling students the physical rules that seem to guide the universe and demonstrating how to use the rules to solve problems. The conceptual presentations are often supported by experimental evidence. This is a very efficient method to transmit information in terms of the time interval needed. We know the concepts and techniques, and students do not. Why not just tell them? Study after study indicates that this expository method is very ineffective—the transmission is efficient but the reception is almost negligible.¹⁻²³

There is growing interest in making instruction at all levels more active, in having students work cooperatively with other students, and in having them help construct their own knowledge.^{3,42-46} An active method of lecture instruction is very foreign to most of us who have seldom if ever taken a math or science class involving active student participation. It is difficult in moderately large lectures to elicit general student discussion above the development and use of physics concepts. Most students are passive observers of the few who contribute to the discussion.

The author has recently tried a method of instruction that easily allows every student in a large lecture to be an active participant. The method relies on a set of sheets called *Active Learning Problem Sheets* (The ALPS kit).⁴⁷ The students participate by interacting with neighboring students while answering questions and solving problems on the sheets. The student eventually gets feedback from the professor.

The sheets can be used for several different purposes. Some sheets help students construct concepts by systematically analyzing a series of thought experiments. Some sheets help students develop special skills—for example, constructing free body diagrams or determining the torque caused by a force. Other sheets challenge misconceptions while at the same time helping students develop qualitative understanding of concepts. Finally, many sheets help develop problem-solving techniques similar to those used by experienced physicists. These sheets encourage students to represent problem situations in terms of sketches, physics-type diagrams, physical concepts in mathematical language, and to evaluate solutions to see if they are reasonable.

The sheets can be used during lectures in several different ways. On some activities, students prefer to work alone. After completing an activity or a part of an activity, they compare their answer with that of a neighboring student. They then try to reconcile differences in their results. Finally, the professor goes through the solution. If the professor can anticipate points of confusion, polling the students and asking them to support their positions seems to be preferable to simply providing the correct answer. The objective is to make students active participants.

On some qualitative questions, one student can serve as the solver while the other student is the listener.⁴⁸ The solver says everything that passes through his or her head

while answering the question. The listener can ask for clarification, but does not ask leading questions. After the solver completes the solution, she or he summarizes the main steps that led to the solution. This active struggle with a problem or question seems to engrave the process and the concepts in the student mind, whereas passively taking lecture notes has little effect on long-term acquisition and retention of the concepts and skills.

In summary, our classroom instruction must provide opportunities for students to (1) be active participants during lectures in constructing concepts, reasoning qualitatively using the concepts, and in solving problems, (2) evaluate their own thinking and that of their classmates, and (3) make unpenalized mistakes while getting immediate feedback from the professor.

B. Repeated exposure over an extended time interval in a variety of contexts

Conventional instruction allows students to see the material in each chapter for about 1 week. Unfortunately, students seldom learn a technique or concept after a brief exposure. If the skill or concept is abandoned after 1 week in order to develop new skills and concepts, then permanent acquisition is unlikely. Skills and concepts require multiple exposures over extended time intervals to become a permanent part of the student knowledge bank.⁴⁹

VI. SUMMARY

The educational system can be thought of as a transformer that helps students acquire conceptual knowledge and analytical skills. When physicists build transformers, considerable care is taken to match impedances at the source and at the load. For the instructional transformer, the load is the minds of students. To make an effective system of instruction, the output of an educational transformer must be attuned to the characteristics of student minds at all times [see Fig. 6(a)].

Many studies indicate that our present form of instruction is mismatched to the characteristics of student minds. Students leave our courses in about the same status as they entered. They have the same preconceptions and misconceptions as when they started. They still use formula-centered problem-solving methods. Their knowledge consists of a small number of facts and equations stored randomly in the mind. They see physics problems as springs, inclined planes, ropes, and pulleys whereas experienced physicists see them as dynamics, work-energy, and momentum conservation problems. This poor acquisition of knowledge and of analytical skills by student minds is expected from an educational transformer that has a significant impedance mismatch at the interface between its output and the student mind [Fig. 6(b)].

To provide a better impedance match, we suggest that our instructional transformer must meet goals such as follows.

(1) To understand basic physical quantities and concepts, students must learn to represent these quantities and concepts using *qualitative* representations and to use these representations to reason *qualitatively* about physical processes.

(2) To help students develop quantitative understanding and problem-solving expertise, their problem solutions should involve multiple representations of the process or

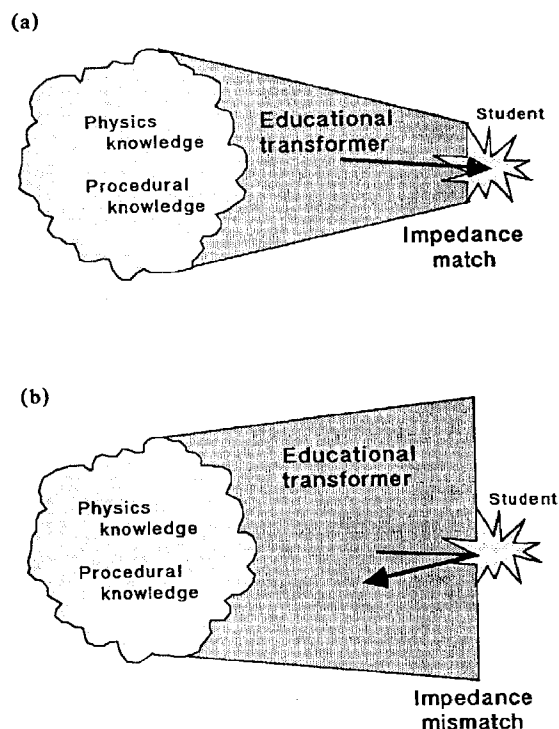


Fig. 6. The educational system can be thought of as a transformer that helps students acquire conceptual and procedural knowledge. (a) If the transformer output is matched to the characteristics of the student mind, then knowledge acquisition is efficient. (b) However, if the transformer output is not matched to the characteristics of the student mind, the knowledge acquisitions is minimal.

event described in the problem. Students must receive explicit instruction and practice with individual skills needed to represent and solve these problems.

(3) Students are more likely to see the world in terms of physical concepts if they have an opportunity to form a knowledge hierarchy, and if they learn cues for accessing that knowledge from general ideas at the top to specific detailed applications at the bottom.

(4) Expository lectures have been very unsuccessful in helping students acquire conceptual and procedural knowledge. Instead, students should become *active* participants during lectures (and in other parts of the course) in constructing concepts, in confronting preconceptions that are misconceptions, in reasoning qualitatively about physical processes, and in learning to use concepts to solve problems.

(5) To become a permanent part of their knowledge, students need to use concepts and skills repeatedly in a variety of contexts over an extended time interval.

Following guidelines such as these, students have a better chance of learning to think like physicists.

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- ¹I. A. Halloun and D. Hestenes, "The initial knowledge state of college physics students," *Am. J. Phys.* **53**, 1043-1055 (1985); "Common sense concepts about motion," *Am. J. Phys.* **53**, 1056-1065 (1985).
- ²L. C. McDermott, "Research on conceptual understanding in mechanics," *Phys. Today* **37**, 24-32 (1984).
- ³Richard R. Hake, "Promoting student crossover to the Newtonian world," *Am. J. Phys.* **55**, 878-884 (1987).
- ⁴John Clement, "Students' preconceptions in introductory mechanics," *Am. J. Phys.* **50**, 66-71 (1982).
- ⁵F. Goldberg and L. C. McDermott, "An investigation of student understanding of the real image formed by a converging lens or concave mirror," *Am. J. Phys.* **55**, 108-119 (1987).
- ⁶D. P. Maloney, "Rule-governed physics; some novice predictions," *Eur. J. Sci. Ed.* **7**, 295-306 (1985); "Rule-governed approaches to physics: Newton's third law," *Phys. Educ.* **19**, 37-42 (1984).
- ⁷M. McCloskey, "Intuitive physics," *Sci. Am.* **248**, 122-130 (1983).
- ⁸A. Caramazza, M. McCloskey, and B. Green, "Naive beliefs in 'sophisticated' subjects: Misconceptions about trajectories of objects," *Cognition* **9**, 117-123 (1981).
- ⁹P. C. Peters, "Even honors students have conceptual difficulties with physics," *Am. J. Phys.* **50**, 501-508 (1982).
- ¹⁰*Proceedings of the International Seminar on Misconceptions in Science and Mathematics*, edited by H. Helm and J. D. Novak (Cornell U.P., Ithaca, NY, 1983).
- ¹¹A. Champagne, L. E. Klopfer, and J. H. Anderson, "Factors influencing the learning of classical mechanics," *Am. J. Phys.* **48**, 1074-1079 (1980).
- ¹²J. Minstrell, "Explaining the 'at rest' condition of an object," *Phys. Teach.* **20**, 10-14 (1982).
- ¹³L. Viennot, "Analyzing students' reasoning: Tendencies in interpretation," *Am. J. Phys.* **53**, 432-436 (1985).
- ¹⁴P. W. Hewson, "Diagnosis and remediation of an alternative conception of velocity using a microcomputer program," *Am. J. Phys.* **53**, 684-690 (1985).
- ¹⁵R. Driver and L. Warrington, "Students use of the principle of energy conservation in problem situations," *Phys. Educ.* **20**, 171-176 (1985).
- ¹⁶R. Gunstone, "Student understanding in mechanics: A large population survey," *Am. J. Phys.* **55**, 691-696 (1987).
- ¹⁷R. Cohen, B. Eylon, and U. Ganiel, "Population difference and current in simple circuits: A study of students' concepts," *Am. J. Phys.* **51**, 407-412 (1983).
- ¹⁸N. Sadanand and J. Kess, "Concepts in force and motion," *Phys. Teach.* **28**, 530-533 (1990).
- ¹⁹J. I. Heller and F. Reif, "Prescribing effective human problem solving processes: Problem description in physics," *Cognit. Instruct.* **1**, 177-216 (1984).
- ²⁰I. A. Halloun and D. Hestenes, "Modeling instruction in mechanics," *Am. J. Phys.* **55**, 455-462 (1987).
- ²¹*Problem Solving and Education: Issues in Teaching and Research*, edited by D. T. Tuma and F. Reif (Erlbaum, Hillsdale, NJ, 1979).
- ²²Jill H. Larkin, John McDermott, Dorteia Simon, and Herbert A. Simon, "Expert and novice performance in solving physics problems," *Science* **208**, 1135-1342 (1980).
- ²³M. T. H. Chi, P. J. Feltovich, and R. Glaser, "Categorization and representation of physics problems by experts and novices," *Cognit. Sci.* **5**, 121-152 (1981).
- ²⁴M. T. H. Chi, R. Glaser, and E. Rees, "Expertise in problem solving," in *Advances in the Psychology of Human Intelligence*, edited by R. J. Sternberg (Erlbaum, Hillsdale, NJ, 1982), Vol. 1, pp. 7-75.
- ²⁵R. Dufresne, W. Gerace, P. T. Hardiman, and J. Mestre, "Hierarchical structured problem solving in elementary mechanics; guiding novices' problem analysis," in *Proceedings of the Second International Seminar on Misconceptions and Educational Strategies in Science and Mathematics*, edited by J. D. Novak (Cornell U.P., Ithaca, NY, 1987), Vol. III, p. 116.
- ²⁶J. H. Larkin, "Processing information for effective problem solving," *Eng. Educ.* **70**, 285-288 (1979).
- ²⁷B. Eylon and F. Reif, "Effects of knowledge based organization on task performance," *Cognit. Instruct.* **1**, 5-44 (1984).
- ²⁸F. deJong and M. G. M. Ferguson-Hessler, "Cognitive structure of good and poor novice problem solvers in physics," *J. Educ. Psychol.* **78**, 279-288 (1986).
- ²⁹J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964), pp. 110-111.
- ³⁰F. Reif and J. I. Heller, "Knowledge structure and problem solving in physics," *Educ. Psychol.* **17**, 102-127 (1982).
- ³¹R. G. Van Ausdal, "Structured problem solving in kinematics," *Phys. Teach.* **26**, 518-522 (1988).
- ³²D. Hestenes, "Toward a modeling theory of physics instruction," *Am. J. Phys.* **55**, 440-454 (1987).
- ³³D. S. Wright and C. D. Williams, "A WISE strategy for introductory physics," *Phys. Teach.* **24**, 211-216 (1986).
- ³⁴F. Reif, "Teaching problem solving—A scientific approach," *Phys. Teach.* **19**, 310-316 (1981).
- ³⁵Alan Van Heuvelen, "Overview, case study physics," *Am. J. Phys.* **59**, 898-907 (1991) (following paper).
- ³⁶Jill Larkin, "Understanding, problem representations, and skill in physics," in *Thinking and Learning Skills*, edited by S. F. Chipman, J. W. Segal, and R. Glaser (Erlbaum, Hillsdale, NJ 1985), Vol. 2, pp. 141-159.
- ³⁷F. Reif, "Interpretation of scientific or mathematical concepts: Cognitive issues and instructional implications," *Cognit. Sci.* **11**, 395-416 (1987).
- ³⁸F. Reif, "Instructional design, cognition, and technology: Application to the teaching of scientific concepts," *J. Res. Sci. Teach.* **24**, 309-324 (1987).
- ³⁹H. Burkhardt, "System physics: A uniform approach to the branches of classical physics," *Am. J. Phys.* **55**, 344-350 (1987) and references therein. The idea of emphasizing systems in all parts of mechanics was first introduced to me during a personal communication with Bruce Sherwood of Carnegie Mellon University.
- ⁴⁰R. K. Thornton, "Tools for scientific thinking—Microcomputer-based laboratories for physics teaching," *Phys. Educ.* **22**, 230-238 (1987).
- ⁴¹S. Tobias, *They're Not Dumb, They're Different* (Research Corporation, Tucson, AZ, 1990), p. 36.
- ⁴²R. J. Light, *The Harvard Assessment Seminars: Explorations with Students and Faculty about Teaching, Learning, and Student Life* (Harvard University Graduate School of Education and Kennedy School of Government, Cambridge, MA, 1990).
- ⁴³J. L. Cooper and R. Mueck, "Student involvement in learning: Cooperative learning and college instruction," *J. Excellence College Teach.* (to be published). Contact James L. Cooper at the Cooperative Learning Center, Office of Faculty Development, California State University—Dominguez Hills, Carson, CA 90747 about a cooperative learning project supported in part by a grant from the U.S. Department of Education Fund for the Improvement of Postsecondary Education.
- ⁴⁴D. P. Maloney, "Forces as interactions," *Phys. Teach.* **28**, 386-390 (1990).
- ⁴⁵D. P. Maloney, "Fill-in problems," *J. Coll. Sci. Teach.* **12**, 104-106 (1982).
- ⁴⁶David Squires, "Providing computer-based experiences for learning physics," *Phys. Educ.* **22**, 239-244 (1987).
- ⁴⁷A small sample of the sheets along with information about ordering the entire set can be obtained by writing: ALPS Kit, Box 4013, University Park Branch, Las Cruces, NM 88003.
- ⁴⁸A. Whimbey and J. Lochhead, *Problem Solving and Comprehension* (Erlbaum, Hillsdale, NJ, 1986).
- ⁴⁹Arnold B. Arons, "Students patterns of thinking and reasoning," *Phys. Teach.* **21**, 576-581 (1983); **22**, 21-26 (1984); **22**, 88-93 (1984).